A FIRST COURSE IN PHYSICS

MILLIKAN AND GALE

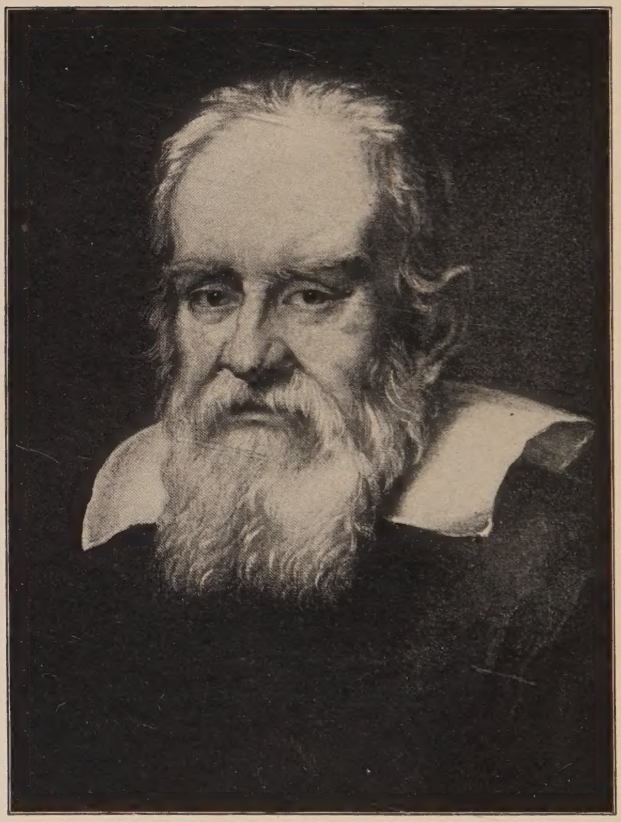
Marion Johnson.
Griswold, Ia.

1.25

Each week
6:45
Quizes Monday Room 6
Thursday Room 6
6:45

Marion Johnson
Murray Iowa

Box 163



GALILEO (1564–1642)

Great Italian physicist, astronomer, and mathematician; "founder of experimental science"; was son of an impoverished nobleman of Pisa; studied medicine in early youth, but forsook it for mathematics and science; was professor of mathematics at Pisa and at Padua; discovered the laws of falling bodies and the laws of the pendulum; was the creator of the science of dynamics; constructed the first thermometer; first used the telescope for astronomical observations; discovered Jupiter's satellites and the spots on the sun. Modern physics begins with Galileo.

A FIRST COURSE IN PHYSICS

BY

ROBERT ANDREWS MILLIKAN, Ph.D.
Associate Professor of Physics in the University of Chicago

AND

HENRY GORDON GALE, Ph.D.
Assistant Professor of Physics in the University of Chicago

GINN & COMPANY
BOSTON · NEW YORK · CHICAGO · LONDON

ENTERED AT STATIONERS' HALL

COPYRIGHT, 1906, BY
ROBERT A. MILLIKAN AND HENRY G. GALE

ALL RIGHTS RESERVED

69.10

The Athenæum Press
GINN & COMPANY · PROPRIETORS · BOSTON · U.S.A.

PREFACE

The course presented in this book, and in the *List of Laboratory Experiments*, which is published in a separate volume, has grown out of the actual needs of the elementary work in physics in the University of Chicago, particularly in the University High School of the School of Education and the affiliated secondary schools. Its most characteristic features have been on trial for three or four years in more than a score of different secondary schools in various parts of the country.

The books represent primarily an attempt to give concrete expression to a rapidly spreading movement to make high-school physics, to a less extent than it has been in the past, either a condensed reproduction of college physics, or a mathematical and mechanical introduction to technical science, and to a greater extent than it has heretofore been, *a simple and immediate presentation, in language which the student already understands, of the hows and whys of the physical world in which he lives.*

A second aim has been to develop a course in which the laboratory and class-room phases of elementary instruction in physics are carefully differentiated and, at the same time, closely correlated. It is hoped that something may thus be done toward remedying the inadequacy which still exists in the laboratory instruction of many of the smaller schools. A very carefully selected and tested list of distinctively class-room demonstrations will be found to run through the book in fine print, while footnotes indicate the location and nature of the laboratory exercises which should be inserted. For the sake of definiteness and simplicity the references are made simply to the authors' manual, though the exercises may be taken from any good laboratory text.

In the chapters on Molecular Motions and Molecular Forces, the authors have sought to bring into their proper relations to one another and to the modern theory of physics a large number of phenomena which are sometimes thrown together in somewhat scrappy and illogical form under the general head, Properties of Matter.

In the treatment of image formation the time-honored fiction of rays has been replaced by the truer, simpler, and more comprehensible view point of change in wave curvature. In the treatment also of surface tension, electro-magnetic induction, and the mechanism of tone production by wind instruments, it is thought that some familiar fictions of physics have been replaced by "causally conditioning" facts.

In the description and illustration of *physical appliances* the course has been made unusually complete. It is not expected that all of the material of this sort which has been introduced will under all circumstances be assigned for recitation purposes. It is inserted because it is precisely what the student is usually most eager to learn, but cannot, in general, obtain from books because their language is too technical for him, nor yet from his teacher because the latter lacks the necessary diagrams. It is thought that it will be read by most pupils whether it is assigned or not.

In the last chapter are presented in some detail the recent epoch-making discoveries which have brought *the electron* into prominence and have so profoundly modified molecular, electrical, and optical theories.

Much attention has been given to the Questions and Problems which are placed at the end of each subdivision of a chapter, so that they may be made, in so far as is possible, a part of each day's assignment.

In the *illustration* of the course an effort has been made to make each of the very large number of figures not in any sense showy, but in the fullest possible sense *educative*. The portraits of sixteen of the great makers of physics have been inserted for the sake of adding human and historic interest.

Finally, *the authors have endeavored to avoid sacrificing comprehensibility to condensation.* Although they have presented a smaller number of subjects than is often found in an elementary text, they have striven to present each subject with sufficient illustration and amplification to make it easily and quickly intelligible. This, together with the large number of figures, has added to the number of pages in the book, although it has actually shortened the course. For the sake, however, of indicating in what directions omissions may be made, if necessary, without interfering with continuity, paragraphs have here and there been thrown into fine print. These paragraphs will easily be distinguished from the class-room experiments, which are in the same type. They are, for the most part, descriptions of physical appliances.

It is quite impossible to make suitable recognition of the assistance which has been derived from the close coöperation of more than a score of men who have taken an active interest in the development of this course. All of the following have read either the whole or large parts of the manuscript or proof, and all of them have made important suggestions which have been incorporated in the text: Dr. C. J. Ling of the Manual Training High School, Denver, Colorado; Superintendent H. O. Murfee of the Marion Military Institute, Marion, Alabama; Mr. C. F. Adams of the Central High School, Detroit, Michigan; J. C. Packard, Submaster of Brookline High School, Brookline, Massachusetts; Dr. T. C. Hebb of the Central High School, St. Louis, Missouri; Professor B. O. Hutchison of Shurtleff College, Upper Alton, Illinois; Mr. C. C. Kirkpatrick of the Seattle High School, Washington; Dr. G. M. Hobbs, Dr. C. J. Lynde, and Mr. F. H. Wescott of the University High School; and Mr. Harry D. Abells of the Morgan Park Academy of the University of Chicago. The part which Dr. Ling has had in the development of the course has been of especial importance.

R. A. MILLIKAN
H. G. GALE

CONTENTS

PAGES

CHAPTER I. MEASUREMENT. Fundamental Units — Construction of Standards — Density 1–11

CHAPTER II. FORCE AND MOTION. Definition and Measurement of Force — Composition and Resolution of Force — Gravitation — Uniformly Accelerated Motion — Newton's Laws . 12–37

CHAPTER III. PRESSURE IN LIQUIDS. Liquid Pressure beneath a Free Surface — Pascal's Law — The Principle of Archimedes 38–57

CHAPTER IV. PRESSURE IN AIR. Barometric Phenomena — Expansibility of Air — Pneumatic Appliances . . . 58–81

CHAPTER V. MOLECULAR MOTIONS. Kinetic Theory of Gases — Molecular Motions in Liquids — Properties of Vapors — Hygrometry — Molecular Motions in Solids 82–106

CHAPTER VI. MOLECULAR FORCES. Elasticity — Capillary Phenomena — Solution and Crystallization — Absorption of Gases 107–128

CHAPTER VII. THERMOMETRY-EXPANSION COEFFICIENTS. Thermometry — Expansion Coefficient of Gases — Expansion of Liquids and Solids — Applications of Expansion . . . 129–145

CHAPTER VIII. WORK AND MECHANICAL ENERGY. Definition of Work — Work and the Pulley — Work and the Lever — The Principle of Work — Power and Energy 146–166

CHAPTER IX. WORK AND HEAT ENERGY. Friction — Efficiency — Mechanical Equivalent of Heat — Specific Heat — Heat Engines 167–195

CHAPTER X. CHANGE OF STATE. Fusion — Vaporization — Artificial Cooling — Industrial Applications 196–215

PAGES

CHAPTER XI. The Transference of Heat. Conduction — Convection — Radiation — Heating and Ventilating . . 216–226

CHAPTER XII. Magnetism. General Properties of Magnets — Terrestrial Magnetism 227–237

CHAPTER XIII. Static Electricity. General Facts of Electrification — Distribution of Charge — Potential and Capacity — Electrical Generators 238–260

CHAPTER XIV. Electricity in Motion. Measurement of Currents — Measurement of Potential Difference — Measurement of Resistance — Primary Cells 261–291

CHAPTER XV. Effects of Electrical Currents. Chemical Effects — Magnetic Properties of Coils — Heating Effects . 292–311

CHAPTER XVI. Induced Currents. The Principle of the Dynamo — Dynamos — The Principle of the Electric Motor — The Induction Coil and Transformer 312–342

CHAPTER XVII. Nature and Transmission of Sound. Speed — Nature — Reflection and Reënforcement — Interference . 343–365

CHAPTER XVIII. Properties of Musical Sounds. Musical Scales — Laws of Vibrating Strings — Fundamentals and Overtones — Musical Properties of Air Chambers — Wind Instruments 366–387

CHAPTER XIX. Nature and Propagation of Light. Transmission — Intensity — Reflection and Refraction — Nature . 388–415

CHAPTER XX. Formation of Images. Images in Plane Mirrors — Images in Convex Mirrors — Images in Concave Mirrors — Images formed by Lenses — Optical Instruments . . . 416–442

CHAPTER XXI. Color Phenomena. Color and Wave Length — Dispersion — Spectra 443–460

CHAPTER XXII. Invisible Radiations. Radiations from a Hot Body — Electrical Radiations — Cathode and X Rays — Radio-activity 461–482

INDEX 483

A FIRST COURSE IN PHYSICS

CHAPTER I

MEASUREMENT

Fundamental Units

1. The historic standard of length. Nearly all civilized nations have at some time employed a unit of length the name of which bore the same significance as does *foot* in English. There can scarcely be any doubt, therefore, that in each country this unit has been derived from the length of the human foot.

But, as might have been expected from such an origin, no two peoples have agreed in the length of their standard. Thus the Greek foot, supposed to represent the length of the foot of Hercules, was 12.14 English inches; the Macedonian foot was 14.08 inches, the Pythian 9.72, and the Sicilian 8.75. In Europe during the Middle Age almost every town had its own characteristic foot; thus in Rome a foot was 11.62 inches, in Milan 13.68, in Brussels 10.86, in Göttingen 11.45, and in Geneva 19.21.

It is probable that in England, after the yard (a unit which is supposed to have represented the length of the arm of King Henry I) became established as a standard, the foot was arbitrarily chosen as one third of this standard yard. The mean length of the male foot in the United States, according to measurements made upon 16,000 men in the United States army, is 10.05 inches.

2. Relations between different units of length. It has also been true, in general, that in a given country the different units of length in common use, such, for example, as the inch, the hand, the foot, the fathom, the rod, the mile, etc., have been derived either from the lengths of different members of the human body or from equally unrelated magnitudes, and in consequence have been connected with one another by no common multiplier. Thus there are 12 inches in a foot, 3 feet in a yard, $5\frac{1}{2}$ yards in a rod, 1760 yards in a mile, etc. Furthermore the multipliers are not only different, but are frequently extremely awkward; e.g. there are $16\frac{1}{2}$ feet, or $5\frac{1}{2}$ yards, in a rod.

3. Relations between units of length, area, volume, and mass. A similar and even worse complexity exists in the relations of the units of length to those of area, capacity, and mass. For example, a square field containing an acre measures 12.649 rods, 69.569 yards, or 208.708 feet on a side; one square rod contains $272\frac{1}{4}$ square feet; there are $57\frac{3}{4}$ cubic inches in a quart, and $31\frac{1}{2}$ gallons in a barrel.

When we turn to the unit of mass we find that the grain, the ounce, the pound, the ton, etc., not only bear different and often very inconvenient relations to one another, but also that none of them bear any simple and logical relations to the units of length. Thus, for example, the pound, instead of being the mass of a cubic inch or a cubic foot of water, or of some other common substance, is the mass of a cylinder of platinum, of inconvenient dimensions, which is preserved in London.

4. Origin of the metric system. At the time of the French Revolution the extreme inconvenience of existing weights and measures, together with the confusion arising from the use of different standards in different localities, led the National Assembly of France to appoint a commission to devise a more logical system. The result of the labors of this commission was the present metric system, which was introduced in France in 1793, and has since been adopted by the governments

of most civilized nations except those of Great Britain and the United States; and even in these countries its use in scientific work is practically universal.

5. The standard meter. The standard *length* in the metric system is called the *meter*. It is the distance, at the freezing temperature, between two transverse parallel lines ruled on a bar of platinum (Fig. 1), which is kept in the Palace of the Archives in Paris.

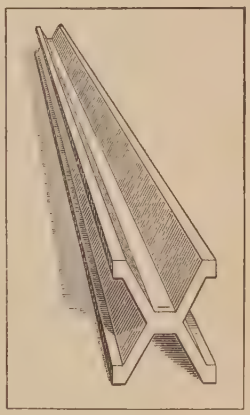


Fig. 1. The standard meter

In order that this standard length might be reproduced if lost, the commission attempted to make it one ten-millionth of the distance from the equator to the north pole, measured on the meridian of Paris. But since later measurements have thrown some doubt upon the exactness of the commission's determination of this distance, we now define the meter, not as any particular fraction of the earth's quadrant, but simply as the distance between the scratches on the above bar. This distance is equivalent to 39.37 inches, or about 1.1 yards.

6. Metric standards of area and capacity. The standard *area* in the metric system is the *are*. It is equal to 100 square meters, or about 119.6 square yards.

The standard unit of capacity is called the *liter*. It is the volume of a cube which is one tenth of a meter (about 4 inches) on a side. It is equivalent to 1.057 quarts. A liter and a quart are therefore roughly equivalent measures.

7. The metric standard of mass. In order to establish a connection between the unit of length and the unit of mass, the commission directed a committee of the French Academy to prepare a cylinder of platinum which should have the same weight as a liter of water at its temperature of greatest density,

namely, 4° Centigrade, or 39° Fahrenheit. This cylinder was deposited with the standard meter in the Palace of the Archives and now represents the standard of mass in the metric system. It is called the *standard kilogram* and is equivalent to about 2.2 pounds. One one-thousandth of this mass was adopted as the fundamental unit of mass and was named the *gram.*

8. The other metric units. The four standard units of the metric system — the meter, the liter, the gram, and the are — have decimal multiples and submultiples, so that every unit of length, area, volume, or mass is connected with the unit of next higher denomination by an invariable multiplier, and that the simplest possible multiplier, — namely, ten.

The names of the multiples are obtained by adding to the name of the standard unit the Greek prefixes, *deka* (ten), *hecto* (hundred), *kilo* (thousand), and *myria* (ten thousand), while the submultiples are formed by adding the Latin prefixes, *deci* (tenth), *centi* (hundredth), and *milli* (thousandth). Thus:

1 dekameter = 10 meters	1 decimeter = $\frac{1}{10}$ meter
1 hectometer = 100 meters	1 centimeter = $\frac{1}{100}$ meter
1 kilometer = 1000 meters	1 millimeter = $\frac{1}{1000}$ meter

The most common of these units, with the abbreviations which will henceforth be used for them, are the following:

meter (m.)	cubic centimeter (cc.)
kilometer (km.)	hectare (ha.)
centimeter (cm.)	gram (g.)
millimeter (mm.)	kilogram (kg.)
liter (l.)	milligram (mg.)

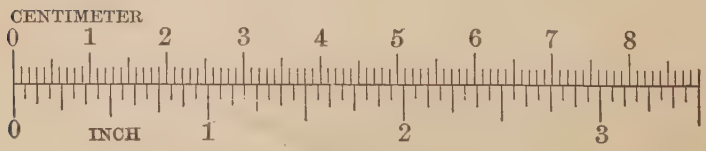


FIG. 2. Centimeter and inch scales

9. Relations between the English and metric units. The following table gives the relation between the most common English and metric units.

1 inch (in.)	= 2.54 cm.	1 cm.	= .3937 in.
1 foot (ft.)	= 30.48 cm.	1 m.	= 1.094 yd. = 39.37 in.
1 mile (M.)	= 1.609 km.	1 km.	= .6214 M.
1 sq. in.	= 6.45 sq. cm.	1 sq. cm.	= .1550 sq. in.
1 sq. ft.	= 929.03 sq. cm.	1 sq. m.	= 1.196 sq. yd.
1 acre	= .405 ha.	1 ha.	= 2.47 acres
1 cu. in.	= 16.387 cc.	1 cc.	= .061 cu. in.
1 cu. ft.	= 28,317 cc.	1 cu. m.	= 1.308 cu. yd.
1 qt.	= .9463 l.	1 l.	= 1.057 qt.
1 grain	= 64.8 mg.	1 g.	= 15.44 grains
1 oz. av.	= 28.35 g.	1 g.	= .0353 oz.
1 lb. av.	= .4536 kg.	1 kg.	= 2.204 lb.

This table is inserted chiefly for reference; but the relations 1 in. = 2.54 cm., 1 m. = 39.37 in., 1 kilo (kg.) = 2.2 lb. should be memorized. On account of its more convenient size, the centimeter, instead of the meter, is universally used for scientific purposes as the fundamental unit of length. Portions of a centimeter and of an inch scale are shown together in Fig. 2.

10. The standard unit of time. The *second* is taken among all civilized nations as the standard unit of time. It is $\frac{1}{86400}$ part of the time from noon to noon.

11. The three fundamental units. It is evident that measurements of both area and volume may be reduced simply to measurements of length; for an area is expressed as the product of two lengths, and a volume as the product of three lengths. Hence one single instrument, namely, the meter stick, is all that is absolutely essential to the determination of any or all of these quantities. For these reasons the units of area and volume are looked upon as *derived* units, depending on one *fundamental* unit, the unit of length.

The *mass* of a body is found by weighing it upon a balance. This operation is something wholly distinct from a measurement

of length and requires a new form of instrument. Also the measurement of time is wholly unlike the measurement of either length or mass, and is made with another distinct kind of instrument, namely, a clock, or watch.

Now it is found that just as measurements of area and of volume can be reduced in the ultimate analysis to measurements of length, so the determination of any measurable quantities, such as the pressure in a steam boiler, the velocity of a moving train, the amount of electricity consumed by an electric lamp, the amount of magnetism in a magnet, etc., can be reduced simply to measurements of length, mass, and time. Hence the units of length, mass, and time are considered as *the three fundamental units*, and the three instruments which measure these three quantities, namely, the meter stick, the balance, and the clock, are considered the most fundamental of all instruments.

Whenever any measurement has been reduced to its equivalent in terms of the units of length, mass, and time, it is said to be expressed in *absolute* units. Furthermore, since in all scientific work the centimeter, the gram, and the second are now universally recognized as the fundamental units of length, mass, and time, reducing a measurement to absolute units consists simply in reducing all lengths involved to centimeters, all masses to grams, and all times to seconds. The measurement is then often said, for short, to be expressed in C.G.S. (Centimeter-Gram-Second) units.

QUESTIONS AND PROBLEMS

1. The Eiffel Tower is 335 m. high. What is its height in feet?

2. A freely falling body, starting from rest, moves 490 cm. during the first second of its fall. Express this distance in feet.

3. A man weighs 160 lb. What is his weight in kilograms?

4. How many kilograms of butter may be bought for $1 if a pound of butter costs 30 ¢?

5. Find the number of millimeters in 5 km. Find the number of inches in 3 mi.

6. Find the number of square rods in a field 200 ft. on a side. Find the number of square meters in a field .3 km. on a side.

7. There are 231 cu. in. in a gallon. How deep must a tank be made which is 4 yd. long and 4 ft. wide if it is to hold 1500 gal.? What must be the depth of a tank which is to hold 6000 l. if it is 4 m. long and 1.5 m. wide?

Construction of Standards

12. Measurement of length. Measuring the length of a body consists simply in comparing its length with that of the standard meter bar kept in Paris. In order that this may be done conveniently, millions of rods of the same length as this standard meter bar have been made and scattered all over the world. They are our common meter sticks. They are divided into 10, 100, or 1000 equal parts, great care being taken to have all the parts of exactly the same length. The method of making a measurement with such a bar is more or less familiar to every one.

13. Measurement of mass. Similarly, measuring the mass of a body consists in comparing its mass with that of the standard cylinder of platinum, the kilogram of the Archives. In order that this might be done conveniently, it was first necessary to construct bodies of the same mass as this kilogram and then to make a whole series of bodies whose masses were $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc., of the mass of this kilogram; in other words, to construct a *set of weights.*

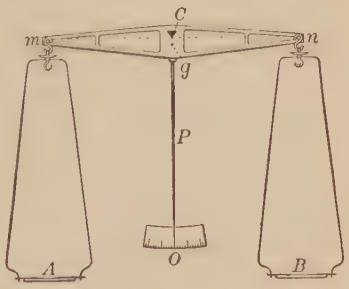


Fig. 3. The simple balance

14. Method of duplicating the standard kilogram. To obtain masses exactly equal to the standard kilogram the method of procedure is as follows. The standard cylinder is placed on one pan A of a balance (Fig. 3), — an instrument which consists

essentially of a beam *mn*, supported on a knife edge *C*, and carrying two pans *A* and *B*. Any convenient objects, such as shot, paper, etc., are then added to the pan *B* until the beam balances in the horizontal position, a condition which is indicated by the coincidence of the pointer *P* with the mark *O*. The standard is then removed from *A* and replaced by the body which it is desired to make equivalent to it. If the pointer is now found to come back exactly to the mark *O*, the body is considered to have a mass of one kilogram. If the pointer does not return to *O*, the body is altered (filed away or added to) until coincidence between *P* and *O* is exact.

15. Method of making a set of weights. To obtain bodies of mass equal to half a kilogram, it is only necessary to take two pieces of metal as nearly alike as possible and file them down together, always keeping them exactly equal to each other, until the balance shows that the two together are exactly equivalent to the standard kilogram. In this way sets of weights may be made which contain any desired masses, e.g. 500 g., 200 g., 100 g., 50 g., 10 g., 1 g., .1 g., .01 g., .001 g., etc.

16. Method of weighing a body of unknown mass. With the aid of such a set of standard weights, the determination of the mass of any unknown body is made by first placing the body upon the pan *A* and counterpoising with shot, paper, etc., then replacing the unknown body by as many of the standard weights as are required to again bring the pointer back to *O*. The mass of the body is equal to the sum of these standard weights. This is the rigorously correct method of making a weighing, and is called the *method of substitution.*

If a balance is well constructed, however, a weighing may usually be made with sufficient accuracy by simply placing the unknown body upon one pan and finding how many standard weights must then be placed upon the other pan to bring the pointer again to *O*. This is the usual method of weighing. It gives correct results, however, only when the knife edge *C* is

exactly midway between the points of support m and n of the two pans. The method of substitution, on the other hand, is independent of the position of the knife edge.

Density

17. Definition of density. When equal volumes of different substances, such as lead, wood, iron, etc., are weighed in the manner described above, they are found to have widely different masses. The term "density" is therefore introduced to denote *the mass of unit volume* of a substance.

Thus, for example, in the English system the cubic foot is taken as the unit of volume and the pound as the unit of mass. Since then one cubic foot of water is found to weigh 62.3 lb., we say that in the English system the density of water is 62.3 lb. per cu. ft.

In the C.G.S. system the cubic centimeter is taken as the unit of volume and the gram as the unit of mass. Hence we say that in this system the density of water is 1 g. per cc., for it will be remembered that the gram was taken as the mass of one cubic centimeter of water. Unless otherwise expressly stated, density is now universally understood to mean density in C.G.S. units, i.e. *the density of a substance is the mass in grams of one cubic centimeter of that substance.* For example, if a block of cast iron 3 cm. wide, 8 cm. long, and 1 cm. thick weighs 177.6 g., then, since there are 24 cc. in the block, the mass of 1 cc., i.e. the density, is equal to $\frac{177.6}{24}$ or 7.4.

The density of some of the most common substances is given in the following table.

Densities of Liquids

(In grams per cc.)

Alcohol	.79	Hydrochloric acid	1.27
Carbon bisulphide	1.29	Mercury	13.6
Glycerine	1.26	Olive oil	.91

Densities of Solids

(In grams per cc.)

Aluminium	2.58	Lead	11.3
Brass	8.5	Nickel	8.9
Copper	8.9	Oak	.8
Cork	.24	Pine	.5
Glass	2.6	Platinum	21.5
Gold	19.3	Silver	10.53
Iron (cast)	7.4	Tin	7.29
Iron (wrought)	7.86	Zinc	7.15

18. Relation between mass, volume, and density. Since the volume of a body is equal to the number of cubic centimeters which it contains, and since its density is by definition the number of grams in one cubic centimeter, its mass, i.e. the total number of grams which it contains, must evidently be equal to its volume times its density. Thus, if the density of iron is 7.4 and if the volume of an iron body is 100 cc., the mass of this body in grams must equal $7.4 \times 100 = 740$. To express this relation in the form of an equation, let M represent the mass of a body, i.e. its total number of grams; V its volume, i.e. its total number of cubic centimeters; and D its density, i.e. the number of grams in one cubic centimeter; then

$$D = \frac{M}{V}, \text{ or } M = V \times D, \text{ or } V = \frac{M}{D}. \qquad (1)$$

This equation is merely the algebraic statement of the definition of density.

19. Distinction between density and specific gravity. The term "specific gravity" is used to denote *the ratio between the weight of a body and the weight of an equal volume of water.* Thus, if a cubic centimeter of iron weighs 7.4 times as much as a cubic centimeter of water, its specific gravity is 7.4. But the density of iron in C.G.S. units is 7.4 g. per cc., for by

definition density in that system is the mass per cubic centimeter. It is clear, then, that *density in C.G.S. units is numerically the same as specific gravity.*

Specific gravity is the same in all systems, since it simply expresses how many times as heavy a body is as an equal volume of water. Density, however, which we have defined as the mass per unit volume, is different in different systems. Thus, in the English system the density of iron is 461 lb. per cubic foot (7.4×62.3), since we have found that water weighs 62.3 lb. per cubic foot and iron weighs 7.4 times as much as an equal volume of water.

Since we shall henceforth use the term "density" to signify exclusively density in the C.G.S. system of units, we shall have little further use in this book for the term "specific gravity."[1]

QUESTIONS AND PROBLEMS

1. A tank is 8 by 4 by 10.5 cm. What weight of water can it hold?

2. If a rectangular block of wood 5 by 4 by 20 cm. weighs 200 g., what is the density of wood?

3. Find the weight of a liter of mercury. (See table, p. 9.)

4. How many cc. in a block of zinc weighing 40 g.?

5. Would you attempt to carry home a block of gold the size of a peck measure? (Consider a peck equal to 8 l.)

6. Find the volume of a block of pine weighing 80 g.

7. The density of a sphere of lead is 11.3. Its radius is 50 cm. What is its weight in metric tons? (A metric ton is 1000 kilos, about 2200 lb.)

8. Find the volume in liters of a block of platinum weighing 45.5 kilos.

9. Find the density of a steel sphere of radius 1 cm. and weight 32.7 g.

10. One kilogram of alcohol is poured into a cylindrical vessel and fills it to a depth of 8 cm. Find the diameter of the cylinder.

11. A capillary glass tube weighs .2 g. A thread of mercury 10 cm. long is drawn into the tube, when it is found to weigh .6 g. Find the diameter of the capillary tube.

12. Find the length of a lead rod 1 cm. in diameter and weighing 1 kg.

[1] Laboratory exercises on length, mass, and density measurements should accompany or follow this chapter. See, for example, Experiments 1, 2, and 3 of the authors' manual.

CHAPTER II

FORCE AND MOTION

Definition and Measurement of Force

20. Distinction between a gram of mass and a gram of force. If a gram of mass is held in the outstretched hand, a downward pull upon the hand is felt. If the mass is 50,000 g. instead of 1, this pull is so great that the hand cannot be held in place. The cause of this pull we assume to be an attractive force which the earth exerts on the matter held in the hand, and we define the gram of force as the amount of the earth's pull at its surface upon one gram of mass.

Unfortunately, in common conversation we often fail altogether to distinguish between the concept of mass and the concept of force, and use the same word *gram* to mean sometimes a certain *amount of matter*, and at other times the *pull of the earth upon this amount of matter*. That the two ideas are, however, wholly distinct is evident from the consideration that the amount of matter in a body is always the same, no matter where the body is in the universe, while the pull of the earth upon that amount of matter decreases as we recede from the earth's surface. It will help to avoid confusion if we reserve the simple term "gram" to denote exclusively an amount of matter, i.e. a mass, and use the full expression "gram of force" wherever we have in mind the pull of the earth upon this mass.

21. Method of measuring forces. When we wish to compare accurately the pulls exerted by the earth upon different masses, we find such sensations as those described in the preceding paragraph very untrustworthy guides. An accurate

method, however, of comparing these pulls is that furnished by the stretch produced in a spiral spring. Thus the pull of the earth upon a gram of mass at its surface will stretch a given spring a given distance *ab* (Fig. 4). The pull of the earth upon two grams of mass is found to stretch the spring a larger distance *ac*, upon three grams a still larger distance *ad*, etc. We have only to place a fixed surface behind the pointer and make lines upon it corresponding to the points to which it is stretched by the pull of the earth upon different masses in order to graduate a spring balance (Fig. 5), so that it will thenceforth measure the values of any pulls exerted upon it, no matter how these pulls may arise. Thus if a man stretch the spring so that the pointer is opposite the mark corresponding to the pull of the earth upon two grams of mass, we say that he exerts two grams of force. If he stretch it the distance corresponding to the pull of the earth upon three grams of mass, he exerts three grams of force, etc. The spring balance thus becomes an instrument for measuring forces.

FIG. 4. Method of measuring forces

FIG. 5. The spring balance

22. The gram of force varies slightly in different localities. With the spring balance it is easy to verify the statement made above that the force of the earth's pull decreases as we recede from the earth's surface; for upon a high mountain the stretch produced by a given mass is indeed found to be slightly less than at the sea level. Furthermore, if the balance is simply carried from point to point over the earth's surface, the stretch is still found to vary slightly. For example, in Chicago it is about one part in 1000 less than it is at Paris, and near the equator it is five parts in 1000 less than it is near the pole. This is due in part to the earth's rotation, and in part to the

fact that the earth is an oblate spheroid, so that in going from the equator toward the pole we are coming closer and closer to the center of the earth. We see, therefore, that the gram of force is not an absolutely invariable unit of force.

Composition and Resolution of Force

23. Graphic representation of force. A force is completely defined when its *magnitude*, its *direction*, and the *point at which it is applied* are given. Since the three characteristics of a straight line are its *length*, its *direction*, and the *point at which it starts*, it is obviously possible to represent forces by means of straight lines. Thus, if we wish to represent the fact that a force of 8 lb., acting in an easterly direction, is applied at the point A (Fig. 6), we draw a line 8 units long, beginning at the point A and extending to the right. The length of this line then represents the magnitude of the force; the direction of the line, the direction of the force; and the starting point of the line, the point at which the force is applied.

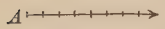


Fig. 6. Graphic representation of a single force

Again, if we wish to represent graphically the fact that two forces are acting simultaneously upon a body at A (Fig. 7), one being a force of 10 lb. acting toward the east, and the other a force of 15 lb. directed toward the north, we have simply to draw two lines from the point A, — one 10 units long and running toward the right, and the other 15 units long and running toward the top of the page. These two lines represent completely the two forces in question.

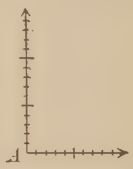


Fig. 7. Graphic representation of two forces at right angles

24. Resultant of two forces acting in the same line. *The resultant of two forces is defined as that single force which will produce the same effect upon a body as is produced by the joint action of the two forces.*

In general, when a single force acts upon a body which is free to move, the body moves in the direction in which the force acts; but if two oppositely directed forces act simultaneously upon the same body, as when two boys pull in opposite directions on a cart, the effect upon the motion of the cart is just the same as though it were acted upon by a single force equal to the difference between the two forces and acting in the direction of the greater force. For example, if one boy pulls back on the cart with a force of 50 lb., while another pulls forward with a force of 75 lb., the effect upon its motion is obviously the same as though it were pulled forward with a single force of magnitude 25 lb.; i.e. *the resultant of two oppositely directed forces applied at the same point is equal to the difference between them, and its direction is that of the greater force.*

If the two forces act in the same direction, the effect upon the motion of the body upon which they act is the same as though one single force equal in magnitude to the sum of the two forces were acting in their common direction; i.e. *the resultant of two similarly directed forces applied at the same point is equal to the sum of the two forces.*

25. The resultant of forces acting at an angle. If a body at A is pulled toward the east with a force of 10 lb. (represented in Fig. 8 by the line AC) and toward the north with a force of 10 lb. (represented in the figure by the line AB), the effect upon the motion of the body must, of course, be the same as though some single force acted somewhere between AC and AB. If the body *moves* under the action of the two equal forces, it may be seen from symmetry that it must move along a line midway between AC and AB, i.e. along the line AR. This line therefore indicates the *direction* of the resultant of the forces AC and AB.

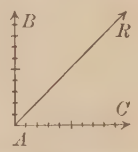


FIG. 8. Direction of resultant of two equal forces at right angles

If the two forces are not equal, then the resultant will lie nearer the larger force. As a matter of fact, the experiment of the following paragraph will show that *if the two given forces are represented in direction and in magnitude by the lines AB and AC (Fig. 9), then their resultant will be exactly represented both in direction and in magnitude by the diagonal AR of the parallelogram of which AB and AC are sides.*

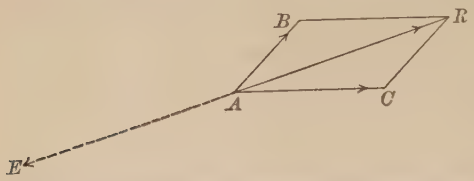


FIG. 9. Resultant of two forces at an angle is represented by the diagonal of the parallelogram of which the forces are sides

26. Equilibrant. When two or more forces act upon a body in such a way that no motion results, there is said to be *equilibrium.* Any single force which will prevent the motion which one or more forces tends to produce is called an *equilibrant.* Hence the equilibrant of two or more forces is a force equal and opposite to their resultant. Thus if *AR* (Fig. 9) is the resultant of the forces *AB* and *AC,* then *AE,* taken equal in length to *AR* but opposite in direction, is the equilibrant of *AB* and *AC.*

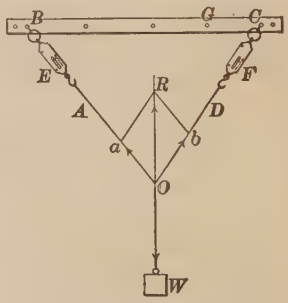


FIG. 10. Experimental proof of parallelogram law

Let the rings of two spring balances be hung over nails *B* and *C* in the rail at the top of the blackboard (Fig. 10), and let a weight *W* be tied near the middle of the string joining the hooks of the two balances. The force of the earth's attraction for the weight *W* is then exactly equal and opposite to the resultant of the two forces exerted by the spring balances; i.e. *OW* is the *equilibrant* of the forces exerted by the balances. Let the lines *OA* and *OD* be drawn upon the blackboard behind the string, and upon these lines let distances *Oa* and *Ob* be laid off which contain as many units of length as there are units of force indicated by the balances *E* and *F*

respectively. Then let a parallelogram be constructed upon Oa and Ob as sides. The diagonal of this parallelogram will be found in the first place to be exactly vertical, i.e. in the *direction* of the resultant, since it is exactly opposite to OW; and in the second place the *length* of the diagonal will be found to contain as many units of length as there are units of force in the earth's attraction for W (W must, of course, be expressed in the same units as the balance readings). Therefore the diagonal OR represents in direction, in magnitude, and in point of application the resultant of the two forces represented by Oa and Ob.

In order to test this conclusion more completely, let balances be hung from B and G (Fig. 10). When the parallelogram is constructed as before, its diagonal will be found to have the same length and the same direction as at first. This was to have been expected, since the resultant of Oa and Ob must be in every case equal and opposite to the force of the earth's attraction upon W.

27. Component of a force. Whenever a force acts upon a body in some other direction than that in which the body is free to move, it is clear that the full effect of the force cannot be spent in producing motion. For example, suppose that a force is applied in the direction OR (Fig. 11) to a car on an elevated track. Evidently OR produces two distinct effects upon the car: on the one hand it moves the car along the track, and on the other it presses it down against the rails. These two effects might be produced just as well by two separate forces acting in the directions OA and OB respectively. The value of the single force which, acting in the direction OA, will produce the same motion of the car on the track as is produced by OR, is called the *component* of OR in the direction OA. Similarly the value of the single force which, acting in the direction OB, will produce the same pressure against the rails as is produced by the force OR, is called the component of OR in the direction OB. In a word, *the component of a force in a given direction is the effective value of the force in that direction.*

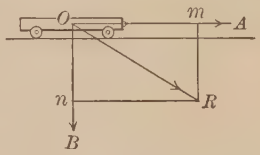


Fig. 11. Component of a force

28. Magnitude of the component of a force in a given direction. Since, from the definition of component just given, the two forces, one to be applied in the direction OA and the other in the direction OB, are together to be exactly equivalent to OR in their effect on the car, their magnitudes must be represented by the sides of a parallelogram of which OR is the diagonal. For in § 25 it was shown that if any one force is to have the same effect upon a body as two forces acting simultaneously, it must be represented by the diagonal of a parallelogram the sides of which represent the two forces. Hence conversely, if two forces are to be equivalent in their joint effect to a single force, they must be sides of the parallelogram of which the single force is the diagonal. Hence the following rule: *To find the component of a force in any given direction, construct upon the given force as a diagonal a rectangle the sides of which are respectively parallel and perpendicular to the direction of the required component. The length of the side which is parallel to the given direction represents the magnitude of the component which is sought.* Thus, in the above illustration, the line Om completely represents the component of OR in the direction OA, and the line On represents the component of OR in the direction OB.

It will be seen from Fig. 11 that as OR becomes more and more nearly parallel to the track, the component of OR along the track becomes larger and larger, while the component perpendicular to the track becomes smaller and smaller. When OR is parallel to the track, the component at right angles to the track becomes zero. When OR is perpendicular to the track, its component parallel to the track becomes zero.

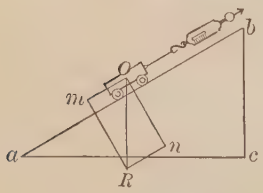


Fig. 12. Component of weight parallel to an inclined plane

29. Component of weight which is parallel to an inclined plane. To apply the test of experiment to the conclusions of the preceding paragraph, let a wagon be placed upon an inclined plane (Fig. 12), the height of which, bc, is

equal to one half its length ab. In this case the force acting on the wagon is the weight of the wagon, and its direction is downward. Let this force be represented by the line OR. Then by the construction of the preceding paragraph, the line Om will represent the value of the force which is pulling the carriage down the plane, and the line On the value of the force which is producing pressure against the plane. Now since the triangle ROm is similar to the triangle abc (for $\angle mOR = \angle abc$, $\angle RmO = \angle acb$, and $\angle ORm = \angle bac$), we have

$$\frac{Om}{OR} = \frac{bc}{ab},$$

i.e. in this case, since bc is equal to one half of ab, Om is one half of OR. Therefore the force which is necessary to prevent the wagon from sliding down the plane should be equal to one half its weight. To test this conclusion, let the wagon be weighed on the spring balance and then placed on the plane in the manner shown in the figure. The pull indicated by the balance will, indeed, be found to be one half of the weight of the wagon.

The equation $Om/OR = bc/ab$ shows that in general *the force which must be applied to a body to hold it in place upon an inclined plane bears the same ratio to the weight of the body that the height of the plane bears to its length.*

30. Component of gravity effective in producing the motion of the pendulum. When a pendulum is drawn aside from its position of rest (Fig. 13), the force acting on the bob is its weight, and the direction of this force is vertical. Let it be represented by the line OR. The component of this force in the direction in which the bob is free to move is On, and the component at right angles to this direction is Om. The second component Om simply produces stretch in the string and pressure upon the point of suspension. The first component On is alone responsible for the motion of the bob. A consideration of the figure shows that this component becomes larger and larger the greater the displacement of the bob. When the bob is directly beneath

FIG. 13. Force acting on displaced pendulum

the point of support the component producing motion is zero. Hence a pendulum can be permanently at rest only when its bob is directly beneath the point of suspension.[1]

QUESTIONS AND PROBLEMS

1. Represent graphically a force of 20 lb. acting southeast and a force of 25 lb. acting southwest at the same point. What will be the magnitude of the resultant, and what will be its approximate direction?

2. The engines of a steamer can drive it 12 mi. an hour. How fast can it go up a stream in which the current is 5 ft. per second? How fast can it come down the same stream?

3. The wind drives a steamer east with a force which would carry it 12 mi. per hour, and its propeller is driving it south with a force which would carry it 15 mi. per hour. What distance will it actually travel in an hour? Draw a diagram to represent the exact path.

4. A boy pulls a loaded sled weighing 200 lb. up a hill which rises 1 ft. in 5. Neglecting friction, how much force must he exert?

5. What force will be required to support a 50-lb. ball on an inclined plane of which the length is 10 times the height?

6. A boy is able to exert a force of 75 lb. How long an inclined plane must he have in order to push a truck weighing 350 lb. up to a doorway 3 ft. above the ground?

Gravitation

31. Newton's law of universal gravitation. In order to account for the fact that the earth pulls bodies toward itself, and at the same time to account for the facts that the moon and planets are held in their respective orbits about the earth and the sun, Sir Isaac Newton (1642–1727) first announced the law which is now known as the law of universal gravitation. This law asserts first that *every body in the universe attracts every other body with a force which varies inversely as the square of the distance between the two bodies.* This means that if the distance between the two bodies considered is doubled, the force

[1] It is recommended that the formal study of the laws of the pendulum be reserved for laboratory work (see Experiment 17, authors' manual).

SIR ISAAC NEWTON (1642–1727)

English mathematician and physicist, "prince of philosophers"; professor of mathematics at Cambridge University; formulated the law of gravitation; invented the method of the calculus; announced the three laws of motion which have become the basis of the science of mechanics; made important discoveries in light; is the author of the celebrated *Principia* (*Principles of Natural Philosophy*), published in 1687.

will become only one fourth as great; if the distance is made three, four, or five times as great, the force will be reduced to one ninth, one sixteenth, or one twenty-fifth of its original value, etc.

The law further asserts that if the distance between two bodies remains the same, *the force with which one body attracts the other is proportional to the product of the masses of the two bodies.* Thus we know that the earth attracts 3 cc. of water with three times as much force as it attracts one, i.e. with a force of three grams. We know also, from the facts of astronomy, that if the mass of the earth were doubled, its diameter remaining the same, it would attract 3 cc. of water with twice as much force as it does at present, i.e. with a force of six grams (multiplying the mass of one of the attracting bodies by 3 and that of the other by 2 multiplies the forces of attraction by 3×2, or 6). In brief, then, Newton's law of universal gravitation is as follows: *Any two bodies in the universe attract each other with a force which is directly proportional to the product of the masses and inversely proportional to the square of the distance between them.*

32. Variation of the force of gravity with distance above the earth's surface. If a body is spherical in shape and of uniform density, it attracts external bodies with the same force as though its mass were concentrated at its center. Since, therefore, the distance from the surface to the center of the earth is about 4000 miles, we learn from Newton's law that a body 4000 miles above the earth's surface would weigh one fourth as much as it does at the surface.

It will be seen, then, that if a body be raised but a few feet or even a few miles above the earth's surface, the decrease in its weight must be a very small quantity, for the reason that a few feet or a few miles is a small distance compared with 4000 miles. As a matter of fact, a body which would weigh 1000 g. at sea level would weigh about 998 g. at the top of a mountain 4 miles high.

33. Center of gravity. From the law of universal gravitation it follows that every particle of a body upon the earth's surface is pulled toward the earth. It is evident that the sum of all these little pulls on the particles of which the body is composed must be equal to the total pull of the earth upon the body, i.e. to the weight of the body. Now it is always possible to find one single point in a body at which a single force equal in magnitude to the weight of the body and directed upward can be applied so that the body will remain at rest in whatever position it is placed. This point is called the *center of gravity* of the body. Since this force counteracts entirely the weight of the body, it must be equal and opposite to the resultant of all the small forces which gravity is exerting upon the different particles of the body. Hence the center of gravity may be defined as the point of application of the resultant of all the little downward forces; i.e. it is *the point at which the entire weight of the body may be considered as concentrated.* The earth's attraction for a body is therefore always considered not as a multitude of little forces but as one single force F (Fig. 14) equal to the weight of the body and applied at its center of gravity G.

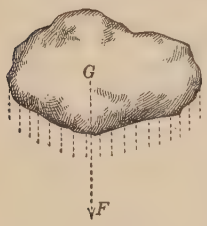


Fig. 14. Center of gravity of an irregular body

34. Method of finding center of gravity experimentally. From the above definition it will be seen that the most direct way of finding the center of gravity of any flat body, like that shown in Fig. 15, is to find the point upon which it will balance.

Let an irregular sheet of zinc be thus balanced on the point of a pencil or the head of a pin. Let a small hole be punched through the zinc at the point of balance, and let a needle be thrust through this hole. When the needle is held horizontally the zinc will be found to remain at rest, no matter in what position it is turned.

To illustrate another method for finding the center of gravity of the zinc, let it be supported from a pin stuck through a hole near its edge, e.g. b (Fig. 15). Let a plumb line be hung from the pin, and let a line bn be drawn through b on the surface of the zinc, parallel to and directly behind the plumb line. Let the zinc be hung from another point a, and another line am drawn in a similar way.

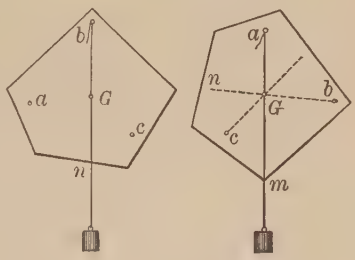


Fig. 15. Locating center of gravity

The point of intersection of the two lines is at the center of gravity. For since the earth's attraction may be considered as a single force applied at the center of gravity, the zinc can remain at rest only when the center of gravity is directly beneath the point of support. It must, therefore, lie somewhere on the line am. For the same reason it must lie on the line bn. But the only point which lies on both of these lines is their point of intersection G.

35. Stable equilibrium. A body is said to be in *stable equilibrium* if it tends to return to its original position when given a slight displacement. A pendulum, a chair, a cube resting on its side, a cone resting on its base, are all illustrations.

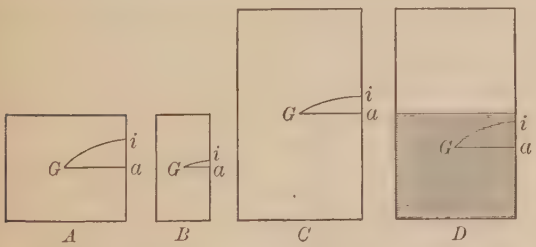


Fig. 16. Illustration of varying degrees of stability

In general, a body is in stable equilibrium whenever a slight displacement tends to raise its center of gravity. Thus, in Fig. 16 all of the bodies A, B, C, D are in stable equilibrium, for in order to overturn any one of them, its center of gravity G must be raised through

the height ai. If the weights are all alike, that one will be most stable for which ai is greatest.

The condition of stable equilibrium for bodies which rest upon a horizontal plane is that a vertical line through the center of gravity shall fall within the base, the base being defined as the polygon formed by connecting the points at which the body touches the plane, as ABC (Fig. 17); for it is clear that in such a case a slight displacement must raise the center of gravity along the arc of which OG is the radius. If the vertical line drawn through the center of gravity fall outside the base, as in Fig. 18, the body must always fall.

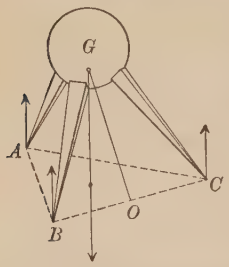


Fig. 17. Body in stable equilibrium

The condition of stable equilibrium for bodies supported from a single point is that the point of support be above the center of gravity. For example, the beam of a balance cannot be in stable equilibrium so that it will return to the horizontal position when slightly displaced, unless its center of gravity g (Fig. 3, p. 7) is below the knife edge C.

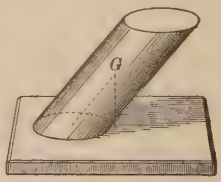


Fig. 18. Body not in equilibrium

36. Neutral equilibrium. A body is said to be in neutral equilibrium when, after a slight displacement, it tends neither to return to its original position nor to move farther from it. Examples of neutral equilibrium are a spherical ball lying on a smooth plane, a cone lying on its side, a wheel free to rotate about a fixed axis through its center, or any body supported at its center of gravity. In general, a body is in neutral equilibrium when a slight displacement neither raises nor lowers its center of gravity.

37. Unstable equilibrium. A body is in unstable equilibrium when after a slight displacement it tends to move farther from its original position. A cone balanced on its point or an egg on its end are examples. In all such cases a slight displacement always lowers the center of gravity and the motion then continues until

the center of gravity is as low as circumstances will permit. The condition for unstable equilibrium in the case of a body supported by a point is that the center of gravity shall be above the point of support. Fig. 19 illustrates the three kinds of equilibrium.

Fig. 19. Stable, unstable, and neutral equilibrium

QUESTIONS AND PROBLEMS

1. A body weighs 100 kg. at the earth's surface. What will it weigh 4000 mi. above the surface? What will it weigh 1000 mi. above the surface? What will it weigh 3 mi. above the surface? (Take the earth's radius as 4000 mi.)

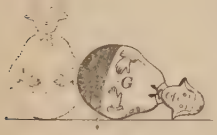


Fig. 20

2. What is the object of ballast in a ship?

3. Explain why the toy shown in Fig. 20 will not lie upon its side, but instead rises to the vertical position. Does the center of gravity actually rise?

4. If a lead pencil is balanced on its point on the finger it will be in unstable equilibrium, but if two knives are stuck into it, as in Fig. 21, it will be in stable equilibrium. Why?

Fig. 21

5. Why does a man lean forward when he climbs a hill?

Uniformly Accelerated Motion

38. Uniform motion. When a body moves over equal distances in succeeding equal intervals of time, its motion is said to be *uniform.* Thus the motion of a train between stations is for the most part nearly uniform.

39. Velocity. When the motion of a body is uniform, its *velocity* is defined as the distance which it traverses per second. When the motion of a body is not uniform, its velocity at any instant is defined as the distance which it would travel in a second if at that instant its motion were to become uniform.

40. Acceleration. If a train starting from rest has a velocity of two feet per second at the end of the first second, a velocity of four feet per second at the end of the second second, of six feet per second at the end of the third second, etc., its motion is said to be *uniformly accelerated.* The *gain* in the *velocity* of such a body *per second* is called its *acceleration;* e.g. in the case above, the acceleration is two feet per second. In general, then, acceleration is defined as *the rate at which velocity changes.* If the motion is uniformly accelerated, its acceleration is equal to the velocity gained per second.

41. Relative distances traversed by a falling body in one, two, three, four, etc., seconds. The simplest case of uniformly accelerated motion is that of a falling body. Since, however, a freely falling body acquires velocity so rapidly that it is difficult to make observations upon it directly, Galileo hit upon the plan of studying the laws of falling bodies by observing the motion of a ball rolling down an inclined plane. He found that a body falls exactly 4 times as far in 2 seconds as in 1, 9 times as far in 3 seconds, 16 times as far in 4 seconds, 25 times as far in 5 seconds, etc.

To test the correctness of these results, let a grooved board about 16 ft. long be supported as in Fig. 22, one end being about a foot and a half above the other. Let supports be introduced near the middle,

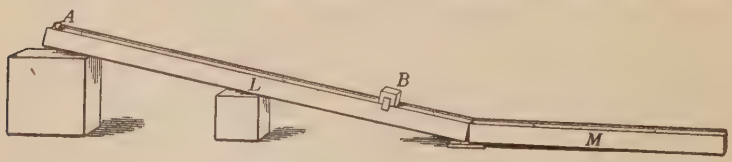


Fig. 22. Spaces traversed and velocities acquired by falling bodies in one, two, three, etc., seconds

if necessary, so that the plane will not sag. Let a métronome, or a clock beating seconds, be started, and the marble A released at the instant of one click of the metronome. Let the block B be placed at such a distance down the incline that the click produced by the impact of the

ball upon it coincides exactly with, for example, the fourth click of the metronome. The time of fall is then three seconds. Let the distance traversed be measured. Then let B be placed at a distance equal to $\frac{4}{9}$ of this distance and the experiment repeated. The ball will strike B exactly at the end of two seconds. At a distance equal to $\frac{1}{9}$ of the first distance the impact will occur at the end of one second, etc. An interesting variation of this experiment is to have three grooves, three marbles, and three blocks B set at distances 1, 4, and 9 from the common starting point. If the marbles are all released at the instant of one click, a marble will strike a block at the exact instant of each of the three succeeding clicks.

42. Velocity acquired per second by the marble. In the last paragraph we investigated the distances traversed in one, two, three, etc., seconds. Let us now investigate the *velocities* acquired on the same inclined plane in one, two, three, etc., seconds.

Let a second grooved board M be placed at the bottom of the incline, in the manner shown in Fig. 22. To eliminate friction it should be given a slight slant, just sufficient to cause the ball to roll along it with uniform velocity. Let the ball be started at a distance D up the incline, D being the distance which it was found in the last experiment to roll during the first second. It will then just reach the bottom of the incline at the instant of the second click. Here it will be freed from the accelerating force of gravity, and will therefore move along the lower board with the velocity which it had at the end of the first second. It will be found that when the block is placed at a distance exactly equal to $2\,D$ from the bottom of the incline, the ball will hit it at the exact instant of the third click of the metronome, i.e. exactly two seconds after starting; hence the velocity acquired in one second is $2\,D$. If the ball is started at a distance $4\,D$ up the incline, it will take it two seconds to reach the bottom, and it will roll a distance $4\,D$ in the next second; i.e. in two seconds it acquires a velocity $4\,D$. In three seconds it will be found to acquire a velocity $6\,D$, etc.

The experiment shows, first, that the increase in velocity each second is the same, namely $2\,D$, and that the motion is therefore uniformly accelerated. Furthermore, it shows that *in uniformly accelerated motion the acceleration (velocity gained per second) is measured by twice the distance passed over in the first second.*

43. Distances traversed during successive seconds. If we subtract from the distance traversed in two seconds the distance traversed in one second, we get $4D - D = 3D$, which is the distance traversed during the second second. Similarly, if we subtract the distance traversed in two seconds from the distance traversed in three seconds, we obtain $9D - 4D = 5D$, which is the distance traversed during the third second. In the same way the distance traversed in the fourth second is $7D$, etc.

44. Tabular statement of the laws of falling bodies. Putting together the results of the last three paragraphs, we obtain the following table, in which D represents the distance fallen the first second.

Number of Seconds (t)	Velocities at the End of Each Second (v)	Spaces fallen Each Second (s)	Total Distance fallen (S)
1	$2D$	$1D$	$1D$
2	$4D$	$3D$	$4D$
3	$6D$	$5D$	$9D$
4	$8D$	$7D$	$16D$
. . .	. . .	. . .	. . .
t	$2tD$	$(2t-1)D$	t^2D

Since D was shown in § 42 to be equal to one half the velocity acquired per second, i.e. one half the acceleration a, we have at once, by substituting $\frac{1}{2}a$ for D in the last row of the table,

$$v = at \quad (1), \qquad s = \tfrac{1}{2}a(2t-1) \quad (2), \qquad S = \tfrac{1}{2}at^2 \quad (3).$$

These formulas are simply the algebraic expression of the facts brought out by our experiment; but the reasons for these facts may be seen as follows.

Since in uniformly accelerated motion the acceleration a is the velocity gained per second, it follows at once that the velocity v gained in t seconds is $v = at$. This is formula (1) above.

To obtain formula (3) we have only to consider that the total distance S traversed by any moving body in t seconds is the *average* velocity multiplied by t, the number of seconds. But the average velocity in uniformly accelerated motion is the *mean* of the initial and final velocities. Hence,

if a body starts from rest and acquires in t seconds a velocity v, its average velocity is $\frac{o+v}{2} = \frac{v}{2}$. Hence the space traversed is given by $S = \frac{v}{2}t$. By substituting in this equation $v = at$ we get, $S = \frac{1}{2}at^2$. To obtain (2) we have only to subtract from the space traversed in t seconds that traversed in $(t-1)$ seconds. Thus $s = \frac{1}{2}at^2 - \frac{1}{2}a(t-1)^2 = \frac{1}{2}a(2t-1)$.

To illustrate the use of these results, suppose that a body rolling down an inclined plane is known to move over 10 cm. the first second, and that we are required to find (1) what velocity it will have at the end of the tenth second, (2) how far it will roll during the tenth second, and (3) how far it will have rolled during the 10 seconds.

Since the acceleration is $2 \times 10 = 20$, the answers are (1) $v = at = 20 \times 10 = 200$ cm. per sec. (2) $s = \frac{1}{2}a(2t-1) = \frac{1}{2} \times 20(20-1) = 190$ cm. (3) $S = \frac{1}{2}at^2 = \frac{1}{2} \times 20 \times 100 = 1000$ cm.

45. Acceleration of a freely falling body. If in the above experiment the slope of the plane be made steeper, the results will be precisely the same, except that the acceleration has a larger value. If the board is tilted until it becomes vertical, the body becomes a freely falling body. In this case the distance traversed the first second is found to be 490 cm., or 16.08 ft. Hence the acceleration expressed in centimeters is 980, in feet 32.16. This acceleration of free fall, called the *acceleration of gravity*, is usually denoted by the letter g.

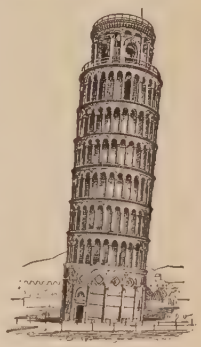


FIG. 23. Leaning tower of Pisa, from which some of Galileo's famous experiments on falling bodies were performed

To illustrate the use of this constant, suppose we wish to know how far a body will fall in 10 seconds. We have

$$S = \frac{1}{2}gt^2 = \frac{1}{2} \times 980 \times 100 = 49{,}000 \text{ cm.} = 490 \text{ m.}$$

46. Rates of fall of different bodies. It is a fact of familiar observation that very light bodies, such as feathers and bits of paper, fall very much more slowly than pieces of wood or iron. Previous to Galileo's time it was taught in the schools that

heavy bodies fall toward the earth with "velocities proportional to their weights." Galileo demonstrated the incorrectness of this view by his famous experiments conducted from the leaning tower of Pisa (Fig. 23). In the presence of the professors and students of the University of Pisa he dropped balls of different sizes and materials from the top of the tower, 180 feet high, and showed that they fell in practically the same time. He showed also that even very light bodies, like paper, fell with velocities which approached more and more nearly those of heavy bodies the more compactly they were wadded together. He inferred from these experiments that all bodies, even the lightest, would fall at the same rate were it not for the resistance offered by the air, — an inference which could not be verified at that time because the air pump had not yet been invented. After its invention, sixty years later, by Otto von Guericke, Galileo's inference was verified in the following way. A feather and a coin were placed in a glass tube four or five feet long, and the air pumped out. When the tube was then inverted the coin and the feather fell side by side from the top to the bottom (Fig. 24).

Fig. 24. Feather and coin fall together in a vacuum

47. The pendulum and its teaching. We can demonstrate the correctness of Galileo's conclusion in still another way — one which he himself employed. If we allow balls of iron and wood, for example, to roll together down the plane of Fig. 22, we shall find that they reach the bottom in almost exactly the same time. This experiment differs from that with the freely falling bodies only in that the resistance of the air is here more nearly negligible because the balls are moving more slowly. In order to make them move still more slowly Galileo suspended them as the bobs of long pendulums and observed that

the periods of pendulums of equal lengths, swinging through short arcs, are completely independent of the weight or material of the bobs. Since in this experiment the bobs, as they pass through any given position, are merely moving very slowly down identical inclined planes (see Fig. 13), it is evident that this equality of periods proves in the most exact way the equality in the rates of fall of different bodies. Indeed, it is from exact measurements on the periods and lengths of pendulums that the value of g (980 cm.) is most accurately determined (see Experiment 17, authors' manual).

48. Velocity acquired in falling from a given height. If we wish to find with what velocity a body which falls from a given height S, say 20,000 cm., will strike the earth, we can first get the time of descent from (3), § 44, and then get the velocity from (1), § 44. Thus from (3),

$$t^2 = \frac{2 \times 20000}{980}, \text{ or } t = \sqrt{\frac{2 \times 20000}{980}},$$

and from (1),

$$v = 980 \times t = 980 \times \sqrt{\frac{2 \times 20000}{980}} = \sqrt{2 \times 980 \times 20000} = 6260 \text{ cm.}$$

If we write the symbols instead of the numbers, we see that the formula connecting v and S is

$$v = \sqrt{2\,gS}. \tag{4}$$

Or, if we wish to find the height S to which a body projected vertically upward will rise, we reflect that the time of ascent must be the initial velocity divided by the upward velocity which the body loses per second, i.e. $t = \frac{v}{g}$, and the height reached must be this multiplied by the average velocity $\frac{v+0}{2}$, i.e.

$$S = \frac{v^2}{2\,g}, \text{ or } v = \sqrt{2\,gS}. \tag{5}$$

Since (5) is the same as (4), we learn that, in a vacuum, the velocity with which a body must be projected upward to rise to a given height is the same as the velocity which it acquires in falling from the same height.

QUESTIONS AND PROBLEMS

1. A body falls from a balloon to the earth in 10 seconds. What is the height of the balloon?

2. A body falls for 8 seconds. With what velocity is it moving at the end of that time?

3. How far will a body fall in half a second?

4. A stone fell from a balloon a kilometer high. With what velocity did it strike the earth?

5. A rifle ball is shot upward with an initial velocity of 300 meters per second. How high will it rise?

6. With what velocity must a ball be shot upward to rise to the height of the Washington Monument (555 ft.)?

7. If the acceleration of a marble rolling down an inclined plane is 20 cm. per second, what velocity will it have at the bottom, the plane being 7 m. long?

8. If a body sliding without friction down an inclined plane moves 40 cm. during the first second of its descent, and if the plane is 500 cm. long and 40.8 cm. high, what is the value of g? (Remember that the acceleration down the incline is simply the component (§ 27) of g parallel to the incline.)

Newton's Laws of Motion

49. First law — Inertia. In 1686 Sir Isaac Newton formulated three statements which embody the results of universal observation and experiment on the relations which exist between force and motion. The statement of the first law is: *Every body continues in its state of rest or uniform motion in a straight line unless impelled by external force to change that state.* This statement is based upon such familiar observations as the following. Bodies on a moving train tend to move toward the forward end when the train stops, and toward the rear end when the train starts; i.e. they tend in each case to continue in their previous state whether that were one of rest or motion. That a moving body also tends to move on in a *straight line in the direction of its motion* is seen from such facts as that mud flies off tangentially from a rotating carriage wheel, or water from a whirling grindstone. This property which all matter possesses of resisting any attempt to start it if at rest, to stop it if in motion, or in any way to change either the direction or amount of its motion, is called *inertia.*

50. Centrifugal force. It is inertia alone which prevents the planets from falling into the sun; which causes a rotating sling

to pull hard on the hand until the stone is released, and which then causes the stone to fly off tangentially. It is inertia which makes rotating liquids move out as far as possible from the axis of rotation (Fig. 25); which makes fly wheels sometimes burst; which makes the equatorial diameter of the earth greater than the polar; which makes the heavier milk move out farther than the lighter cream in the dairy separator, etc. Inertia manifesting itself in this tendency of the parts of rotating systems to move away from the center of rotation is called *centrifugal force.*

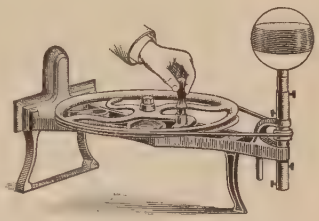


FIG. 25. Illustrating centrifugal force

51. Momentum. The quantity of motion possessed by a moving body is defined as the product of the mass and the velocity of the body. It is commonly called *momentum.* Thus a ten-gram bullet moving 50,000 cm. per second has 500,000 units of momentum. A thousand-kilogram pile driver moving 1000 cm. per second has 1,000,000,000 units of momentum, etc. We shall always express momentum in C.G.S. units, i.e. as a product of grams by centimeters per second.

52. Second law. Newton's second law is stated thus: *Rate of change of momentum is proportional to the force acting, and takes place in the direction in which the force acts.* While the first law asserted that no change in the momentum of any body ever takes place unless a force acts upon it, the second law goes a step farther and asserts that two units of force will produce in one second exactly twice as much momentum as does one unit, one half as much as does four units, etc. Now every one knows from his experience that if he pulls for a second upon a sled, a boat, or any object free to move, the velocity imparted is greater, the greater the pull. That the velocity imparted is *directly proportional* to the pull is the essence of

the assertion contained in the second law, and this can be proved only by careful experiments like the following.

Let the grooved inclined plane shown in Fig. 22, p. 26, be raised a distance *ab* (Fig. 26), just sufficient to cause the ball to roll down it with uniform velocity. Then let the same end be raised 20 cm. higher and the distance which the ball rolls in three seconds be measured with the aid of a metronome, as in § 41. In this case the force which is urging the ball down the incline is the component of the weight of the ball, parallel to the incline. But we proved in § 29 that this is the same fraction of the weight of the body that the height of the plane is of its length; e.g. if the length is 500 cm. the force acting to move the ball is $\frac{20}{500}$, or $\frac{1}{25}$, of the weight of the body. Now let the plane be lifted until *d* is 40 cm. higher than *b*. The force is now twice as great as before, since it is $\frac{2}{25}$ of the weight of the ball. Let the stop *B* (Fig. 22) be placed twice as far down the incline. The ball will be found to reach it again in exactly three seconds.

Fig. 26. Acceleration is proportional to force

We learn, then, that doubling the force without changing the mass has doubled the momentum acquired in a given interval of time, since it has doubled the distance which the body has traveled in that length of time. If now we were to double the size of the ball but keep the height of the plane constant, we should find no change in the velocity acquired per second. This is indeed nothing but Galileo's experiment which proved that, barring atmospheric resistance, all bodies fall with the same acceleration. Hence, since the earth pulls two grams with twice as much force as it pulls one, doubling the mass without changing the velocity involves a doubling of the acting force. The two experiments taken together therefore furnish very satisfactory proof of the statement that, whatever be the mass of a body, the momentum acquired by it per second is strictly proportional to the acting force.

53. The dyne. Since the gram of force varies somewhat with locality, it has been found convenient for scientific purposes to take the above law as the basis for the definition of a new unit of force. It is called an absolute, or C.G.S. unit, because it is based upon the fundamental units of length, mass, and time, and is therefore independent of gravity. It is named the *dyne*, and is defined as *the force which acting for one second upon any body imparts to it one unit of momentum.*

54. A gram of force equivalent to 980 dynes. A gram of force was defined as the pull of the earth upon one gram of mass. Since this pull is capable of imparting to this mass in one second a velocity of 980 cm. per second, i.e. a momentum of 980 units, and since a dyne is the force required to produce in one second one unit of momentum, it is clear that the gram of force is equivalent to 980 dynes of force. The dyne is therefore a very small unit, about equal to the force with which the earth attracts a cubic millimeter of water.

55. Algebraic statement of the second law. If a force f acts for t seconds on a mass of m grams, and in so doing gives it a velocity of v cm. per sec., then, since the total momentum imparted in a time t is mv, the momentum imparted per second is $\frac{mv}{t}$; and since force in dynes is equal to momentum imparted per second, we have

$$f = \frac{mv}{t}. \tag{6}$$

But since $\frac{v}{t}$ is the velocity gained per second, it is equal to the acceleration a. Equation (6) may therefore be written

$$F = ma. \tag{7}$$

This is merely stating in the form of an equation that force is measured by rate of change in momentum. Thus if an engine, after pulling for thirty seconds on a train having a mass of 2,000,000 kg., has given it a velocity of 60 cm. per second, the force of the pull is $2{,}000{,}000{,}000 \times \frac{60}{30} = 4{,}000{,}000{,}000$ dynes. To reduce this force to grams we divide by 980, and to reduce it to kilos we divide further by 1000. Hence the pull exerted by the engine on the train $= \frac{4000000000}{980000}$ $= 4081$ kg., or 4.081 metric tons.

56. Third law. Newton stated his third law thus: *To every action there is an equal and opposite reaction.* Since force is measured by rate at which momentum changes, this is only another way of saying that whenever one body acquires momentum some other body always acquires an equal and opposite momentum. Thus when a man jumps from a boat to the shore, we all know that the boat experiences a backward thrust; when a bullet is shot from a gun the gun recoils, or "kicks." The essence of the assertion of the third law is that the mass of the man times his velocity is equal to the mass of the boat times its velocity, and that the mass of the bullet times its velocity is equal to the mass of the gun times its velocity. The truth of this assertion has been established by a great variety of careful experiments. The law may be illustrated as follows.

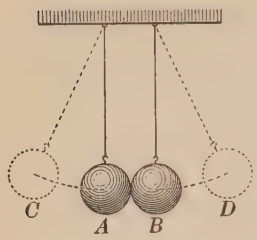


Fig. 27. Illustration of third law

Let a steel ball A (Fig. 27) be allowed to fall from a position C against another exactly similar ball B. In the impact A will lose all of its velocity and B will move on to a position D which is at the same height as C. Hence the velocity acquired by B in the impact is the same as that which A possessed before impact. B has therefore taken away from A exactly the same amount of momentum as A has communicated to B.

It is not always easy to see at first that setting one body into motion involves imparting an equal and opposite motion to some other body. For example, when a gun is held against the earth and a bullet shot upward we are conscious only of the motion of the bullet. The other body is in this case the earth and its momentum is the same as that of the bullet. On account, however, of the greatness of the earth's mass its velocity is infinitesimal.

QUESTIONS AND PROBLEMS

1. Why does a fly wheel cause machinery to run more steadily?

2. What principle is applied when one tightens the head of a hammer by pounding on the handle?

3. What keeps a pendulum moving when the bob reaches the bottom?

4. Why does pounding a carpet free it from dust?

5. Suspend a weight by a string (Fig. 28). Attach a piece of the same string to the bottom of the weight. If the lower string is pulled with a sudden jerk, it breaks; but if the pull is steady, the upper string will break. Explain.

Fig. 28

6. A pull of 1 dyne acts for 3 seconds on a mass of 1 gram. What velocity does it impart?

7. A steamboat weighing 20,000 metric tons is being pulled by a tug which exerts a pull of 2 metric tons. (A metric ton is equal to 1000 kg.) If the friction of the water were negligible, what velocity would the boat acquire in 4 minutes? (Reduce mass to grams, force to dynes, and remember that $F = mv/t$.)

8. If a train of cars weighs 200 metric tons, and the engine in pulling 5 seconds imparts to it a velocity of 2 meters per second, what is the pull of the engine in metric tons?

9. If the motions of the earth and moon were to cease, they would rush together. The earth's mass is 80 times that of the moon. Compare the velocities of the two at the instant of impact.

10. If the earth were to cease rotating, would bodies on the equator weigh more or less than now? Why?

11. How is the third law involved in the action of the rotary lawn sprinkler?

12. The modern way of drying clothes is to place them in a large cylinder with holes in the sides, and then to set it in rapid rotation. Explain.

13. If one ball is thrown horizontally from the top of a tower and another dropped at the same instant, which will strike the earth first? (Remember that the acceleration produced by a force is in the direction in which the force acts and proportional to it, whether the body is at rest or in motion. See second law.)

A laboratory exercise on the composition of force should be performed during the study of this chapter. See e.g. Experiment 4 of the authors' manual.

CHAPTER III

PRESSURE IN LIQUIDS

Liquid Pressure beneath a Free Surface

57. Proof of the existence of a force beneath the surface of a liquid. If a long tube closed at the bottom is pushed down into a cylinder of water in the manner shown in Fig. 29, and then left to itself, it will be seen to spring instantly upward.

Fig. 29. Upward pressure in a liquid

Evidently, then, the liquid must exert an upward force upon the bottom of the tube. A moment's thought will show that no special experiment was necessary to demonstrate the existence of this force, for a boat or any other body could not float on water if the liquid did not push up against its bottom with sufficient force to neutralize its weight.

58. Relation between force and depth. To investigate more fully the nature of this force, we shall use a pressure gauge of the form shown in Fig. 30. If the rubber diaphragm which is stretched across the mouth of a thistle tube A is pressed in lightly with the finger, the drop of ink B will be observed to move forward in the tube T, but it will return again to its first position as soon as the finger is removed. If the pressure of the finger is increased, the drop will move forward a greater distance than before. We may therefore take the amount of motion of the drop as a measure of the amount of force acting on the diaphragm.

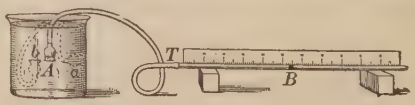


Fig. 30. Gauge for measuring liquid pressure

Now let A be pushed down first 2, then 4, then 8 cm. below the surface. The motion of the index B will show that the force continually increases as the depth increases.

Careful quantitative measurements made in the laboratory on the exact relation between the force and the depth will show that doubling the depth doubles the force, tripling the depth triples the force, etc.; in other words, that *the force is directly proportional to the depth.*[1]

To state this relationship algebraically, let F_1 represent the force at some depth D_1, and F_2 the force at some other depth D_2; then

$$\frac{F_1}{F_2} = \frac{D_1}{D_2}. \tag{1}$$

59. Force independent of direction. That there is a lateral as well as a vertical force beneath the surface of a liquid is shown from the fact that water will rush into a boat through a hole in the side as well as through a hole in the bottom.

To compare the amounts of these two forces on a given surface, let the diaphragm A (Fig. 30) be pushed down to some convenient depth, e.g. 10 cm., and the position of the index noted. Then let it be turned sidewise so that its plane is vertical (see a, Fig. 30), and adjusted in position until its center is exactly 10 cm. beneath the surface, i.e. until the *average* depth of the diaphragm is the same as before. The position of the index will show that the force is also exactly the same as before.

Let the diaphragm then be turned to the position b, so that the gauge measures the *downward* force at a depth of 10 cm. The index will show that this force is again the same.

We conclude, therefore, that *at a given depth a liquid presses up and down and sidewise with exactly the same force.*

60. The magnitude of the force. In order to determine the exact magnitude of the force exerted by a liquid against a sur-

[1] It is recommended that quantitative laboratory work on the law of depths and on the use of manometers precede this discussion (see e.g. Experiments 5 and 6 of the authors' manual).

face, we shall perform a simple experiment with the apparatus shown in Fig. 31.

AB is a thin ground glass plate which is pressed against the bottom of the glass cylinder *AD*. It is the upward force on the surface *AB* which we desire to measure. If we pour colored water carefully into the top of the cylinder, the weight of this water will press down on *AB* and tend to counteract this upward force. When the downward force is equal to the upward force the glass plate *AB* will drop from the end of the cylinder.

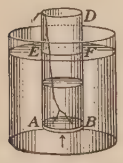


Fig. 31. Upward pressure on a surface

If the plate is thin, so that its own weight is very small, it will be found to drop almost exactly at the instant at which the level of the water within the cylinder is the same as the level of the water outside. But at this instant the downward force on *AB* is evidently the weight of the column of water *ABFE*. Hence the upward force which originally acted on *AB* was also equal to the weight of the column of water *ABFE*. In other words, *the upward force on any horizontal surface beneath the free surface of a liquid is equal to the weight of a column of the liquid whose base is the given surface and whose height is the depth of the given surface beneath the free surface of the liquid.*

61. Magnitude of the force on any surface. In § 59 we proved that the force on a given surface is independent of the direction in which that surface is turned, so long as the depth of its center is kept the same. Hence, by combining this result with that of § 60, we arrive at the conclusion that the force acting on any surface beneath the free surface of a liquid is equal to the weight of the column of the liquid whose base is the given surface and whose altitude is the *average* depth, i.e. the depth of the center of the surface beneath the free surface of the liquid.

To put this conclusion into algebraic form, let A represent the area of the given surface, h the *mean* depth of the surface beneath the free surface of the liquid, d the density of the liquid, and F

the value of the force which the liquid exerts against the surface A. Then the weight of the column of liquid whose base is A and whose height is h is Ahd (§ 18, p. 10). Hence the algebraic statement of the above rule is

$$F = Ahd. \tag{2}$$

62. The hydrostatic paradox. We may infer from the preceding paragraph that the downward force exerted on the bottom of a vessel by a liquid which fills it has nothing whatever to do with the shape of the vessel, but depends only on the area of the base and on the depth and density of the liquid [see formula (2)]. Thus, if the three vessels of Fig. 32 have bases of the same area and are filled to the same depths with liquids of the same density, the forces exerted upon the bases by the liquids should be exactly the same in all three vessels, for by the preceding paragraph they should all be equal to the weight of a column of liquid of the size $ABCD$.

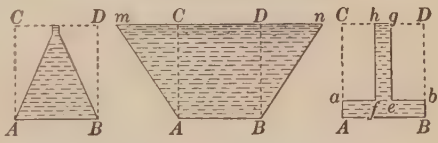


FIG. 32. Downward force on the bottoms of vessels of different shapes

This conclusion is known as the hydrostatic paradox, because at first sight it seems unreasonable to suppose that the little liquid contained in the third vessel can press down on the bottom with the same force as the large amount of liquid contained in the second vessel. The following experiment, however, will furnish a complete demonstration of the correctness of the conclusion, and will prove experimentally that the downward force on the bottom of a vessel has nothing to do with the shape of the vessel.

Let the funnel ABD [Fig. 33, (1)] be closed at the bottom by the same glass plate which was used in the experiment of Fig. 31. At a given depth beneath the free surface of the liquid the upward force

acting against the lower side of the plate AB must, of course, be the same as it was before, when the cylinder was used, i.e. it is equal to the weight of the column of water $ABEF$ (§ 60). Now let water be poured carefully into the top of the funnel until the plate AB is forced off. Just as in the experiment of § 60, this will be found to occur exactly when the level of the water inside of the funnel has risen to the height of the water outside. Hence the liquid within the funnel ABD must exert the same downward force on AB as did the liquid within the cylindrical tube $ABEF$ in the experiment of § 60.

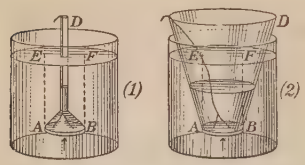


Fig. 33. Illustrating hydrostatic paradox

Let the experiment now be tried with a vessel of the shape shown in Fig. 33, (2). Again the plate will be found to fall when the levels inside and outside are the same, notwithstanding the fact that the water in the vessel of Fig. 33, (2), weighs several times as much as the water in the cylinder of Fig. 31, and perhaps a hundred times as much as the water in the funnel of Fig. 33, (1).

63. Explanation of the hydrostatic paradox. A moment's consideration will show that there is no real inconsistency in the fact that the third vessel of Fig. 32 exerts a force on the bottom so much greater than its own weight, and that the second vessel exerts a force so much less than its own weight. For the law discovered in § 59, that the force at a given depth beneath the free surface of a liquid acts equally in all directions upon all equal surfaces, means that while the liquid in the third vessel does indeed exert a downward force on AB which is equal to the weight of the column of water $ABCD$, it also exerts an upward force on the surfaces af and eb which is equal to the weight of the water which would fill the spaces $afhC$ and $ebDg$. Hence the net or resultant force which is acting down is the difference between the downward force on AB and the upward forces on af and eb, and this will be seen at once from the figure to be simply the weight of the liquid in the vessel, as of course it must be.

Similarly in the second vessel of Fig. 32, while the force acting directly upon the bottom is only the weight of the column of water $ABCD$, the downward force upon the sides Am and Bn amounts, in all, exactly to the weight of the remainder of the water in the vessel, i.e. to the weight of the water contained in the spaces AmC and BnD.

64. Pressure in liquids. Thus far attention has been confined to the total force exerted by a liquid against the *whole* of a given surface. It is often more convenient to consider the surface divided into square centimeters and to confine the attention to the force exerted upon one of these square centimeters. In physics the word "pressure" is used exclusively to denote this *force per unit area.* Thus, if the weight of the column of liquid $ABCD$ in Fig. 32 is 100 g., and if the area of the surface AB is 20 sq. cm., then the force per square centimeter acting on AB is 5 g. Hence we say that the *pressure* on AB is 5 g. Pressure is thus seen to be a measure of the *intensity* of the force acting on a surface, and not to depend at all upon the *area* of the surface.

It is clear, then, that in order to obtain pressure, we divide the total force acting by the area of the surface against which it acts. Or, algebraically stated, if we represent pressure by p, force by F, and area by A, we have

$$p = \frac{F}{A},$$

or since [see equation (2)] $F = Ahd$, we have

$$p = hd. \tag{3}$$

In other words, *the liquid pressure existing at any depth h beneath the free surface of any liquid of density d is equal to the product of this depth by the density of the liquid;* i.e. it is the weight of a column of liquid whose height is equal to the given depth, and the area of whose cross section is unity. It is important to remember this technical use of the word "pressure."

65. Levels of liquids in connecting vessels. It is a perfectly familiar fact that when water is poured into a teapot it stands at exactly the same level in the spout as in the body of the teapot; or if it is poured into a number of connected tubes, like those shown in Fig. 34, the surfaces of the liquid in the various tubes lie in the same horizontal plane. These facts follow as a necessary consequence of the law, discovered above, that the pressure beneath the surface of a liquid depends simply upon the *depth* and not at all upon the shape and size of the vessel.

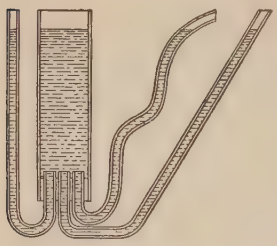


Fig. 34. Water level in communicating vessels

Thus, in accordance with the above rule, in Fig. 35 the pressure acting at *o* to drive water to the left is equal to the density of the liquid times the height *hs*; and the pressure acting at *e* to drive water to the right is equal to the same density times the height *eg*. Hence these two pressures will be balanced and the liquids will be at rest only when these two heights are the same, i.e. when the free surfaces in the two vessels are in the same horizontal plane.

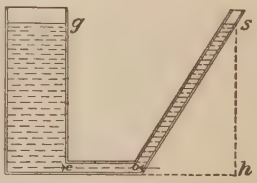


Fig. 35. Why water seeks its level

If water is poured in at *s* so that the height *hs* is increased, the pressure to the left at *o* becomes greater than the pressure to the right at *e*, and a flow of water takes place to the left until the heights are again equal.

QUESTIONS AND PROBLEMS

1. Find the pressure which exists at a depth of 1 km. beneath the surface of the ocean, the density of salt water being assumed to be 1.026.

2. Find the force acting on the bottom of a box 3 m. long, 2 m. wide, and 4 m. deep, filled with water.

3. Find the total force acting against each of the sides and ends of this box.

4. A cone-shaped vessel filled with water has a base of 200 sq. cm. and a height of 100 cm. Find the force acting on the bottom.

5. Would the force required to lift this vessel be greater or less than the total force exerted by the liquid against the bottom? Explain.

6. A cubical box 10 cm. on a side is filled with mercury. Find the total force exerted on the bottom and on each of the sides.

7. At what depth in oil (density .9) is the pressure the same as it is 10 cm. below the surface of mercury?

8. A hole 5 cm. square is made in a ship's bottom 7 m. below the water line. What force in kilograms is required to hold a board above the hole?

9. A house is supplied with water from a reservoir the surface of which is 280 ft. above the level of the ground. What will be the pressure in pounds per square inch at a tap 50 ft. above the ground? (Call 1 ft. = 30 cm. and 1 kg. = 2.2 lb.)

Pascal's Law

66. Transmission of pressure by liquids. From the fact that pressure within a free liquid depends simply upon the depth and density of the liquid, it is possible to deduce a very surprising conclusion, which was first stated by the famous French scientist, mathematician, and philosopher, Pascal (1623–1662).

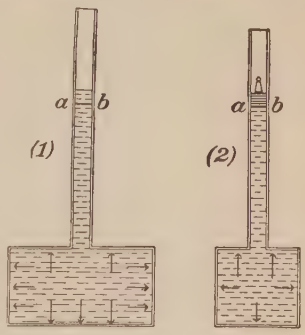


Fig. 36. Proof of Pascal's law

Let us imagine a vessel of the shape shown in Fig. 36, (1), to be filled with water up to the level ab. For simplicity let the upper portion be assumed to be 1 sq. cm. in cross section. Since the density of water is 1, the force with which it presses against any square centimeter of the interior surface which is h cm. beneath the level ab is h grams. Now let one gram of water (i.e. 1 cc.) be poured into the tube. Since each square centimeter of surface which was before h cm. beneath the level of the water in the tube is now $h + 1$ cm. beneath this level, the new pressure which the

water exerts against it is $h + 1$ g.; i.e. applying 1 g. of force to the square centimeter of surface ab has added 1 g. to the force exerted by the liquid against each square centimeter of the interior of the vessel. Obviously it can make no difference whether the pressure which was applied to the surface ab was due to a weight of water or to a piston carrying a load, as in Fig. 36, (2), or to any other cause whatever. We thus arrive at Pascal's conclusion that *pressure applied anywhere to a body of confined liquid is transmitted by the liquid so as to act with undiminished force on every square centimeter of the containing vessel.*

67. Multiplication of force by the transmission of pressure by liquids. Pascal himself pointed out that with the aid of the principle stated above we ought to be able to transform a very small force into one of unlimited magnitude. Thus if the area of the cylinder ab, Fig. 37, is 1 sq. cm., while that of the cylinder AB is 1000 sq. cm., a force of 1 kg. applied to ab would be transmitted by the liquid so as to act with a force of 1 kg. on each square centimeter of the surface AB. Hence the total upward force exerted against the piston AB by the one kilo applied at ab would be 1000 kg. Pascal's own words are as follows: "A vessel full of water is a new principle in mechanics, and a new machine for the multiplication of force to any required extent, since one man will by this means be able to move any given weight."

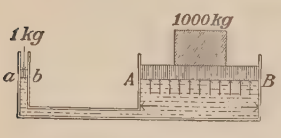


FIG. 37. Multiplication of force by transmission of pressure

68. The hydraulic press. The experimental proof of the correctness of the conclusions of the preceding paragraph is furnished by the hydraulic press, an instrument now in common use for subjecting to enormous pressures paper, cotton, etc.; for punching holes through iron plates, testing the strength of iron beams, extracting oil from seeds, making dies, embossing metal, etc.

Such a press is represented in section in Fig. 38. As the small piston p is raised, water from the cistern C enters the piston chamber through the valve v. As soon as the down stroke begins the valve v closes, the valve v' opens, and the pressure applied on the piston p is transmitted through the tube K to the large reservoir, where it acts on the large cylinder P with a force which is as many times that applied to p as the area of P is times the area of p.

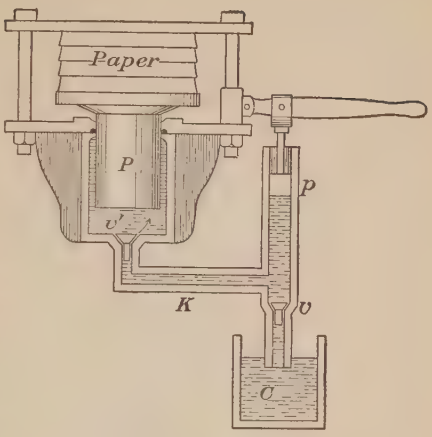


Fig. 38. Diagram of a hydraulic press

Hand presses similar to that shown in Fig. 39 are often made which are capable of exerting a compressing force of from 500 to 1000 tons.

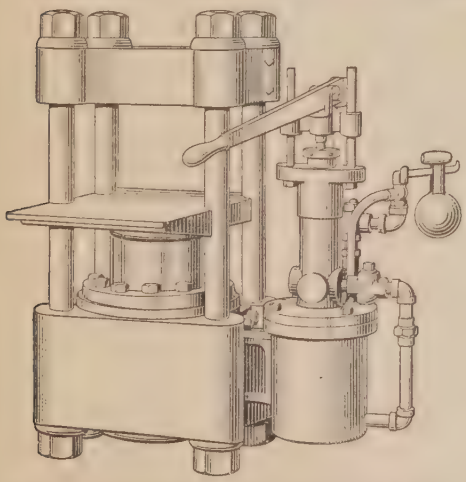


Fig. 39. Seventy-five-ton hydraulic die press

69. No gain in the product of force times distance. It should be noticed that, while the force acting on AB (Fig. 37) is 1000 times as great as the force acting on ab, the distance through which the piston AB is pushed up in a given time is but $\frac{1}{1000}$ of the distance through which the piston ab moves down. For, forcing ab down a distance of 1 cm. crowds but 1 cc. of water over into the large

cylinder, and this additional cubic centimeter can raise the level of the water there but $\frac{1}{1000}$ cm. We see therefore that the product of the force acting by the distance moved is precisely the same at both ends of the machine. This important conclusion will be found in our future study to apply to all machines.

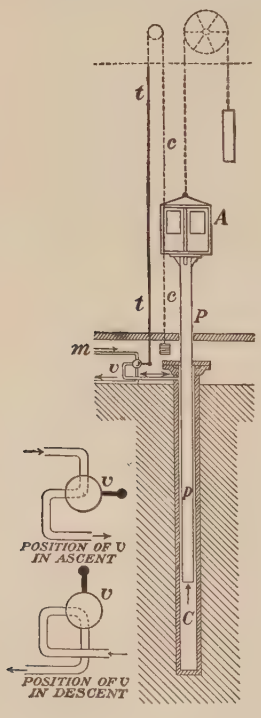


FIG. 40

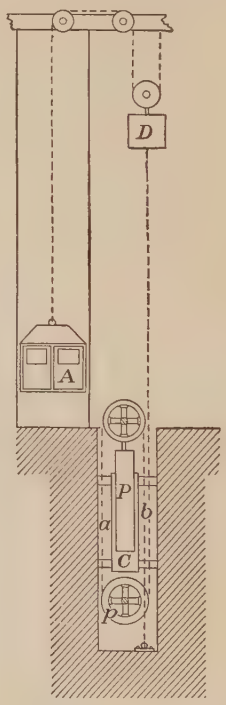


FIG. 41

Diagrams of hydraulic elevators

70. The hydraulic elevator. Another very common application of the principle of transformation of pressure by liquids is found in the hydraulic elevator. The simplest form of such an elevator is shown in Fig. 40. The cage A is borne on the top of a long piston P which runs in a cylindrical pit C of the same depth as the height to which the

carriage must ascend. Water enters the pit either directly from the water mains m of the city's supply, or, if this does not furnish sufficient pressure, from a special reservoir on top of the building. When the elevator boy pulls *up* on the cord cc, the valve v opens so as to make connection from m into C. The elevator then ascends. When cc is pulled *down*, v turns so as to permit the water in C to escape into the sewer. The elevator then descends.

Where speed is required the motion of the cylinder is communicated indirectly to the cage by a system of pulleys like that shown in Fig. 41. With this arrangement a foot of upward motion of the cylinder P causes the counterpoise D of the cage to descend 2 ft., for it is clear from the figure that when the cylinder goes up 1 ft. enough rope must be pulled over the fixed pulley p to lengthen each of the two strands a and b 1 ft. Similarly, when the counterpoise descends 2 ft. the cage ascends 4 ft. Hence the cage moves four times as fast and four times as far as the cylinder. The elevators in the Eiffel Tower in Paris are of this sort. They have a total travel of 420 ft. and are capable of lifting 50 people 400 ft. per minute.

71. City water supply. Fig. 42 illustrates the method by which a city is often supplied with water from a distant source. The aqueduct from the lake a passes under a road r, a brook b, a hill H, and into a reservoir e, from which it is forced by the pump p into the standpipe P, whence it is distributed to

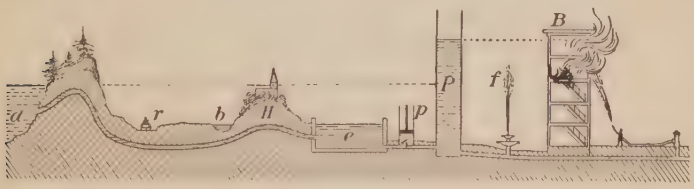


Fig. 42. City water supply from lake

the houses of the city. If a static condition prevailed in the whole system, then the water level in e would of necessity be the same as that in a, and the level in the pipes of the building B would be the same as that in the standpipe P. But when the water is flowing the friction of the mains causes the level in e

to be somewhat less than that in a, and that in B less than that in P. It is on account of the friction both of the air and of the pipes that the fountain f does not actually rise nearly as high as the ideal limit shown in the figure (see dotted line).

72. Artesian wells. It is in the principle of transmission of pressure by liquids that artesian wells find their explanation. Fig. 43 is an ideal section of what geologists call an artesian basin. The stratum A is composed of some porous material such as sand, open-textured sandstone, or broken rock, through which the water can percolate easily. Above and below it are strata C and B of clay, slate, or some other material impervious to water. The porous layer is filled with water which finds entrance at the outcropping margins. As soon as a boring

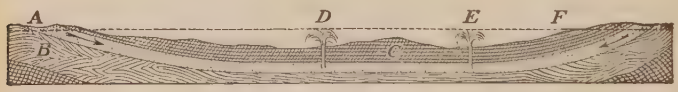


Fig. 43. Artesian wells

is made through the layer C the water gushes forth because of the transmission of pressure from the higher levels. A well of this sort exists near Kissingen, Germany, which is 1800 ft. deep and which throws a stream of water 58 ft. high. The deepest artesian well in existence is near Berlin. It is 4194 ft. deep. Many artesian wells have been bored in the desert of Sahara and an abundant water supply found at a depth of 200 ft. Great numbers of artesian wells exist in the United States. Notable ones are located at Chicago, Louisville (Kentucky), and Charleston (South Carolina). The artesian basins in which the wells are found are often a hundred miles or more in width.

QUESTIONS AND PROBLEMS

1. If the water pressure in the city mains is 80 lb. to the square inch, how high above the town is the top of the water in the standpipe? (1 cu. ft. of water weighs 62.3 lb.)

2. To what total force in pounds is a diver subjected who dives to an average depth of 10 ft., if the area of his body is 20 sq. ft.?

3. The water pressure in the city mains is 80 lb. to the square inch. The diameter of the piston of a hydraulic elevator of the type shown in Fig. 40 is 10 in. If friction could be disregarded, how heavy a load could the elevator

lift? If 30 per cent of the ideal value must be allowed for frictional loss, what load will the elevator lift?

4. Fig. 44 represents an instrument commonly known as the hydrostatic bellows. If the base C is 20 in. square and the tube is filled with water to a depth of 5 ft. above the top of C, what is the value of the weight which the bellows can support?

5. A hydraulic press having a piston 1 in. in diameter exerts a force of 10,000 lb. when 10 lb. are applied to this piston. What is the diameter of the large piston?

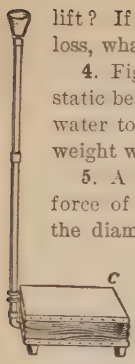


Fig. 44. Hydrostatic bellows

The Principle of Archimedes[1]

73. Loss of weight of a body in a liquid. The preceding experiments have shown that an upward force acts against the bottom of any body immersed in a liquid. If the body is a boat, cork, piece of wood, or any body which floats, it is clear that, since it is in equilibrium, this upward force must be equal to the weight of the body. Even if the body does not float, everyday observation shows that it still loses a portion of its natural weight, for it is well known that it is easier to lift a stone under water than in air; or again, that a man in a bath tub can support his whole weight by pressing lightly against the bottom with his fingers. It was indeed this very observation which first led the old Greek philosopher, Archimedes (287–212 B.C.), to the discovery of the exact law which governs the loss of weight of a body in a liquid.

Hiero, the tyrant of Syracuse, had ordered a gold crown made, but suspected that the artisan had fraudulently used silver as well as gold in its construction. He ordered Archimedes to discover whether or not this were true. How to do so without destroying the crown was at first a puzzle to the old philosopher. While in his daily bath, noticing the loss of weight of his own

[1] A laboratory exercise on the experimental proof of Archimedes' principle should precede this discussion. See e.g. Experiment 7 of the authors' manual.

body, it suddenly occurred to him that *any body immersed in a liquid must lose a weight equal to the weight of the displaced liquid.* He is said to have jumped at once to his feet and rushed through the streets of Syracuse crying, "Eureka, eureka!" (I have found it, I have found it!)

74. Theoretical proof of Archimedes' principle. It is probable that Archimedes, with that faculty which is so common among men of great genius, saw the truth of his conclusion without going through any logical process of proof. Such a proof, however, can easily be given. Thus, since the upward force on the bottom of the block *abcd* (Fig. 45) is equal to the weight of the column of liquid *obce*, and since the downward force on the top of this block is equal to the weight of the column of liquid *oade*, it is clear that the upward force must exceed the downward force by the weight of the column of liquid *abcd*; i.e. *the buoyant force exerted by the liquid is exactly equal to the weight of the displaced liquid.*

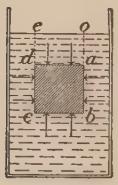
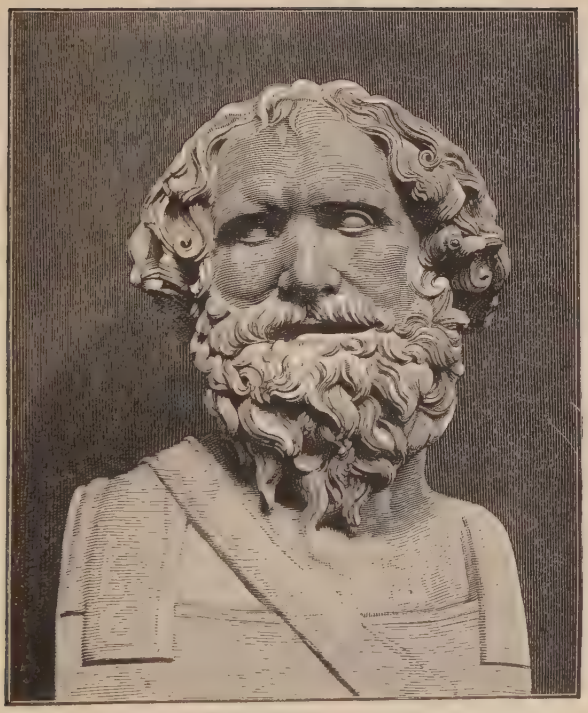


Fig. 45. Proof that an immersed body is buoyed up by a force equal to the weight of the displaced liquid

The reasoning is exactly the same no matter what may be the nature of the liquid in which the body is immersed, nor how far the body may be beneath the surface. Further, if the body weighs more than the liquid which it displaces, it must sink, for it is urged down with the force of its own weight, and up with the lesser force of the weight of the displaced liquid. But if it weighs less than the displaced liquid, then the upward force due to the displaced liquid is greater than its own weight, and consequently it must rise to the surface. When it reaches the surface the downward force on the top of the block, due to the liquid, becomes zero. The body must, however, continue to rise until the upward force on its bottom is equal to its own weight. But this upward force is always equal to the weight of



ARCHIMEDES (287–212 B.C.)

The celebrated geometrician of antiquity; lived at Syracuse, Sicily; first made a determination of π and computed the area of the circle; discovered the laws of the lever and was author of the famous saying, "Give me where I may stand and I will move the world"; discovered the laws of flotation; invented various devices for repelling the attacks of the Romans in the siege of Syracuse; on the capture of the city, while in the act of drawing geometrical figures in a dish of sand (the blackboard of that day), he was killed by a Roman soldier to whom he cried out, "Don't spoil my circle." (Bust in Naples Museum.)

the displaced liquid, i.e. to the weight of the column of liquid *mbcn* (Fig. 46).

Hence *a floating body must displace its own weight of the liquid in which it floats.* This statement is embraced in the original statement of Archimedes' principle, for a body which floats has lost its whole weight.

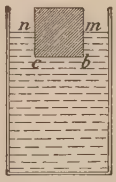


FIG. 46. Proof that a floating body is buoyed up by a force equal to the weight of the displaced liquid

75. Experimental proof of Archimedes' principle. To test experimentally the truth of Archimedes' principle, we weigh a body of known volume first in air, then in some liquid (Fig. 47). If the principle is correct, the difference between these two weights should be exactly equal to the product of the volume of the body by the density of the liquid, since this product is the weight of the displaced liquid. If the liquid is water of density 1, then the loss of weight should be numerically equal to the volume of the body.

To test the principle for a floating body, we measure the immersed portion of the volume of a floating cylinder like that shown in Fig. 49. If the liquid is water, this volume should be numerically equal to the weight of the floating cylinder. Tests of this sort are best performed by the pupil in the laboratory.

FIG. 47. Method of weighing a body under water

76. Density of a heavy solid. The density of a body is by definition its mass divided by its volume. It is always possible to obtain the mass of a body by weighing it, but it is not, in general, possible to obtain the volume of an irregular body from measurements of its dimensions. Archimedes' principle, however, furnishes an accurate and easy method for obtaining the volume of any solid, however irregular, for by the preceding paragraph this volume is *numerically* equal to the loss of weight in water. Hence the equation which defines density, namely,

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

becomes in this case

$$\text{Density} = \frac{\text{Mass}}{\text{Loss of weight in water}}. \tag{4}$$

77. Density of a solid lighter than water. If the body is too light to sink of itself, we may still obtain its volume by forcing it beneath the surface with a sinker. Thus suppose w_1 represents the weight on the right pan of the balance when the body is in air and the sinker in water, as in Fig. 48; while w_2 is the weight on the right pan when both body and sinker are under water. Then $w_1 - w_2$ is obviously the buoyant effect of the water on the body alone, and is therefore equal to the weight of the displaced water which is numerically equal to the volume of the body.

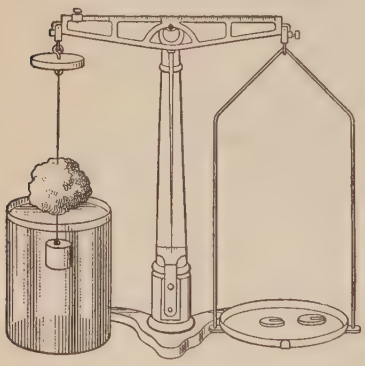


FIG. 48. Method of finding density of a light solid

78. Density of liquids by hydrometer method. Archimedes' principle also furnishes an easy method for finding the density of any liquid. For suppose a uniform cylinder like that of Fig. 49 is floated in water and is found to sink a distance l_1; then, if A represents the area of the cross section of the cylinder, the volume of the displaced water is Al_1; and since the density of water is 1, the weight of the displaced water is also Al_1. By Archimedes' principle this is equal to the weight

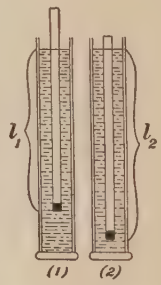


FIG. 49. Method of finding density of a liquid

of the floating body. Next suppose that the same cylinder is floated in the liquid whose density d_2 is sought [Fig. 49, (2)]. It will now sink some distance l_2. The volume of the displaced liquid will be Al_2, and its weight will be Al_2d_2. By Archimedes' principle this is again equal to the weight w of the floating body. Hence

$$Al_2d_2 = Al_1, \text{ or } d_2 = \frac{l_1}{l_2}; \tag{5}$$

i.e. *the density of the unknown liquid is simply the ratio of the depth l_1, which the cylinder sinks in water, to the depth l_2, which it sinks in the unknown liquid.*

79. Commercial form of hydrometer. The commercial constant-weight hydrometer such as is now in common use for testing the density of alcohol, milk, acids, sugar solutions, etc., instead of being a cylinder like that shown in Fig. 49, is of the form shown in Fig. 50. The stem is calibrated so that the density of any liquid may be read upon it directly. The advantage of this form over that of Fig. 49 is that it is much more suitable for detecting very slight differences between the densities of two liquids. The reason for this will be clear when it is remembered that the instrument must always sink until it displaces its own weight of the liquid, and that if the stem is made very narrow in comparison with the lower portion, the sinking of a considerable portion of the stem will add but very little to the total volume of the liquid displaced. By making the cylinder exceedingly long the same sensitiveness could of course be obtained with the cylindrical form, but it would then be inconvenient to use.

Fig. 50. Constant-weight hydrometer

80. Density of liquids by "loss-of-weight" method. If any heavy body is weighed first in air, then in water, and lastly in a liquid of unknown density d_2, then, since the weight of the water displaced by the body is its volume V times its density 1, and since the weight of the unknown liquid displaced is the same volume V times the density d_2, we have by Archimedes' principle, if L_1 represents the loss of weight in water and L_2 the loss in the unknown liquid,

$$L_1 = V \times 1, \text{ and } L_2 = Vd_2.$$

Dividing the second equation by the first gives

$$d_2 = \frac{L_2}{L_1}; \tag{6}$$

i.e. the density of the unknown liquid is the loss of weight in that liquid divided by the loss of weight in water.[1]

QUESTIONS AND PROBLEMS

1. The hull of a modern battle ship is made almost entirely of steel, its walls being of steel plates from 6 to 18 in. thick. Explain how it can float.

2. If a barge 30 ft. by 15 ft. sank 4 in. when an elephant was taken aboard, what was the elephant's weight?

3. Will the water line of a boat rise or fall as it passes from fresh into salt water?

4. If the density of ice is .917 and that of sea water 1.026, what is the total height of a mass of ice of uniform cross section which rises 100 ft. above water? In general, what fraction of an iceberg is above water?

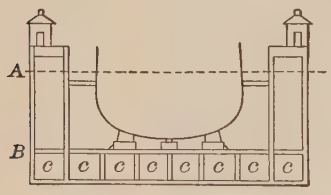


Fig. 51. Floating dock

5. If each boat of a pontoon bridge is 100 ft. long and 75 ft. wide at the water line, how much will it sink when a locomotive weighing 100 tons passes over it?

6. A block of wood 10 in. high sinks 6 in. in water. Find the density of the wood.

7. If this block sank 7 in. in oil, what would be the density of the oil?

8. To what depth would it sink in turpentine of density .87?

9. A graduated glass cylinder contains 190 cc. of water. An egg weighing 40 g. is dropped into the glass; it sinks to the bottom and raises the water to the 225 cc. mark. Find the density of the egg.

10. A cube of iron 10 cm. on a side weighs 7500 g. What will it weigh in alcohol of density .82?

11. A floating dock is shown in Fig. 51. When the chambers *c* are filled with water the dock sinks until the water line is at *A*. The vessel is then floated into the dock. As soon as it is in place the water is pumped from the chambers until the water line is as low as *B*. Workmen can then

[1] Laboratory experiments in determination of densities of solids and liquids should follow or accompany the discussion of this chapter. See e.g. Experiments 8 and 9 of the authors' manual.

get at all parts of the bottom. If each of the chambers is 10 ft. high and 10 ft. wide, what must be the length of the dock if it is to be available for the *Celtic*, of 21,000 tons weight?

Fig. 52

12. The density of stone is about 2.5. If a boy can lift 120 lb., how heavy a stone can he lift to the surface of a pond?

13. A block of cork weighs 60 g. A sinker which weighs 300 g. in water will just keep the cork immersed. What is the density of the cork?

14. A diver with his diving suit weighs 100 kg. It requires 15 kg. of lead to sink him. If the density of lead is 11.3, what is the volume of the diver and his suit?

15. A body loses 25 g. in water, 23 g. in oil, and 20 g. in alcohol. Find the density of the oil and of the alcohol.

16. A cubical block of iron (density 7.8) floats on mercury (density 13.6). What fractional part of the iron is immersed?

17. A platinum ball weighs 330 g. in air, 315 g. in water, and 303 g. in sulphuric acid. Find the density of the platinum, the density of the acid, and the volume of the ball.

Fig. 53

18. Fig. 52 shows a common carpenter's level. The tube containing the alcohol and air bubble is curved, as in Fig. 53, although this may not be apparent to the eye. Why is this necessary?

19. What fraction of the total volume of an irregular block of wood of density .6 will float above the surface of alcohol of density .8?

CHAPTER IV

PRESSURE IN AIR

Barometric Phenomena

81. The weight of air. To ordinary observation air is scarcely perceptible. It appears to have no weight and to offer no resistance to bodies passing through it. But if a bulb be balanced as in Fig. 54, then removed and filled with air under pressure by a few strokes of a bicycle pump, it will be found, when again placed on the balance, to be heavier than it was before. On the other hand, if the bulb be connected with an air pump and exhausted, it will be found to have lost weight. Evidently, then, air can be put into and taken out of a vessel, weighed, and handled, just like a liquid or a solid.

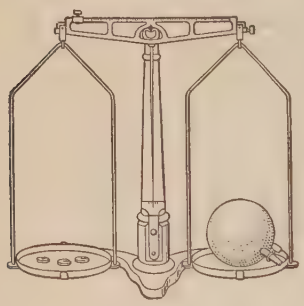


Fig. 54. Proof that air has weight

We are accustomed to say that bodies are "as light as air," yet careful measurement shows that it takes but 12 cu. ft. of air to weigh a pound, so that a single large room contains more air than an ordinary man can lift. Thus the air in a room 60 ft. by 30 ft. by 15 ft. weighs more than a ton. The exact weight of air at the freezing temperature and under normal atmospheric conditions is .001293 g. per cc., i.e. 1.293 g. per liter.

82. Proof that air exerts pressure. Since air has weight, it is to be inferred that it, like a liquid, exerts force against any surface immersed in it. The following experiments prove this.

Let a rubber membrane be stretched over a glass vessel, as in Fig. 55. As the air is exhausted from beneath the membrane the latter will be observed to be more and more depressed until it will finally burst under the pressure of the air above.

Again, let a tin can be partly filled with water and the water boiled. The air will be expelled by the escaping steam. While the boiling is still going on let the can be tightly corked, then placed in a sink or tray and cold water poured over it. The steam will be condensed and the weight of the air outside will crush the can.

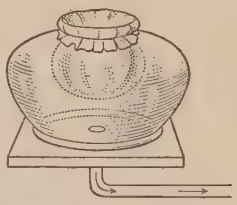


Fig. 55. Rubber membrane stretched by weight of air

83. Cause of the rise of liquids in exhausted tubes. If the lower end of a long tube be dipped into water and the air exhausted from the upper end, water will rise in the tube. We prove the truth of this statement every time we draw lemonade through a straw. The old Greeks and Romans explained such phenomena by saying that "nature abhors a vacuum," and this explanation was still in vogue in Galileo's time. But in 1640 the Duke of Tuscany had a deep well dug near Florence, and found to his surprise that no water pump which could be obtained would raise the water higher than about 32 feet above the level in the well. When he applied to the aged Galileo for an explanation the latter replied that evidently "nature's horror of a vacuum did not extend beyond thirty-two feet." It is quite likely that Galileo suspected that the pressure of the air was responsible for the phenomenon, for he had himself proved before that air had weight, and, furthermore, he at once devised another experiment to test, as he said, the "power of a vacuum." He died in 1642 before the experiment was performed, but suggested to his pupil, Torricelli, that he continue the investigation.

84. Torricelli's experiment. Torricelli argued that if water would rise 32 ft., then mercury, which is about 13 times as

heavy as water, ought to rise but $\frac{1}{13}$ as high. To test this inference he performed in 1643 the following famous experiment.

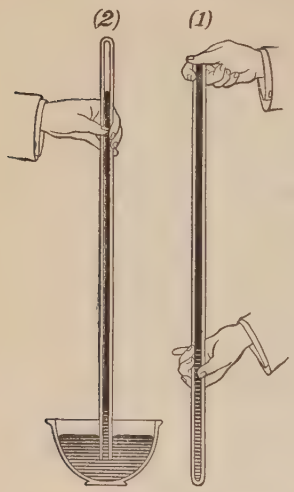


Fig. 56. Torricelli's experiment

Let a tube about 4 ft. long, which is sealed at one end, be completely filled with mercury, as in Fig. 56, (1), then closed with the thumb and inverted, and the bottom then immersed in a dish of mercury, as in Fig. 56, (2). When the thumb is removed from the bottom of the tube, the mercury will fall away from the upper end of the tube in spite of the fact that in so doing it will leave a vacuum above it, and its upper surface will in fact stand about $\frac{1}{13}$ of 32 ft., i.e. 29 or 30 in. above the mercury in the dish.

Torricelli concluded from this experiment that the rise of liquids in exhausted tubes is due to an outside pressure exerted by the atmosphere on the surface of the liquid, and not to any mysterious sucking power created by the vacuum.

85. Further decisive tests. An unanswerable argument in favor of this conclusion will be furnished if the mercury in the tube falls as soon as the air is removed from above the surface of the mercury in the dish.

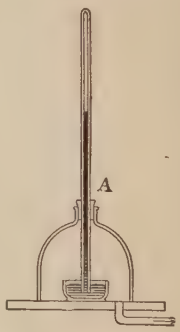


Fig. 57. Barometer falls when air pressure on the mercury surface is reduced

To test this point, let the dish and tube be placed on the table of an air pump, as in Fig. 57, the tube passing through a tightly fitting rubber stopper A, in the bell jar. As soon as the pump is started the mercury in the tube will, in fact, be seen to fall. As the pumping is continued it will fall nearer and nearer to the level in the dish, although it will not usually reach it for

the reason that an ordinary vacuum pump is not capable of producing as good a vacuum as that which exists in the top of the tube. As the air is allowed to return to the bell jar the mercury will rise in the tube to its former level.

86. Amount of the atmospheric pressure. Torricelli's experiment shows exactly how great the atmospheric pressure is, since this pressure is able to balance a column of mercury of definite length. In accordance with Pascal's law the downward pressure exerted by the atmosphere on the surface of the mercury in the dish (Fig. 58) is transmitted as an exactly equal upward pressure on the layer of mercury inside the tube at the same level as the mercury outside. But the downward pressure at this point within the tube is equal to hd, where d is the density of mercury and h is the depth below the surface b. Since the average height of this column at sea level is found to be 76 cm., and since the density of mercury is 13.6, the downward pressure inside the tube at a is equal to 76 times 13.6 or 1033.6 g. per sq. cm. Hence the atmospheric pressure acting on the surface of the mercury in the dish is 1033.6 g., or roughly 1 kg., per sq. cm. This amounts to about 15 lb. per sq. in.

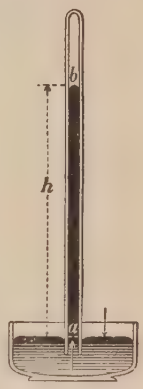


Fig. 58. Air column to top of atmosphere balances the mercury column ab

87. Pascal's experiment. Pascal thought of another way of testing whether or not it were indeed the weight of the outside air which sustains the column of mercury in an exhausted tube. He reasoned that, since the pressure in a liquid diminishes on ascending toward the surface, atmospheric pressure ought also to diminish on passing from sea level to a mountain top. As no mountain existed near Paris, he carried Torricelli's apparatus to the top of a high tower and found, indeed, a slight fall in the

height of the column of mercury. He then wrote to his brother-in-law Perrier, who lived near Puy de Dome, a mountain in the south of France, and asked him to try the experiment on a larger scale. Perrier wrote back that he was "ravished with admiration and astonishment" when he found that on ascending 1000 m. the mercury sank about 8 cm. in the tube. This was in 1648, five years after Torricelli's discovery.

Fig. 59. The barometer

At the present day geological parties actually ascertain differences in altitude by observing the change in the barometric pressure as they ascend or descend. A fall of 1 mm. in the column of mercury corresponds to an ascent of about 12 m.

88. The barometer. The modern barometer (Fig. 59) is essentially nothing more nor less than Torricelli's tube. Taking a barometer reading consists simply in accurately measuring the height of the mercury column. This height varies from 73 to 76.5 cm. in localities which are not far above sea level, the reason being that disturbances in the atmosphere affect the pressure at the earth's surface in the same way in which eddies and high waves in a tank of water would affect the liquid pressure at the bottom of the tank.

The barometer does not directly foretell the weather, but it has been found that a low or rapidly falling pressure is usually accompanied, or soon followed, by stormy conditions. Hence the barometer, although not an infallible weather prophet, is nevertheless of considerable assistance in forecasting weather conditions some hours ahead. Further, by comparing at a central station the telegraphic reports of barometer readings made every few hours at stations all over the country, it is possible to determine in what direction the atmospheric eddies which

cause barometer changes and stormy conditions are traveling, and hence to "forecast" the weather even a day or two in advance.

89. The first barometers. Torricelli actually constructed a barometer not essentially different from that shown in Fig. 59, and used it for observing changes in the atmospheric pressure; but perhaps the most interesting of the early barometers was that set up about 1650 by the famous old German physicist Otto von Guericke of Magdeburg (1602–1686). He used for his barometer a water column the top of which passed through the roof of his house. A wooden image which floated on the upper surface of the water appeared above the house top in fair weather but retired from sight in foul, a circumstance which led his neighbors to charge him with being in league with Satan.

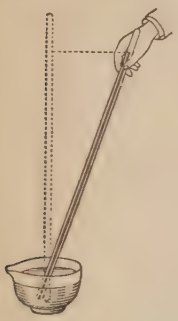


Fig. 60. Effect of inclining a barometer tube

90. Effect of inclining a barometer. If a barometer tube is inclined in the manner shown in Fig. 60, the top of the mercury will be found to remain always in the same horizontal plane. Explain, remembering that pressure equals height times density (Fig. 35).

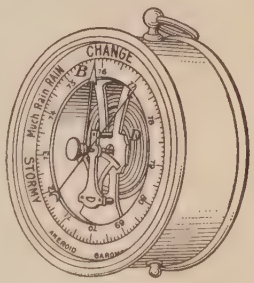


Fig. 61. An aneroid barometer

91. The aneroid barometer. Since the mercurial barometer is somewhat long, and inconvenient to carry, geological and surveying parties commonly use an instrument called the *aneroid barometer* (Fig. 61). It consists essentially of an air-tight cylindrical box *D*, the top of which is a metallic diaphragm which bends slightly under the influence of change in the atmospheric pressure. This motion of the top of the box is multiplied by a delicate system of levers and communicated to a hand *B* which moves over a dial whose readings are made to correspond to the readings of a mercury barometer. These instruments are made so sensitive as to indicate a change in pressure when they are moved no farther than from a table to the floor.

QUESTIONS AND PROBLEMS

1. Find the weight of the air contained in a room 18 × 11 × 4.5 meters.

2. If a barometer were sunk in water so that the lower mercury surface stood one meter below the surface of the water, what would be the reading of the barometer, the barometric height at the surface being 75.42 cm.?

3. If a circular piece of wet leather, having a string attached to the middle, is pressed down on a flat smooth stone (as in Fig. 62), the latter may often be lifted by pulling on the string. Explain.

Fig. 62

4. Why does not the ink run out of a pneumatic inkstand like that shown in Fig. 63?

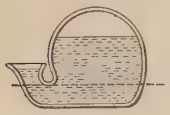


Fig. 63

5. Set fire to a loose roll of paper which floats on a saucer of water, and then quickly place a tumbler over the burning paper, the edges of the tumbler extending beneath the surface of the water. The water will be sucked up out of the saucer into the tumbler. Explain.

6. If the variation of the height of a mercury barometer is 2 in., how far did the image rise and fall in Otto von Guericke's water barometer (see § 89)?

7. If a tumbler is filled level full of water, and a piece of writing paper is placed over the top, it may be inverted, as in Fig. 64, without spilling the water. Explain. What is the function of the paper?

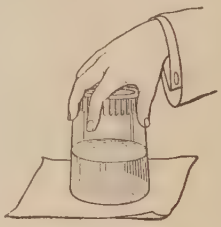


Fig. 64

8. Magdeburg hemispheres (Fig. 65) are so called because they were invented by Otto von Guericke, who was mayor of Magdeburg. When the lips of the hemispheres are placed in contact and the air exhausted from between them, it is found very difficult to pull them apart. Why?

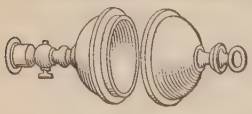
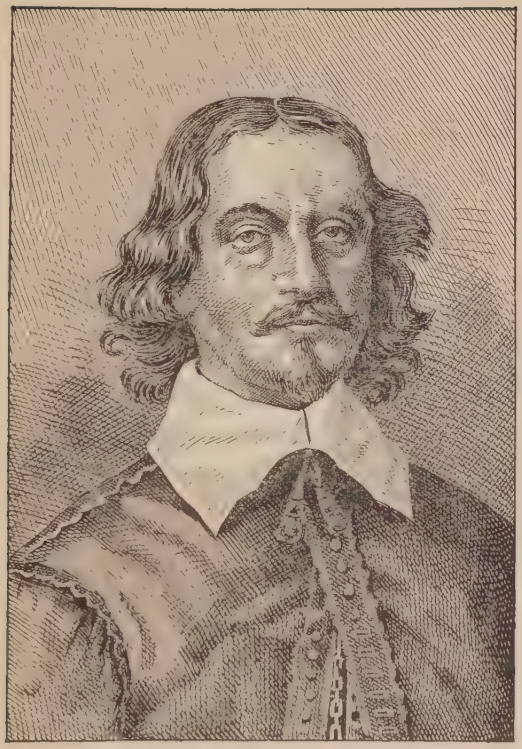


Fig. 65. Magdeburg hemispheres

9. Von Guericke's original hemispheres are still preserved in the museum at Berlin. Their interior diameter is 22 inches. On the cover of the book which describes his experiments is a picture which represents 4 teams of horses on each side of the hemispheres trying to separate them. The experiment was actually performed in this way before the German emperor Ferdinand III. If the air was all removed from the interior of the hemispheres, what force in pounds was in fact required to pull them apart? (Find the atmospheric force on a circle of 11 in. radius.)



Otto von Guericke (1602-1686)

German physicist, astronomer, and man of affairs; mayor of Magdeburg; invented the air pump in 1650, and performed many new experiments with liquids and gases; discovered electrostatic repulsion; constructed the famous Magdeburg hemispheres.

state of readiness to expand as soon as the pressure is diminished. The ordinary air of the room will expand in the same way if the pressure to which it is subjected is relieved.

Thus let a bladder or a toy balloon be filled with air under ordinary conditions and then tied up air-tight and placed under the receiver of an air pump. As soon as the pump is set into operation the inside air will expand with sufficient force to burst the bladder, or to greatly distend the balloon, as shown in Fig. 66.

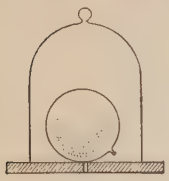


Fig. 66

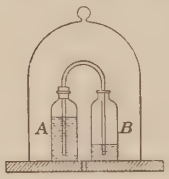


Fig. 67

Illustrations of the expansibility of air

Again, let two bottles be arranged as in Fig. 67, one being stoppered air-tight, while the other is uncorked. As soon as the two are placed under the receiver of an air pump and the air exhausted, the water in *A* will pass over into *B*. When the air is readmitted to the receiver the water will flow back. Explain.

94. Why hollow bodies are not crushed by atmospheric pressure. The preceding experiments show why the walls of hollow bodies are not crushed in by the enormous forces which the weight of the atmosphere exerts against them. For the air inside such bodies presses their walls out with as much force as the outside air presses them in. In the experiment of § 82 the inside air was removed by the escaping steam. When this steam was condensed by the cold water, the inside pressure became very small and the outside pressure then crushed the can. In the experiment shown in Fig. 66 it was the outside pressure which was removed by the air pump, and the pressure of the inside air then burst the bladder.

95. Boyle's law. The first man to investigate the exact relation between the change in the pressure exerted by a confined body of air and its change in volume was Robert Boyle, an Irishman (1626–1691). We shall repeat a modified form of his experiment much more carefully in the laboratory; but

the following will illustrate the method by which he discovered one of the most important laws of physics.

Let mercury be poured into a bent glass tube until it stands at the same level in the closed arm AC as in the open arm BD (Fig. 68). There is now confined in AC a certain volume of air under the pressure of one atmosphere. Call this pressure P_1. Let the length AC be measured and called V_1. Then let mercury be poured into the long arm until the level in this arm is as many centimeters above the level in the short arm as there are centimeters in the barometer height. The confined air is now under a pressure of two atmospheres. Call it P_2. Let the new volume A_1C ($= V_2$) be measured. It will be found to be just half its former value.

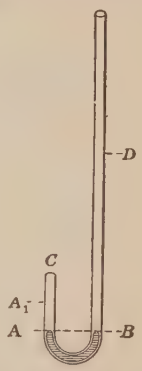


FIG. 68. Method of proving Boyle's law

Hence we learn that doubling the pressure exerted upon a body of air halves its volume. If we had tripled the pressure, we should have found the volume reduced to one third its initial value, etc. That is, *the pressure which a given quantity of air at constant temperature exerts against the walls of the containing vessel is inversely proportional to the volume occupied.* This is algebraically stated thus

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}, \text{ or } P_1V_1 = P_2V_2. \tag{1}$$

This is Boyle's law. It may also be stated in slightly different form. Doubling, tripling, or quadrupling the pressure must double, triple, or quadruple the *density*, since the volume is made only one half, one third, or one fourth as much, while the mass remains unchanged. Hence *the pressure which air exerts is directly proportional to its density, or, algebraically,*

$$\frac{P_1}{P_2} = \frac{D_1}{D_2}.^{1} \tag{2}$$

[1] A laboratory experiment on Boyle's law should follow this discussion. See e.g. Experiment 10, authors' manual.

96. Extent and character of the earth's atmosphere. From the facts of compressibility and expansibility of air we may know that the air, unlike the sea, must become less and less dense as we ascend from the bottom toward the top. Thus at the top of Mont Blanc, where the barometer height is but 38 cm., or one half of its value at sea level, the density also must, by Boyle's law, be just one half as much as at sea level.

No one has ever ascended higher than 7 mi., which was approximately the height attained in 1862 by the two daring English aëronauts, Glasier and Coxwell. At this altitude the barometric height is but about 7 in. and the temperature about $-60°$ F. Both aëronauts lost the use of their limbs and Mr. Glasier became unconscious. Mr. Coxwell barely succeeded in grasping with his teeth the rope which opened a valve and caused the balloon to descend. Again, on July 31, 1901, the French aëronaut M. Berson rose without injury to a height of about 7 mi. (35,420 ft.), his success being due to the artificial inhalation of oxygen.

By sending up self-registering thermometers and barometers in balloons which burst at great altitudes, the instruments being protected by parachutes from the dangers of rapid fall, the atmosphere has been explored to a height of 22,290 m. (13.8 mi.), this being the height attained on December 4, 1902, by a little rubber balloon 76 in. in diameter which was sent up from the Strasburg (Germany) observatory. These extreme heights are calculated from the indications of the self-registering barometers. Fig. 69 shows, in the right-hand column, the densities of air at various heights in terms of its density at sea level. In the next column are shown the corresponding barometer heights in inches, while the left-hand column indicates heights in miles.

It will be seen that at a height of 35 mi. the density is estimated to be but $\frac{1}{30000}$ of its value at sea level. By calculating how far below the horizon the sun must be when the last

traces of color disappear from the sky, we find that at a height as great as 45 mi. there must be air enough to reflect some light. How far beyond this an extremely rarefied atmosphere may extend, no one knows. It has been estimated at all the way from 100 to 500 mi. These estimates are based on observations of the height at which meteors first become visible, on the

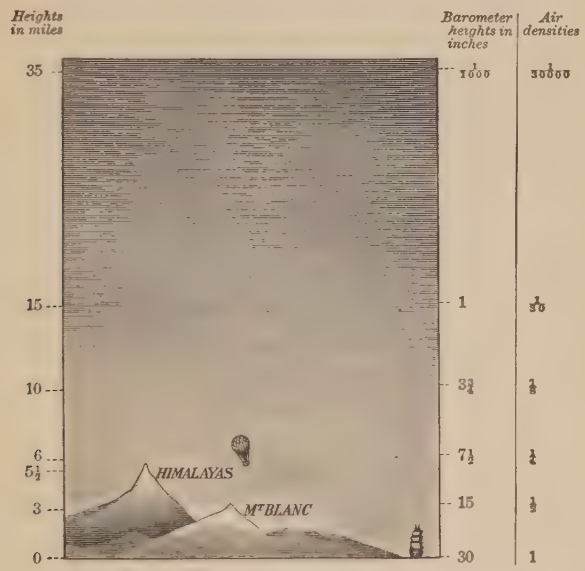


FIG. 69. Extent and character of atmosphere

height of the aurora borealis, and on the darkening of the surface of the moon just before it is eclipsed by the shadow of the solid earth.

97. Height of the "homogeneous atmosphere." Although, then, we cannot tell to what height the atmosphere extends, we do know with certainty that the weight of a column of air 1 sq. cm. in cross section and reaching from the earth's surface to the extreme limits of the atmosphere will just balance a column of mercury 76 cm. high, for this was shown by Torricelli's

experiment. Since 1 cc. of air at the earth's surface weighs about 1.2 mg., i.e. since the density of air is about .0012, or one eight-hundredth that of water, and since mercury is about 13.6 times as heavy as water, it follows that if the air had the same density at all altitudes which it has at the earth's surface, its height would be $76 \times 13.6 \times 800$ cm., i.e. 8.2 km., or about 5 mi. The tops of the Himalayas would therefore rise above it. This height of 5 mi., which is the height to which the air would extend if it, like the ocean, had the same density throughout, is called the *height of the homogeneous atmosphere.*

98. Density of air below sea level. The same cause which makes air diminish rapidly in density as we ascend above sea level must produce a rapid increase in its density as we descend below this level. It has been calculated that if a boring could be made in the earth 35 mi. deep, the air at the bottom would be one thousand times as dense as at the earth's surface. Therefore wood and even water would float in it.

QUESTIONS AND PROBLEMS

1. Under ordinary conditions a gram of air occupies about 800 cc. Find what volume a gram will occupy at the top of Mont Blanc (altitude 15,810 feet), where the barometer indicates that the pressure is only about one half what it is at sea level.

2. The mean density of the air at sea level is about .0012. What is its density at the top of Mont Blanc? What fractional part of the earth's atmosphere has one left beneath him when he ascends to the top of this mountain?

3. If Glasier and Coxwell rose in their balloon until the barometric height was only 18 cm., how many inhalations were they obliged to make in order to obtain the same amount of air which they could obtain at the surface in one inhalation?

4. With the aid of the experiment in which a rubber bag was expanded under the exhausted receiver of an air pump, explain why high mountain climbing often causes pain and bleeding in the ears and nose. Why does deep diving produce similar effects?

5. Blow as hard as possible into the tube of the bottle shown in Fig. 70. Then withdraw the mouth and explain all of the effects observed.

FIG. 70

6. If a bottle or cylinder is filled with water and inverted in a dish of water, with its mouth beneath the surface (see Fig. 71), the water will not run out. Why?

7. If a bent rubber tube is inserted beneath the cylinder and air blown in at *o*, it will rise to the top and displace the water. This is the method regularly used in collecting gases. Explain (1) what forces the gas up into it, and (2) what causes the water to descend in the tube as the gas rises.

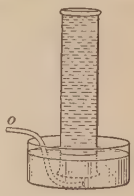


FIG. 71

8. Why must the bung be removed from a cider barrel in order to secure a proper flow from the faucet?

9. When a bottle full of water is inverted, the water will gurgle out instead of issuing in a steady stream. Why?

10. There is a pressure of 70 cm. of mercury on 1000 cc. of gas. What pressure must be applied to reduce the volume to 600 cc., if the temperature is kept constant?

11. What sort of a change in volume do the bubbles of air which escape from a diver's suit experience as they ascend?

PNEUMATIC APPLIANCES

99. The siphon. Let a rubber or glass tube be filled with water and then placed in the position shown in Fig. 72. Water will be found to flow through the tube from vessel *A* into vessel *B*. If, then, *B* be raised until the water in it is at a higher level than that in *A*, the direction of flow will be reversed. This instrument, which is called the *siphon*, is very useful for removing liquids for vessels which cannot be overturned, or for drawing off the upper layers of a liquid without disturbing the lower layers.

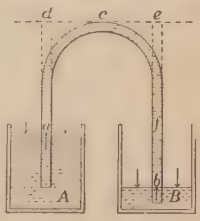


FIG. 72. The siphon

The explanation of the siphon's action is readily seen from Fig. 72. Since the tube *acb* is full of water, water must evidently flow through it if the force which pushes it one way is greater than that which pushes it the other way. Now the upward pressure at *a* is equal to atmospheric

pressure minus the downward pressure due to the water column ad; while the upward pressure at b is the atmospheric pressure minus the downward pressure due to the water column be. Hence the pressure at a exceeds the pressure at b by the pressure due to the water column fb. The siphon will evidently cease to act when the water is at the same level in the two vessels, since then $fb = 0$, and the forces acting at the two ends of the tube are therefore equal and opposite. It will also cease to act when the bend c is more than 34 ft. above the surface of the water in A, since then a vacuum will form at the top, atmospheric pressure being unable to raise water to a height greater than this in either tube.

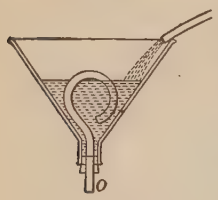


Fig. 73. Intermittent siphon

Would a siphon flow in a vacuum?

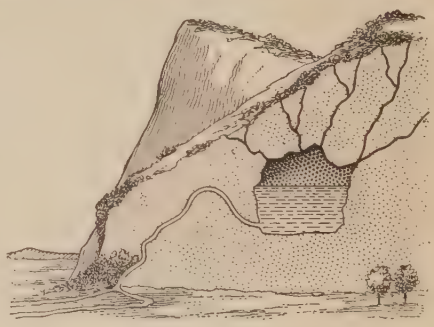


Fig. 74. Intermittent spring

100. The intermittent siphon. Fig. 73 represents an intermittent siphon. If the vessel is at first empty, to what level must it be filled before the water will flow out at o? To what level will the water then fall before the flow will cease?

The intermittent spring sometimes found in nature is nothing but a natural siphon of this kind. Its action may be understood from Fig. 74.

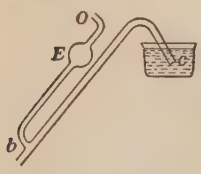


Fig. 75. Aspirating siphon

101. The aspirating siphon. It is clear from the theory of siphon action that the flow cannot start unless the tube is initially full of the liquid. Fig. 75 represents a so-called aspirating siphon, an instrument designed to minimize the inconvenience and danger incident upon starting the flow when it is desired to siphon off acids

or other disagreeable or poisonous liquids. The open end b is first closed; the tube is then filled by sucking on the end O while the end c is immersed in the liquid to be siphoned off. The bulb E is made so large that there is no danger of inadvertently sucking liquid into the mouth.

102. The air pump. The air pump was invented in 1650 by Otto von Guericke, mayor of Magdeburg, Germany, who deserves the greater credit, since he was apparently wholly without knowledge of the discoveries which Galileo, Torricelli, and Pascal had made a few years earlier regarding the character of the earth's atmosphere. A simple form of such a pump is shown in Fig. 76. When the piston is raised the air from the receiver R expands into the cylinder B through the valve A. When the piston descends it compresses this air, and thus closes the valve A and opens the exhaust valve C. Thus with each double stroke a certain fraction of the air in the receiver is transferred from R through the cylinder to the outside.

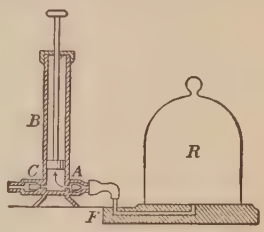


FIG. 76. A simple air pump

In many pumps the valve C is in the piston itself.

103. The compression pump. A compression pump is nothing but an exhaust pump with the valves reversed, so that A closes and C opens on the upstroke, and A opens and C closes on the downstroke. In its cheaper forms, e.g. the common bicycle pump, the valve C is often replaced by a very simple device called a cup valve. This valve consists of a disk of leather a little larger than the barrel of the pump, attached to a loosely fitting metal piston. When the piston is raised the air passes in around the leather, but when it is lowered the leather is crowded closely against the walls, so that there is no escape for the air (Fig. 77).

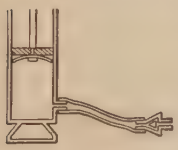


FIG. 77. The cup valve

Compressed air finds so many applications in such machines as air drills (used in mining), air brakes, air motors, etc., that the compression pump must be looked upon as of much greater importance industrially than the exhaust pump.

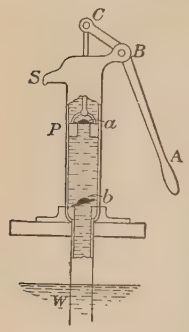


Fig. 78. The lift pump

104. The lift pump. The common water pump, shown in Fig. 78, has been in use at least since the time of Aristotle (fourth century B.C.). It will be seen from the figure that it is nothing more nor less than a simplified form of air pump. In fact, in the earlier strokes we are simply exhausting air from the pipe below the valve *b*. Water could never be obtained at *S*, even with a perfect pump, if the valve *b* were not within 34 ft. of the surface of the water in *W*. Why? On account of mechanical imperfections this limit is usually about 28 ft. instead of 34. Let the student analyze, stroke by stroke, the operation of pumping water from a well with the pump of Fig. 78. Why will pouring in a little water at the top, i.e. "priming," often assist greatly in starting such a pump?

105. The force pump. Fig. 79 illustrates the construction of the force pump, a device commonly used when it is desired to deliver water at a point higher than the position at which it is convenient to place the pump itself. Let the student analyze the action of the pump from a study of the diagram.

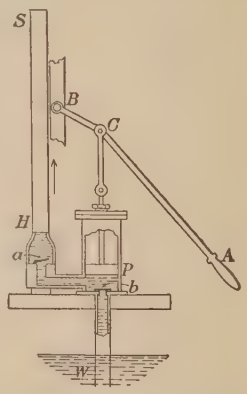


Fig. 79. The force pump

It will be seen that the discharge from such an arrangement as that shown in Fig. 79 must be intermittent, since no water can flow up the pipe *HS* when the

piston P is ascending. In order to make the flow continue during the upstroke an air chamber, such as that shown in Fig. 80, is always inserted between the valve a (Fig. 79) and the discharge point. As the water is forced violently into this chamber it compresses the confined air. It is, then, the reaction of this compressed air which is immediately responsible for the flow in the discharge tube, and as this reaction is continuous the flow is also continuous.

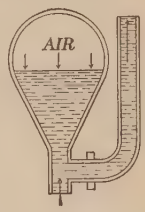


Fig. 80. The air dome of a force pump

Fig. 81 represents one of the most familiar types of force pump, the double-acting steam fire engine. Let the student analyze the action of the pump from a study of the diagram.

106. The Cartesian diver. Descartes (1596–1650), the great French philosopher, invented an odd device which illustrates at the same time the principle of the transmission of pressure by

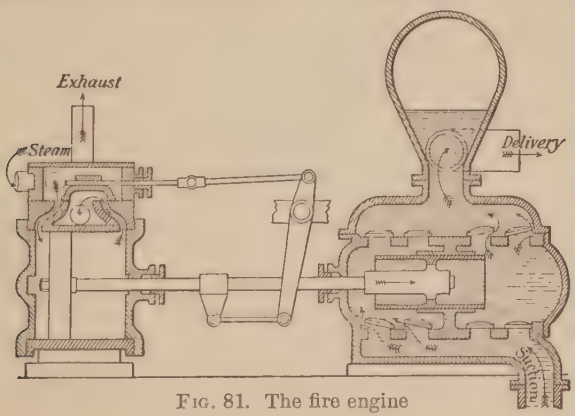


Fig. 81. The fire engine

liquids, the principle of Archimedes, and the compressibility of gases. A hollow glass image in human shape [Fig. 82, (1)] has an opening in the lower end. It is partly filled with water and partly with air, so that it will just float. By pressing on the

rubber diaphragm at the top of the vessel it may be made to sink or rise at will. Explain. If the diver is not available a small bottle or test tube [see Fig. 82, (2)] may be used instead. It works equally well, and brings out the principle even better.

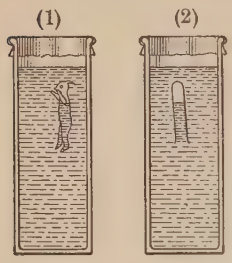


Fig. 82. The Cartesian diver

107. The balloon. A reference to the proof of Archimedes' principle (§ 74, p. 52) will show that it must apply as well to gases as to liquids. *Hence any body immersed in air is buoyed up by a force which is equal to the weight of the displaced air.* The body will therefore rise if its own weight is less than the weight of the air which it displaces.

A balloon is a large silk bag (Fig. 83) varnished so as to be air-tight, and filled either with hydrogen or with common illuminating gas. The former gas weighs about .09 kg. per cubic meter and common illuminating gas weighs about .75 kg. per cubic meter. It will be remembered that ordinary air weighs about 1.20 kg. per cubic meter. It will be seen, therefore, that the lifting power of hydrogen per cubic meter, namely $1.20-.09=1.11$, is more than twice the lifting power of illuminating gas, $1.20-.75=.45$. Nevertheless, on account of the comparative cheapness of the latter gas, its use is very much more common.

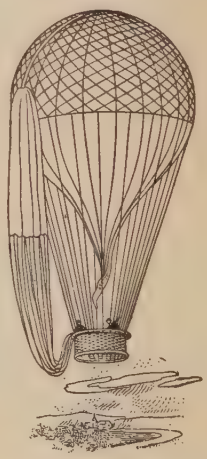


Fig. 83. The balloon

From the weights given above it is easy to calculate the lifting power of any balloon whose volume is known. Glasier and Coxwell's balloon had a volume of 90,000 cu. ft., and was able to carry a load of about 600 lb.

Ordinarily a balloon is not completely filled at the start, for if it were, since the outside pressure is continually diminishing as it ascends, the pressure of the inside gas would subject the bag to enormous strain, and would surely burst it before it reached any considerable altitude. But if it is but partially inflated at the start, it can increase in volume as it ascends by simply inflating to a greater extent.

The parachute seen hanging from the side of the balloon in Fig. 83 is a huge umbrella-like affair, which, after opening as in Fig. 84, descends very slowly on account of the enormous surface exposed to the air. The hole in the top allows air to escape slowly and thus keeps the parachute upright.

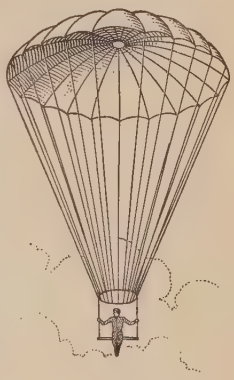


Fig. 84. The parachute

108. The diving bell. The diving bell (Fig. 85) is a heavy bell-shaped body with rigid walls, which sinks of its own weight. Formerly the workmen who went down in the bell had at their disposal only the amount of air confined within it, and the water rose to a certain height within the bell on account of the compression of the air. But in modern practice the air is forced in from the surface through a connecting tube (*a*, Fig. 86) by means of a force pump *h*. This arrangement, in addition to furnishing a continual supply of fresh air, makes it possible to force the water down to the level of the bottom of the bell. In practice a continual stream of bubbles is kept flowing out from the lower edge of the bell, as shown in Fig. 86.

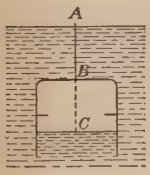


Fig. 85. The diving bell

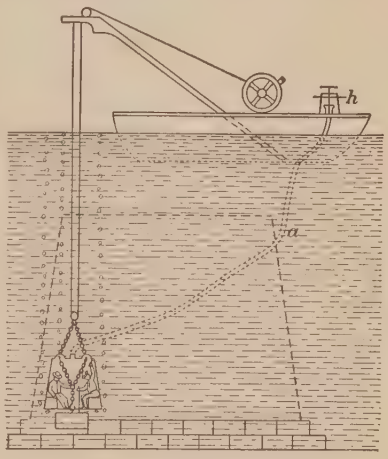


Fig. 86. Laying foundations of piers with the diving bell

The pressure of the air within the bell must, of course, be the pressure existing within the water at the depth

of the level of the water inside the bell, i.e. in Fig. 85 at the depth AC. Thus at a depth of 34 ft. the pressure is 2 atmospheres. Diving bells are used for putting in the foundations of bridge piers, doing subaqueous excavating, etc. The so-called *caisson*, much used in bridge building, is simply a huge stationary diving bell, which the workmen enter through compartments provided with air-tight doors. Air is pumped into it precisely as in Fig. 86.

Fig. 87. The diving suit

109. The diving suit. For most purposes, except those of heavy engineering, the diving suit has now replaced the diving bell. This suit is made of rubber with a metal helmet. The diver is sometimes connected with the surface by a tube (Fig. 87) through which air is forced down to him. It passes out into the water through the valve v in his suit. But more commonly the diver is entirely independent of the surface, carrying air under a pressure of about 40 atmospheres in a tank on his back. This air is allowed to escape gradually through the suit and out into the water through the valve v as fast as the diver needs it. When he wishes to rise to the surface he simply admits enough air to his suit to make him float.

In all cases the diver is subjected to the pressure existing at the depth at which the suit or bell communicates with the outside water. Divers seldom work at depths greater than 60 ft., and 80 ft. is usually considered the limit of safety. But in building the bridge over the Mississippi at St. Louis, Missouri, the bells with their divers were sunk to a depth of 110 ft., while a case is on record of a diver who, in investigating a wreck off the coast of South America, sank to a depth of 201 ft.

The diver experiences pain in the ears and above the eyes when he is ascending or descending, but not when at rest. This is because it requires some time for the air to penetrate into the interior cavities of the body and establish equal pressure in both directions.

110. The air brake. Fig. 88 is a diagram which shows the essential features of the Westinghouse air brake. P is an air pipe leading to the engine, where a compression pump maintains air in the main cylinder

under a pressure of about 70 lb. to the square inch. R is an auxiliary reservoir which is placed under each car, and which connects with P through the triple valve V. So long as the pressure from the engine is on in P, the valve V is open in such a way that there is direct communication between P and R. But as soon as the pressure in P is diminished, either by the engineer or by the accidental breaking of the hose coupling k, which connects P from car to car, the compressed air in R operates the valve in V so as to shut off connection between R and P and to open connection between R and the cylinder C. The piston H is thus driven powerfully to the left and sets the brake shoes against the wheels through the operation of levers attached to H. When it is desired to take off the brakes, pressure is again turned on in P. This operation opens V in such a way as to permit the compressed air in C to escape, and the spring S then pulls back the brake shoes from the wheels.

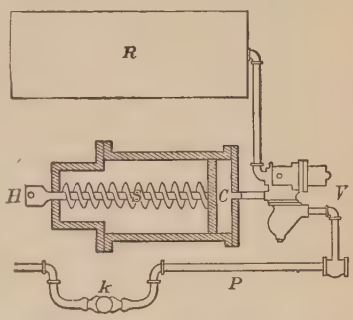


FIG. 88. The Westinghouse air brake

111. The bellows. Fig. 89 shows the construction of the ordinary blacksmith's bellows. When the handle a rises and the point b in consequence falls, the valve v opens and air from the outside enters the lower compartment C_1. When a is pulled down and b thus made to ascend, v at once closes, and as soon as the pressure within C_1 has risen to the same value as that maintained in C_2 by the weights W, the valve v' opens and air passes from C_1 to C_2. With this arrangement it will be seen that the current of air

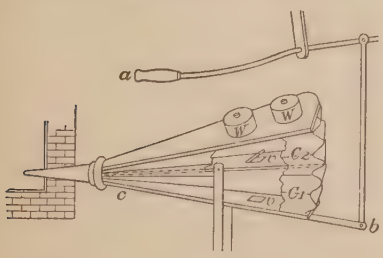


FIG. 89. A blacksmith's bellows

which issues from C_2 through the nozzle is continuous rather than intermittent, as it would be if there were but one compartment and one valve.

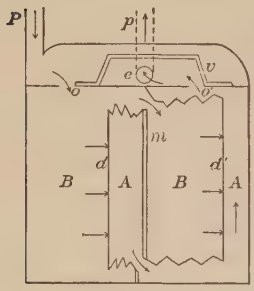


Fig. 90. The gas meter

112. The gas meter. The gas meter is a device which differs little in principle from the blacksmith's bellows. Gas from the city supply enters the meter through P (Fig. 90), and passes through the opening o into the compound compartment B of the meter. Here its pressure forces in the diaphragm d, at the same time forcing out the diaphragm d'. Each of these operations diminishes the size of the compartment A, for the diaphragm m is immovable. The gas already contained in A is therefore pushed out to the burners through the openings o' and e and the pipe p. As soon as compartment B is full, a lever which is worked by the movement of the diaphragms causes the slide valve v to move to the left, thus closing o and shutting off connection between P and B, but at the same time opening o' and allowing the gas from P to enter compartment A through o'. The gas in B is now forced out through the openings o and e and the pipe p. The movement of the diaphragms is recorded by a clockwork device, the dials of which (Fig. 91) indicate the number of cubic feet of gas which have passed through the meter.

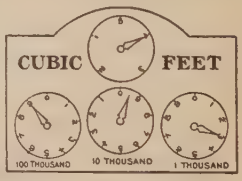


Fig. 91. Dial of gas meter

QUESTIONS AND PROBLEMS

1. Let a siphon of the form shown in Fig. 92 be made by filling a flask one third full of water, closing it with a cork through which pass two pieces of glass tubing, as in the figure, and then inverting so that the lower end of the straight tube is in a dish of water. If the bent arm is of considerable length, the fountain will play forcibly and continuously until the dish is emptied. Explain.

2. Pneumatic dispatch tubes are now used in many large stores for the transmission of small packages. An exhaust pump is attached to one end of the tube in which a tightly fitting carriage moves, and a compression

pump to the other. If the air is half exhausted on one side of the carriage, and has twice its normal density on the other, find the propelling force acting on the carriage when the area of its cross section is 50 sq. cm.

3. Pascal proved by an experiment that a siphon would not run if the bend in the arm were more than 34 ft. above the upper water level. He made it run, however, by inclining it sidewise until the bend was less than 34 ft. above this level. Explain.

4. If the cylinder of an air pump is of the same size as the receiver, what fractional part of the air is removed by one complete stroke? What fractional part is left after 3 strokes? after 10?

5. If the cylinder of an air pump is one third the size of the receiver, what fractional part of the original air will be left after 5 strokes? What will a barometer within the receiver read, the outside pressure being 76?

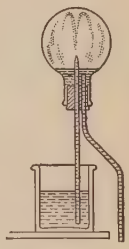


Fig. 92

6. Theoretically, can a vessel ever be completely exhausted by an air pump, even if mechanically perfect?

7. Why, in pumping water, is more and more force required at each succeeding stroke until the water begins to flow?

8. If the air in the air dome of a fire engine is reduced to one tenth of its normal volume, under what pressure is the water at the mouth of the nozzle?

9. What is the lifting power of a balloon which is filled with hydrogen and has a volume of 1000 cu. m.? (Take the weight of air as 1.2 g. per liter and that of hydrogen as one fourteenth that of air.)

10. During the siege of Paris in 1871, 64 balloons left the city carrying with them, in addition to passengers, about 3,000,000 letters, the whole weighing about 10 tons. If the passengers carried by each balloon weighed 400 lb., the balloon and car 600 lb., ballast and supplies 1000 lb., what must have been the minimum capacity of each balloon if filled with coal gas of density .6 that of air?

11. When will a balloon cease to rise?

12. If a diving bell (Fig. 85) is sunk until the level of the water within it is 1033 cm. beneath the surface, to what fraction of its initial volume has the inclosed air been reduced?

13. If a diver's tank has a volume of 2 cu. ft. and contains air under a pressure of 40 atmospheres, to what volume will the air expand when it is released at a depth of 34 ft. under water?

14. If the water within a diving bell is at a depth of 1033 cm. beneath the surface of a lake, what is the density of the air inside, if at the surface the density of air is .0012 and its pressure 76 cm.? What would be the reading of a barometer within the bell?

CHAPTER V

MOLECULAR MOTIONS

Kinetic Theory of Gases

113. Molecular constitution of matter. In order to account for some of the simplest facts in nature, — e.g. the fact that two substances often apparently occupy the same space at the same time, as when two gases are crowded together in the same vessel, or when sugar is dissolved in water, — it is now universally assumed that all substances are composed of very minute particles called *molecules*. Spaces are supposed to exist between these molecules, so that when one gas enters a vessel which is already full of another gas, the molecules of the one scatter themselves about between the molecules of the other. Since molecules cannot be seen with the most powerful microscopes, it is evident that they must be very minute, and the number of them contained in a cubic centimeter of any substance must be enormous. Probably it would take as many as a thousand molecules laid side by side to make a speck long enough to be seen with the best microscopes.

114. Evidence for molecular motions in gases. Certain very simple observations lead us to the conclusion that the molecules of gases, even in a still room, must be in continual and quite rapid motion. Thus, if a little chlorine, or ammonia, or any gas of powerful odor is introduced into a room, in a very short time it will have become perceptible in all parts of the room. This shows clearly that enough of the molecules of the gas to affect the olfactory nerves must have found their way across the room.

Again, chemists tell us that if two globes, one containing hydrogen and the other carbon dioxide gas, be connected as in Fig. 93 and the stopcock between them opened, after a few hours chemical analysis will show that each of the globes contains the two gases in exactly the same proportions, — a result which is at first sight very surprising, since carbon dioxide gas is about twenty-two times as heavy as hydrogen. This mixing of gases in apparent violation of the laws of weight is called *diffusion.*

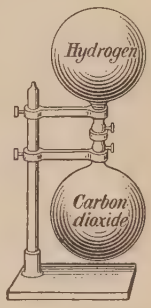


Fig. 93. Illustrating the diffusion of gases

We see then that such simple facts as the transference of odors and the diffusion of gases furnish very convincing evidence that the molecules of a gas are not at rest, but are continually moving about.

115. Molecular motions and the indefinite expansibility of a gas. Perhaps the most striking property which we have found gases to possess is the property of indefinite or unlimited expansibility. The existence of this property was demonstrated by the fact that we were able to obtain a high degree of exhaustion by means of an air pump. No matter how much air was removed from the bell jar, the remainder at once expanded and filled the entire vessel. In fact, it was only because of this property that the air pump was able to perform its functions at all.

In order to explain these facts it used to be assumed that the molecules of gases exert mutual repulsion upon one another. This theory has now, however, been completely abandoned, for it has been conclusively shown that no such repulsions exist. The motions of the molecules alone furnish a thoroughly satisfactory explanation of the phenomenon. As soon as the piston of the air pump is drawn up, some of the molecules follow it because they were already moving in that direction, and not on account

of any repulsion exerted upon them by the molecules below. The phenomenon is precisely the same as that illustrated in Fig. 93 where the carbon dioxide molecules moved up into the globe containing hydrogen; only in the latter case the operation took much more time because the upward motion of the carbonic acid molecules was hindered by collisions with the hydrogen molecules.

The fact that, however rapidly the piston of the air pump is drawn up, gas always appears to follow it instantly, leads us to the conclusion that the natural velocity possessed by the molecules of gases must be very considerable.

116. Molecular motions and gas pressures. If the molecules of gases do not repel one another, how are we to account for the fact that gases exert such pressures as they do against the walls of the vessels which contain them? We have found that in an ordinary room the air presses against the walls with a force of 15 lb. to the square inch. Within an automobile tire this pressure may amount to as much as 100 lb., and the steam pressure within the boiler of an engine is often as high as 240 lb. per square inch. Yet in all these cases we may be certain that the molecules of the gas are separated from each other by distances which are large in comparison with the diameters of the molecules; for when we reduce steam to water it shrinks to $\frac{1}{1600}$ of its original volume, and when we reduce air to the liquid form it shrinks to about $\frac{1}{800}$ of its ordinary volume.

The explanation is at once apparent when we reflect upon the *motions* of the molecules. For just as a stream of water particles from a hose exerts a continuous force against a wall on which it strikes, so the blows which the innumerable molecules of a gas strike against the walls of the containing vessel must constitute a continuous force tending to push out these walls. Indeed, when we give up the wholly untenable notion of molecular repulsions, there is no other way in which we can account for the fact that vessels containing only gas—

balloons, for example — do not collapse under the enormous external pressures to which we know them to be subjected.

117. Explanation of Boyle's law. It will be remembered that it was discovered in the last chapter that when the density of a gas is doubled, the temperature remaining constant, the pressure is found to double also. When the density was trebled, the pressure was trebled, etc. This, in fact, was the assertion of Boyle's law. Now this is exactly what would be expected if the pressure which a gas exerts against a given surface is due to blows struck by an enormous number of swiftly moving molecules; for doubling the number of molecules in the given space — i.e. doubling the density — would simply double the number of blows struck per second against that surface, and hence would double the pressure. While the kinetic theory of gases which is here presented accounts in this simple way for Boyle's law, the theory of molecular repulsions cannot be reconciled with it.

118. Molecular velocities. From the known weight of a cubic centimeter of air under normal conditions, and the known force which it exerts per square centimeter, — viz. 1033 g., — it is possible to calculate the velocity which its molecules must possess in order that they may produce by their collisions against the walls this amount of force. Further, since a cubic centimeter of hydrogen which is in condition to exert the same pressure as a cubic centimeter of air weighs only one fourteenth as much as the air, it is evident that the hydrogen molecules must be moving much more rapidly than the air molecules, or else they could not exert the same pressure. The result of the calculation gives to the air molecules under normal conditions a velocity of about 445 m. per second, while it assigns to the hydrogen molecules the enormous speed of 1700 m. per second. The speed of a cannon ball is seldom greater than 800 m. (2500 ft.) per second. It is easy to see then, since the molecules of gases are endowed with such speeds, why air, for example, expands

instantly into the space left behind by the rising piston of the air pump, and why any gas always fills completely the vessel which contains it.

119. Diffusion of gases through porous walls. Strong evidence for the correctness of the above views is furnished by the following experiment.

Let a porous cup of unglazed earthenware be closed with a rubber stopper through which a glass tube passes, as in Fig. 94. Let the tube be dipped into a dish of colored water, and a jar containing hydrogen placed over the porous cup, or let the jar simply be held in the position shown in the figure, and illuminating gas passed into it by means of a rubber tube connected with a gas jet. The rapid passage of bubbles out through the water will show that the gaseous pressure inside the cup is rapidly increasing. Now let the bell jar be lifted, so that the hydrogen is removed from the outside. Water will at once begin to rise in the tube, showing that the inside pressure is now rapidly decreasing.

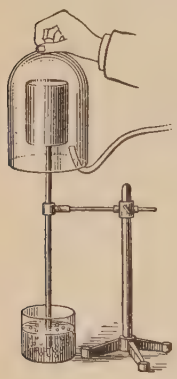


Fig. 94. Diffusion of hydrogen through porous cup

The explanation is as follows. We have learned that the molecules of hydrogen have about four times the velocity of the molecules of air. Hence, if there are as many hydrogen molecules per cubic centimeter outside the cup as there are air molecules per cubic centimeter inside, the hydrogen molecules will strike the outside of the wall four times as frequently as the air molecules will strike the inside. Hence, in a given time, the number of hydrogen molecules which pass into the interior of the cup through the little holes in the porous material will be four times as great as the number of air particles which pass out. Since the inside is thus gaining molecules faster than it is losing them, and since the pressure of a gas at a given temperature is determined solely by the

JAMES CLERK-MAXWELL (1831-1879)

One of the greatest of mathematical physicists; born in Edinburgh, Scotland; professor of natural philosophy at Marischal College, Aberdeen, in 1856, of physics and astronomy in Kings College, London, in 1860, and of experimental physics in Cambridge University from 1871 to 1879; one of the most prominent figures in the development of the kinetic theory of gases, and the mechanical theory of heat; author of the electro-magnetic theory of light, — a theory which has become the basis of nearly all modern theoretical work in electricity and optics (see p. 469).

number of molecules which are bombarding the wall, the inside pressure must increase until the number per cubic centimeter inside is so much larger than the number outside that molecules pass out as fast as they pass in. When the bell jar is removed the hydrogen which has passed inside now begins to pass out faster than the outside air passes in, and hence the inside pressure is diminished.

120. Temperature and molecular velocity. The effects which are observed when a gas is heated furnish further evidence that its molecules are in motion.

Let a bulb of air B be connected with a water manometer m, as in Fig. 95. If the bulb is warmed by holding a Bunsen burner beneath it, or even by placing the hand upon it, the water at m will at once begin to descend, showing that the pressure exerted by the air contained in the bulb has been increased by the increase in its temperature. If B is cooled with ice or ether the water will rise at m.

Fig. 95. Expansion of air by heat

Now if gas pressure is due to the bombardment of the walls by the molecules of the gas, since the number of molecules in the bulb can scarcely have been changed by slightly heating it, we are forced to conclude that the increase in pressure is due to an increase in the *velocity* of the molecules which are already there. The temperature of a given gas, then, from the standpoint of the kinetic theory, is determined simply by the mean velocity of the gas molecules. To increase the temperature is to increase the average velocity of the molecules, and to diminish the temperature is to diminish this average molecular velocity. The theory thus furnishes a very simple and natural explanation of the fact of the expansion of gases with a rise in temperature.

QUESTIONS AND PROBLEMS

1. A liter of air at a pressure of 76 cm. is compressed so as to occupy 430 cc. What is the pressure against the walls of the containing vessel?

2. If an open vessel contains 250 g. of air when the barometric height is 750 mm., what weight will the same vessel contain at the same temperature when the barometric height is 740 mm.?

3. The density of air is .001293 when the temperature is 0° C. and the pressure 76 cm. How large must a vessel be to contain a kilogram of air when the temperature is 0° C. and the pressure 75 cm.?

4. On a day on which the barometric height is 76 cm. the volume of the space above the mercury in a Torricelli tube is 10 cc., and the mercury in the tube stands 74 cm. high. How high will the mercury stand above the cistern if the tube is pulled up out of the dish so that the space above is 20 cc.?

5. Find the pressure to which the diver was subjected who descended to a depth of 201 ft. Find the density of the air in his suit, the density at the surface being .00118 and the temperature being assumed to remain constant. Take the pressure at the surface as 75 cm.

6. A bubble of air which escaped from this diver's suit would increase to how many times its volume on reaching the surface?

Molecular Motions in Liquids

121. Molecular motions in liquids and evaporation. Evidence that the molecules of liquids as well as those of gases are in a state of perpetual motion is found, first, in the familiar facts of evaporation.

We know that the molecules of a liquid in an open vessel are continually passing off into the space above; for it is only a matter of time when the liquid completely disappears and the vessel becomes dry. Now it is hard to imagine a way in which the molecules of a liquid thus pass out of the liquid into the space above, unless these molecules, while in the liquid condition, are in motion. As soon, however, as such a motion is assumed, the facts of evaporation become perfectly intelligible. For it is to be expected that in the jostlings and collisions of rapidly moving liquid molecules an occasional molecule will acquire a velocity much greater than the average. This molecule may

then, because of the unusual speed of its motion, break away from the attraction of its neighbors and fly off into the space above. This is indeed the mechanism by which we now believe that the process of evaporation goes on.

122. Molecular motions and the diffusion of liquids. One of the most convincing arguments for the motions of molecules in gases was found in the fact of diffusion. But precisely the same sort of phenomena are observable in liquids.

Fig. 96. Diffusion of liquids

Let a few lumps of blue litmus be pulverized and dissolved in water. Let a tall glass cylinder be half filled with this water and a few drops of ammonia added. Let the remainder of the litmus solution be turned red by the addition of one or two cubic centimeters of nitric acid. Then let this acidulated water be introduced into the bottom of the jar through a thistle tube (Fig. 96). In a few minutes the line of separation between the acidulated water and the blue solution will be fairly sharp; but in the course of a few hours, even though the jar is kept perfectly quiet, the red color will be found to have spread considerably toward the top of the jar, showing that the acid molecules have gradually found their way toward the top.

Certainly, then, the molecules of a liquid must be endowed with the power of independent motion.

123. Molecular motions and the expansion of liquids. The fact of the expansion of gases with a rise of temperature was looked upon as evidence that the molecules of gases are in motion, the velocity of this motion increasing with an increase in temperature. But precisely the same property belongs to liquids also.

Thus, let the bulb (Fig. 97) be heated with a Bunsen burner. The contained liquid will be found to expand and rise in the tube.

It is natural to infer that the cause of this increase in volume is the same as before; i.e. the velocity of the molecules of the

liquid has been increased by the rise in temperature, and they have therefore jostled one another farther apart, and thus caused the whole volume to be enlarged. According to this view, then, an increase in temperature in a liquid, as in a gas, means an increase in the mean velocity of the molecules, and conversely a decrease in temperature means a decrease in this average velocity.

Fig. 97. Expansion of a liquid

124. Evaporation and temperature. If it is true that increase in temperature means increase in the mean velocity of molecular motion, then the number of molecules which chance in a given time to acquire the velocity necessary to carry them into the space above the liquid, ought to increase as the temperature increases; i.e. evaporation ought to take place more rapidly at high temperatures than at low. Common observation teaches that this is true. Damp clothes become dry under a hot flat-iron but not under a cold one; the sidewalk dries more readily in the sun than in the shade; we put wet objects near a hot stove or radiator when we wish them to dry quickly.

Properties of Vapors

125. Saturated vapor. If a liquid is placed in an open vessel, there ought to be no limit to the number of molecules which can be lost by evaporation, for as fast as the molecules emerge from the liquid they are carried away by air currents. As a matter of fact, experience teaches that water left in an open dish does waste away until the dish is completely dry.

But suppose that the liquid is evaporating into a closed space, such as that shown in Fig. 98. Since the molecules which leave the liquid cannot escape from the space S, it is clear that as time goes on the number of molecules which have passed off from

the liquid into this space must continually increase; in other words, the density of the vapor in S must grow greater and greater. The question which at once suggests itself is, "Is there any limit to the density which this vapor can attain?" i.e. "Will evaporation go on indefinitely into the space S, so that a vessel of liquid placed in it will ultimately dry up?" Experiment has very positively answered this question in the negative. A vessel of water placed in an air-tight bell jar will never waste away. Hence there must be a limit to the possible amount of evaporation into a closed space above a liquid, i.e. to the density which the vapor can attain. When this limit is reached the vapor is said to be *saturated.*

FIG. 98. A saturated vapor

126. Explanation of saturation. The kinetic theory furnishes a very simple explanation of the facts of saturation. The molecules which have escaped into S (Fig. 98) are moving about in all directions within this space. Whenever one of them in its motions chances to strike the surface of the liquid, it reënters and does not again escape unless it chances to acquire again the velocity which is necessary for the escape of *any* molecule from the liquid. It is clear that the more molecules there are present in the space above the liquid, the more frequently will some of them strike the surface of the liquid and return to it permanently in the manner just described. In fact, if we double the number of molecules in the space S, we must double the number which strike the surface of the liquid per second, and hence double the number which will return to the liquid per second. Evidently, then, as the natural process of evaporation causes the vapor to become more and more dense in S, a condition must soon be reached when the number of molecules which return per second from the vapor to the liquid is equal to the number which pass out of the liquid per second into the space S; for the number which pass out of the liquid per second depends

simply upon how many acquire the velocity necessary for escape, and has nothing to do with the amount of vapor above the liquid. When this condition of saturation has been reached there will be a continual exchange of molecules between the liquid and the vapor; but the liquid will no longer waste away and the vapor will no longer increase in density. The vapor is then in the *saturated* condition.

127. Pressure of a saturated vapor. We have learned that any gas or vapor presses out against the walls of the containing vessel because of blows which its moving molecules strike against these walls. We have learned also from Boyle's law that the pressure which a gas or vapor exerts is directly proportional to its density, i.e. to the number of molecules which are present per cubic centimeter to strike such blows. The pressure which the vapor in the space *S* exerts against the walls of *S* must therefore increase in just the proportion in which the density of the vapor increases, and reach a maximum when the density reaches a maximum. This maximum pressure which a vapor can exert at a given temperature is called *the pressure of the saturated vapor.*

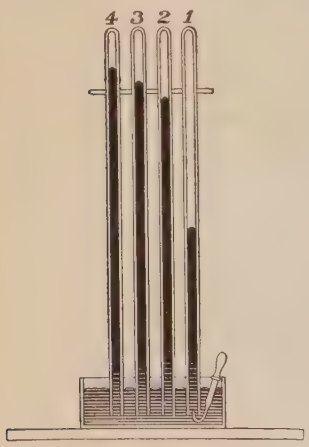


FIG. 99. Vapor pressures of saturated vapors

128. Measurement of the pressure of a saturated vapor. Let four Torricellian tubes be set up as in Fig. 99, and with the aid of a curved pipette (Fig. 99) let a drop of ether be introduced into the bottom of tube 1. This drop will at once rise to the top and a portion of it will evaporate into the vacuum which exists above the mercury. The pressure of this vapor will push down the mercury column, and the number of centimeters of this depression will of course be a measure of the pressure of the vapor. It will be observed that the mercury will

fall almost instantly to the lowest level which it will ever reach, — a fact which indicates that it takes but a very short time for the condition of saturation to be attained. In the same way let alcohol and water be introduced into tubes 2 and 3 respectively.

While the pressure of the saturated ether vapor at the temperature of the room will be found to be as much as 40 cm., that of alcohol will be found to be but 4 or 5 cm., and that of water only 1 or 2 cm.

129. No change in the volume of a saturated vapor can affect its density or pressure. Suppose that after the condition of saturation has been reached in the space S (Fig. 98) — i.e. after the number of molecules which return from the vapor to the liquid per second has become equal to the number which pass from the liquid to the vapor per second — the volume of the space S were to be suddenly decreased so as to increase momentarily the number of molecules per cubic centimeter in the space above the liquid. This would increase the number of vapor molecules which strike the liquid surface per second, and thus increase the rate at which molecules return to the liquid without changing in any way their rate of emergence. Hence the vapor would necessarily grow less and less dense because of this uncompensated loss of molecules, until the number entering per second was again reduced to the number emerging per second, — i.e. until the vapor density in S became the same as at first. We conclude, then, that the density of a vapor in contact with its liquid cannot be permanently increased by compressing it so long as the temperature remains the same.

If, on the other hand, the density of the vapor above the liquid is momentarily diminished by suddenly increasing the volume of the space S, more molecules will emerge per second from the liquid than enter it from the vapor. Consequently the density of the vapor must increase until it reaches the old equilibrium value. In a word, then, if we decrease the volume of a saturated vapor, it should condense until the former density

is restored; and if we increase the volume, more liquid should evaporate until the first condition is again regained. In order to verify this conclusion let the following striking experiment be performed.

Fig. 100. Pressure of a saturated vapor cannot be changed by a change in volume of vessel

Let two Torricellian tubes be placed in a long cistern of mercury, as in Fig. 100, and let a drop of ether be admitted into one, while enough air is allowed to pass into the other to reduce the mercury height to about the same level in the two tubes. Let the tubes be pushed down into the cistern so as to diminish the volume of the gases in the upper part. In the air tube this operation will be found to decrease the height of the mercury column *db*, showing that the pressure of the air within the tube has been increased, as of course it ought to be in accordance with Boyle's law, the volume having been diminished. But in the ether tube the height *ab* will be found to have been only momentarily changed by either lowering or raising the tube, thus showing that the pressure, and therefore the density, of the vapor remains constant for all changes in volume. An increase in the volume simply causes more of the liquid to evaporate, while a decrease causes some of the vapor to condense.[1]

130. Influence of temperature on the density and pressure of a saturated vapor. Let a Bunsen flame be passed quickly to and fro across the tubes of Fig. 100, near the upper level of the mercury. The heights *ab* and *db* will fall in both, but the fall will be found to be much greater in the ether tube than in the air tube. Since the two tubes have been about equally heated, there must have been about the same relative increase in molecular velocity in each.

[1] If enough mercury is not at hand to perform the experiment as indicated in Fig. 100, this property of the saturated vapor may be illustrated almost as well by simply inclining the vapor tubes of Fig. 99. This will decrease the volume, but the upper level of the mercury will remain at the same distance above the table, showing that the pressure has undergone no change.

Hence the excess of pressure which the heating has produced in the ether tube must be due to increased evaporation, i.e. to an increase in the number of molecules per cubic centimeter in the ether vapor.

The experiment proves that both the pressure and the density of a saturated vapor increase rapidly with the temperature. This was to have been expected from our theory; for increasing the temperature of the liquid increases the mean velocity of its molecules and hence increases the number which attain each second the velocity necessary for escape.

Let a piece of ice be held about the tubes near the top of the mercury. The mercury will rise in both, but much more rapidly in the ether tube than in the air tube, thus showing that the ether vapor is condensing.

The experiment shows that if the temperature of a saturated vapor is diminished, it condenses until its density is reduced to that corresponding to saturation at the lower temperature. How rapidly the density and pressure of saturation increase with temperature may be seen from the following table.

Table of Constants of Saturated Water Vapor

The table shows the pressure P, in millimeters of mercury, and the density D of aqueous vapor saturated at temperatures t° C.

t.	P.	D.	t.	P.	D.	t.	P.	D.
– 10°	2.2	.0000023	4°	6.1	.0000064	18°	15.3	.0000152
– 9°	2.3	.0000025	5°	6.5	.0000068	19°	16.3	.0000162
– 8°	2.5	.0000027	6°	7.0	.0000073	20°	17.4	.0000172
– 7°	2.7	.0000029	7°	7.5	.0000077	21°	18.5	.0000182
– 6°	2.9	.0000032	8°	8.0	.0000082	22°	19.6	.0000193
– 5°	3.2	.0000034	9°	8.5	.0000087	23°	20.9	.0000204
– 4°	3.4	.0000037	10°	9.1	.0000093	24°	22.2	.0000216
– 3°	3.7	.0000040	11°	9.8	.0000100	25°	23.5	.0000229
– 2°	3.9	.0000042	12°	10.4	.0000106	26°	25.0	.0000242
– 1°	4.2	.0000045	13°	11.1	.0000112	27°	26.5	.0000256
0°	4.6	.0000049	14°	11.9	.0000120	28°	28.1	.0000270
1°	4.9	.0000052	15°	12.7	.0000128	30°	31.5	.0000301
2°	5.3	.0000056	16°	13.5	.0000135	35°	41.8	.0000393
3°	5.7	.0000060	17°	14.4	.0000144	40°	54.9	.0000509

131. The influence of air on evaporation. We observed that when a drop of ether was inserted into a Torricellian tube the mercury fell *very suddenly* to its final position, showing that in a vacuum the condition of saturation is reached almost instantly. This was to have been expected from the great velocities which we found the molecules of gases and vapors to possess.

In order to see what effect the presence of air has upon evaporation, let a drop of ether be introduced into the air tube of the last experiment (Fig. 100). The mercury will not now be found to sink instantly to its final level as it did before, but although it will fall rapidly at first, it will continue to fall slowly for several hours. At the end of a day, if the temperature has remained constant, it will show a depression which indicates a vapor pressure of the ether just as great as that existing in a tube which contains no air.

The experiment leads, then, to the rather remarkable conclusion that *just as much liquid will evaporate into a space which is already full of air as into a vacuum.* The air has no effect except to retard greatly the *rate of evaporation.*

132. Explanation of the retarding influence of air on evaporation. This retarding influence of air on evaporation is easily explained by the kinetic theory; for while in a vacuum the molecules which emerge from the surface fly at once to the top of the vessel, when air is present the escaping molecules collide with the air molecules before they have gone any appreciable distance away from the surface (probably less than .00001 cm.), and only work their way up to the top after an almost infinite number of collisions. Thus, while the space immediately above the liquid may become saturated very quickly, it requires a long time for this condition of saturation to reach the top of the vessel. That ultimately, however, as much liquid will evaporate into a space containing air as into a vacuum is to be expected from the fact that evaporation ceases only when as many molecules of the liquid substance return to the liquid per second as escape per second. This number which returns depends

simply on the number of molecules of the liquid which are present per cubic centimeter in the space above, and not at all on how many molecules of other gases may be present there.

It must not be forgotten, however, that at a given temperature the *pressure* existing within a vessel containing gases is simply due to the total number of molecules per cubic centimeter which are striking blows against each square centimeter of the wall. Therefore, when a liquid evaporates into a closed vessel already containing air, the pressure gradually increases, and *is ultimately equal to the air pressure plus the pressure of the saturated vapor.* When a liquid evaporates in an open vessel, — i.e. under constant pressure, — its molecules crowd out an equal number of molecules of air.

QUESTIONS AND PROBLEMS

1. If the inside of a barometer tube is wet when it is filled with mercury, will the height of the mercury be the same as in a dry tube?

2. At a temperature of 15° C., what will be the error in the barometric height indicated by a barometer which contains moisture? (See the table of constants of saturated water vapor, p. 95.)

3. Why do clothes dry more quickly on a windy than on a quiet day?

4. If dry air were placed in a closed vessel when the barometer was 76 cm., and a dish of water then introduced within the closed space, what pressure would finally be attained within the vessel if the temperature were kept at 18° C.?

5. How many grams of water will evaporate at 20° C. into a closed room 18 × 20 × 4 m.? (See table, p. 95, for density of saturated water vapor at 20° C.)

Hygrometry, or the Study of Moisture Conditions in the Atmosphere[1]

133. Condensation of water vapor from the air. Were it not for the retarding influence of air upon evaporation we should be obliged to live in an atmosphere which would be always completely saturated with water vapor; for the evaporation from

[1] It is recommended that this subject be preceded by a laboratory determination of dew-point, humidity, etc. See e.g. Experiment 11 of the authors' manual.

oceans, lakes, and rivers would almost instantly saturate all the regions of the earth. This condition — one in which moist clothes would never dry, and in which all objects would be perpetually soaked in moisture — would be exceedingly uncomfortable, if not altogether unendurable.

But on account of the slowness with which, as the last experiment showed, evaporation takes place into air, the water vapor which always exists in the atmosphere is usually far from saturated, even in the immediate neighborhood of lakes and rivers. Since, however, the amount of vapor which is necessary to produce saturation rapidly decreases with a fall in temperature, if the temperature decreases continually in some unsaturated locality, it is clear that a point must soon be reached at which the amount of vapor already existing in a cubic centimeter of the atmosphere is the amount corresponding to saturation. Then, in accordance with the facts discovered in § 130, if the temperature still continues to fall, the vapor must begin to condense. Whether it condenses as dew, or cloud, or fog, or rain will depend upon how and where the cooling takes place.

134. The formation of dew. If the cooling is due to the natural radiation of heat from the earth at night after the sun's warmth is withdrawn, the atmosphere itself does not fall in temperature nearly as rapidly as do solid objects on the earth, such as blades of grass, trees, stones, etc. The layers of air which come into immediate contact with these cooled bodies are themselves cooled, and as they thus reach a temperature at which the amount of moisture which they already contain is in a saturated condition, they begin to deposit this moisture, in the form of dew, upon the cold objects. The drops of moisture which collect on an ice pitcher in summer illustrate perfectly the whole process.

135. The formation of fog. If the cooling at night is so great as not only to bring the grass and trees below the temperature at which the vapor in the air in contact with them is in a state

of saturation, but also to lower the whole body of air near the earth below this temperature, then the condensation takes place not only on the solid objects but also on dust particles suspended in the atmosphere. This constitutes a fog.

136. The formation of clouds, rain, hail, and snow. When the cooling of the atmosphere takes place at some distance *above* the earth's surface, as when a warm current of air enters a cold region, if the resultant temperature is below that at which the amount of moisture already in the air is sufficient to produce saturation, this excessive moisture immediately condenses about floating dust particles and forms a *cloud.* If the cooling is sufficient to free a considerable amount of moisture, the drops become large and fall as *rain.* If this falling rain passes through cold regions, it freezes into *hail.* If the temperature at which condensation begins is below freezing, the condensing moisture forms into *snowflakes.*

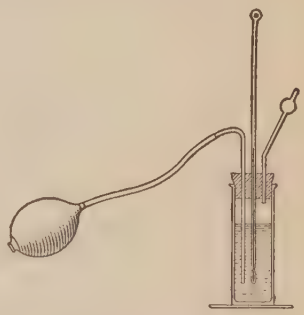


Fig. 101. Apparatus for determining dew-point

137. The dew-point. The temperature to which the atmosphere must be cooled in order that condensation may begin is called the *dew-point.* This temperature may be found by partly filling with water a brightly polished vessel of 200 or 300 cc. capacity and dropping into it little pieces of ice, stirring thoroughly at the same time with a thermometer. The dew-point is the temperature indicated by the thermometer at the instant a film of moisture appears upon the polished surface. In winter the dew-point is usually below freezing, and it will therefore be necessary to add salt to the ice and water in order to make the film appear. The experiment may be performed equally well by bubbling a current of air through ether contained in a polished tube (Fig. 101).

138. Humidity of the atmosphere. From the dew-point and the table given in § 130, p. 95, we can easily find what is commonly known as the *relative humidity,* or the *degree of saturation* of the atmosphere. This quantity is defined as *the ratio between the amount of moisture actually present in the air per cubic centimeter, and the amount which would be present if the air were completely saturated.* This is precisely the same as the ratio between the pressure which the water vapor present in the air exerts, and the pressure which it would exert if it were present in sufficient quantity to be in the saturated condition. An example will make clear the method of finding the relative humidity.

Suppose that the dew-point were found to be 15° C. on a day on which the temperature of the room was 25° C. The amount of moisture actually present in the air then saturates it at 15° C. We see from the *P* column in the table that the pressure of saturated vapor at 15° C. is 12.7 mm. This is then the pressure exerted by the vapor in the air at the time of our experiment. Running down the table, we see that the amount of moisture required to produce saturation at the temperature of the room, i.e. at 25°, would exert a pressure of 23.5 mm. Hence at the time of the experiment the air contains 12.7/23.5, or .54, as much water vapor as it might hold. We say, therefore, that the air is 54 per cent saturated, or that the relative humidity is 54%.

139. Practical value of humidity determinations. From humidity determinations it is possible to obtain much information regarding the likelihood of rain or frost. Such observations are continually made for this purpose at all meteorological stations. Further, they are made in greenhouses to see that the air does not become too dry for the welfare of the plants, and also in hospitals and public buildings, and even in private dwellings, in order to insure the maintenance of hygienic living conditions. For the most healthful conditions the relative humidity should be kept at from 50% to 60%.

140. Cooling effect of evaporation. Let three shallow dishes be partly filled, the first with water, the second with alcohol, and the third

with ether, the bottles from which these liquids are obtained having stood in the room long enough to acquire its temperature. Let three students carefully read as many thermometers, first before their bulbs have been immersed in the respective liquids and then after. In every case the temperature of the liquid in the shallow vessel will be found to be somewhat lower than the temperature of the air, the difference being greatest in the case of ether and least in the case of water.

It appears from this experiment that an evaporating liquid assumes a temperature somewhat lower than its surroundings, and that the substances which evaporate the most readily, i.e. those which have the greatest vapor pressures at a given temperature (see § 128), assume the lowest temperatures.

Another way of establishing the same truth is to place a few drops of each of the above liquids in succession on the bulb of the arrangement shown in Fig. 95, and observe the rise of water in the stem; or, more simply still, to place a few drops of each liquid on the back of the hand, and notice that the order in which they evaporate — namely, ether, alcohol, water — is the order of greatest cooling.

141. Explanation of the cooling effect of evaporation. The kinetic theory furnishes a simple explanation of the cooling effects of evaporation. We saw that in accordance with this theory evaporation means an escape from the surface of those molecules which have acquired velocities considerably above the average. But such a continual loss from a liquid of its most rapidly moving molecules involves, of course, a continual diminution of the average velocity of the molecules left behind in the liquid state, and this means a decrease in the temperature of the liquid (see §§ 120 and 123).

Again, we should expect the amount of cooling to be proportional to the rate at which the liquid is losing molecules. Hence, of the three liquids studied, ether should cool most rapidly, since it shows the highest vapor pressure at a given temperature and therefore the highest rate of emission of molecules. The alcohol should come next in order, and the water last, as was in fact observed.

142. Freezing by evaporation. In § 131 it was shown that a liquid will evaporate much more quickly into a vacuum than into a space containing air. Hence if we place a liquid under the receiver of an air pump and exhaust rapidly, we ought to expect a much greater fall in temperature than when the liquid evaporates into air. This conclusion may be strikingly verified as follows.

Let a thin watch glass be filled with ether and placed upon a drop of cold water, preferably ice water, which rests upon a thin glass plate. Let the whole arrangement be placed underneath the receiver of an air pump and the air rapidly exhausted. After a few minutes of pumping the watch glass will be found frozen to the plate.

It was by evaporating liquid hydrogen in this way that Professor James Dewar of London, in 1900, attained the lowest temperature which has ever been reached, viz. $-260°$ C. or $-436°$ F.

143. Effect of air currents upon evaporation. Let four thermometer bulbs, the first of which is dry, the second wetted with water, the third with alcohol, and the fourth with ether, be rapidly fanned and their respective temperatures observed. The results will show that in all of the wetted thermometers the fanning will considerably augment the cooling, but the dry thermometer will be wholly unaffected.

The reason that fanning thus facilitates evaporation, and therefore cooling, is that it removes the saturated layers of vapor which are in immediate contact with the liquid and replaces them by unsaturated layers into which new evaporation may at once take place. From the behavior of the dry-bulb thermometer, however, it will be seen that fanning produces cooling only when it can thus hasten evaporation. A dry body at the temperature of the room is not cooled in the slightest degree by blowing a current of air across it.

144. The wet-and-dry-bulb hygrometer. In the wet-and-dry-bulb hygrometer (Fig. 102) the principle of cooling by evaporation finds a very useful application. This instrument consists

of two thermometers, the bulb of one of which is dry, while that of the other is kept continually moist by a wick dipping into a vessel of water. Unless the air is saturated the wet bulb indicates a lower temperature than the dry one, for the reason that evaporation is continually taking place from its surface. How much lower will depend on how rapidly the evaporation proceeds; and this in turn will depend upon the relative humidity of the atmosphere. Thus in a completely saturated atmosphere no evaporation whatever takes place at the wet bulb, and it consequently indicates the same temperature as the dry one. By comparing the indications of this instrument with those of the dew-point hygrometer (Fig. 101), tables have been constructed which enable one to determine at once from the readings of the two thermometers both the relative humidity and the dew-point. On account of their convenience instruments of this sort are used almost exclusively in practical work. They are not very reliable unless the air is made to circulate about the wet bulb before the reading is taken.

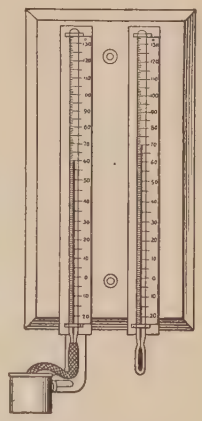


Fig. 102. Wet-and-dry-bulb hygrometer

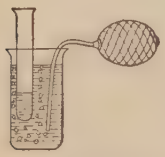


Fig. 103. Freezing water by the evaporation of ether

145. Effect of increased surface upon evaporation. Let a small test tube containing water be dipped into a larger tube, or a small glass, containing ether, as in Fig. 103, and let a current of air be forced rapidly through the ether with an aspirator. The water within the test tube will be frozen in a few minutes.

The effect of passing bubbles through the ether is simply to increase enormously the evaporating surface, for the ether

molecules which could before escape only at the upper surface can now escape into the air bubbles as well.

146. Factors affecting evaporation. The above results may be summarized as follows: The rate of evaporation depends (1) on the nature of the evaporation liquid; (2) on the temperature of the evaporating liquid; (3) on the humidity of the space into which the evaporation takes place; (4) on the density of the air or other gas above the evaporating surface; (5) on the rapidity of the circulation of the air above the evaporating surface; (6) on the extent of the exposed surface of the liquid.

Molecular Motions in Solids

147. Evidence for molecular motion in solids. We have inferred that the molecules of liquids are in motion, in part at least, from the fact that liquids increase in volume when the temperature is raised, and from the fact that molecules of the liquid can usually be detected in a gaseous condition above the surface. Both of these reasons apply just as well in the case of solids.

Thus the facts of expansion of solids with an increase in temperature may be seen on every side. Railroad rails are laid with spaces between their ends so that they may expand during the heat of summer without crowding each other out of place. Wagon tires are made smaller than the wheels which they are to fit. They are then heated until they become large enough to be driven on, and in cooling they shrink again and thus grip the wheels with immense force. A common lecture-room demonstration of expansion is the following.

Let the ball B, which when cool just slips through the ring R, be heated in a Bunsen flame. It will now be found too large to pass through the ring; but if the ring is heated, or if the ball is again cooled, it will pass through easily (see Fig. 104).

148. Evaporation of solids. That the molecules of a solid substance are found in a vaporous condition above the surface of the solid, as well as above that of a liquid, is proved by the often observed fact that ice and snow evaporate even though they are kept constantly below the freezing point. Thus wet clothes dry in winter after freezing. An even better proof is the fact that the odor of camphor can be detected many feet away from the camphor crystals. The evaporation of solids may be rendered visible by the following striking experiment.

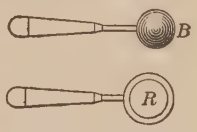


Fig. 104. Expansion of solids

Let a few crystals of iodine be placed on a watch glass and heated gently with a Bunsen flame. The visible vapor of iodine will appear above the crystals though none of the liquid is formed. A great many substances at high temperatures pass thus from the solid to the gaseous condition without passing through the liquid stage at all. This process is called *sublimation*.

149. Diffusion of solids. It has recently been demonstrated that if a layer of lead is placed upon a layer of gold, molecules of gold may in time be detected throughout the whole mass of the lead. This diffusion of solids into one another at ordinary temperatures has been shown only for these two metals, but at higher temperatures, e.g. 500° C., all of the metals show the same characteristics to quite a surprising degree.

The evidence for the existence of molecular motions in solids is then no less strong than in the case of liquids.

150. The three states of matter. Although it has been shown that in accordance with current belief the molecules of all substances are in very rapid motion, and that the temperature of a given substance, whether in the solid, liquid, or gaseous condition, is determined by the average velocity of its molecules, yet differences exist in the kind of motion which the molecules in the three states possess. Thus in the solid state it is probable that the molecules oscillate with great rapidity about

certain fixed points, always being held by the attractions of their neighbors, i.e. by the *cohesive forces* (see § 158), in practically the same positions with reference to other molecules in the body. In rare instances, however, as the facts of diffusion show, a molecule breaks away from its constraints. In liquids, on the other hand, while the molecules are, in general, as close together as in solids, they slip about with perfect ease over one another and thus have no fixed positions. This assumption is necessitated by the fact that liquids adjust themselves readily to the shape of the containing vessel. In gases the molecules are comparatively far apart, as is evident from the fact that a cubic centimeter of water occupies about 1600 cc. when it is transformed into steam; and furthermore they exert practically no cohesive force upon one another, as is shown by the indefinite expansibility of gases.

QUESTIONS AND PROBLEMS

1. Why does sprinkling the street on a hot day make the air cooler?

2. Why is the heat so oppressive on a very damp day in summer?

3. Would fanning produce any feeling of coolness if there were no moisture on the face?

4. If there were moisture on the face, would fanning produce any feeling of coolness in a saturated atmosphere?

5. If a glass beaker and a porous earthenware vessel are filled with equal amounts of water at the same temperature, in the course of a few minutes a noticeable difference of temperature will exist between the two vessels. Which will be the cooler and why? Will the difference in temperature between the two vessels be greater in a dry or in a moist atmosphere?

6. Why are icebergs frequently surrounded with fog?

7. What weight of water is contained in a room $5 \times 5 \times 3$ m. if the relative humidity is 60% and the temperature 20° C.? (See table, p. 95.)

8. Why will an open, narrow-necked bottle containing ether not show as low a temperature as an open shallow dish containing the same amount of ether?

9. A morning fog generally disappears before noon. Explain the reason for its disappearance.

10. Dew will not usually collect on a pitcher of ice water standing in a warm room on a cold winter day. Explain.

CHAPTER VI

MOLECULAR FORCES[1]

Molecular Forces in Solids. Elasticity

151. Proof of the existence of molecular forces in solids. The fact that a gas will expand without limit as the volume of the containing vessel is increased seems to show very conclusively that the molecules of gases do not exert any appreciable attractive forces upon one another. In fact, all of the experiments of the last chapter upon gases showed that such substances certainly behave as they would if they consisted of *independent molecules* moving hither and thither with great velocities and influencing each other's motions only at the instants of collision. Between collisions the molecules doubtless move in straight lines. It must not, however, be thought that the distances moved by a single molecule between successive collisions are large. In ordinary air these distances probably do not average more than .00009 mm. Small, however, as this distance is, it is at least one hundred times the diameter of a molecule.

But that the molecules of *solids*, on the other hand, cling together with forces of great magnitude is proved by some of the simplest facts of nature; for solids not only do not expand like gases, but it often requires enormous forces to pull their molecules apart. Thus a rod of cast steel 1 cm. in diameter may be loaded with a weight of 8.8 tons before it will be pulled in two.

[1] This chapter should be preceded by a laboratory experiment in which Hooke's law is discovered by the pupil for certain kinds of deformation easily measured in the laboratory. See, for example, Experiment 12 of the authors' manual.

152. Tenacities, or tensile strengths. In order to compare the strengths of the forces which hold together the molecules of different substances, let three wires, all of the same diameter, e.g. .25 mm. (number 30), but consisting of three different materials, such as steel, brass, and aluminum, be wrapped side by side about a cylindrical rod, as in Fig. 105, and weights added successively to the wires until they break. The breaking weights will be found to differ greatly for the three wires.

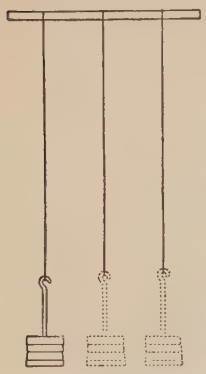


Fig. 105. Tensile strength of wires

Tests made by methods similar to the above show that the *tenacities*, or *tensile strengths*, of wires of the same material are directly proportional to the cross sections. This was to have been expected, since doubling the cross section doubles the number of molecules which must be pulled apart. The following are the weights in kilograms necessary to break drawn wires of different materials, 1 sq. mm. in cross section.

Lead, 2.6	Copper, 51	Iron, 77
Silver, 37	Platinum, 43	Steel, 91

153. Elasticity. We can obtain additional information about the molecular forces existing in different substances by studying what happens when the weights applied are not large enough to break the wires.

Thus let a long steel wire, e.g. number 26 piano wire, be suspended from a hook in the ceiling, and let the lower end be wrapped tightly about one end of a meter stick, as in Fig. 106. Let a fulcrum c be placed in a notch in the stick at a distance of about 5 cm. from the point of attachment to the wire, and let the other end of the stick be provided with a knitting needle, one end of which is opposite the vertical mirror scale S. Let enough weights be applied to the pan P to place the wire under slight tension; then let the reading of the pointer p on the scale S be taken. Let three or four kilogram weights be added successively to the pan and the corresponding positions of the pointer read. Then let the readings be taken again as the weights are

successively removed. In this last operation the pointer will probably be found to come back exactly to its first position.

This characteristic which the steel has shown in this experiment, of returning to its original length when the stretching weights are removed, is an illustration of a property possessed to a greater or less extent by all solid bodies. It is called *elasticity.*

154. The measure of elasticity. The relative amounts of elasticity possessed by different substances are found by subjecting wires of exactly the same dimensions, but of different materials, to tests like that used on the steel wire above. One substance is said to have twice as high an elastic constant, or simply to be twice as *elastic* as another, when it requires twice as much force to produce the same stretch; or, to state the same thing in a slightly different way, when with the same stretch it tends to spring back with twice the force. Thus it is found that if it requires 20 kg. to stretch a given steel wire through 1 mm., it will require but 12 kg. to stretch an exactly similar copper wire through 1 mm., and 6 kg. to produce the same stretch in a similar wire of aluminum. Steel is therefore about 1.7 times as elastic as copper and 3.3 times as elastic as aluminum. It will be seen that when elasticity is measured in this way India rubber has a very small elastic constant, for it requires only a very small force to produce a considerable stretch.

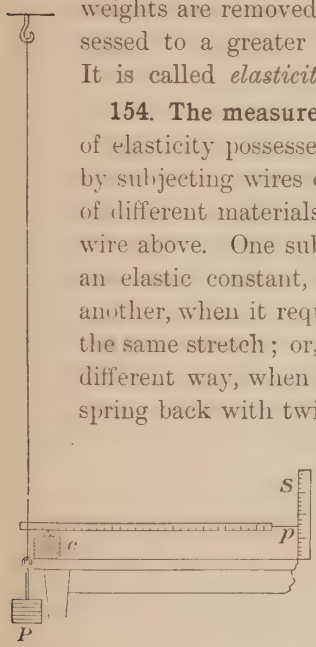


Fig. 106. Elasticity of a steel wire

155. Limits of perfect elasticity. If a sufficiently large weight is applied to the end of the wire of Fig. 106, it will be found that the pointer does not return exactly to its original position when the weight is removed. We say, therefore, that

steel is *perfectly elastic* only so long as the distorting forces are kept within certain limits, and that, as soon as these limits are overstepped, it no longer shows perfect elasticity.* Different substances differ very greatly in the amount of distortion which they can sustain before they show this failure to return completely to the original shape. Thus a drawn copper wire 1 mm. in diameter shows perfect elasticity until the stretching force exceeds about 12 kg., while a similar steel wire returns completely to its original length so long as the stretching force is less than 42 kg. Since, according to the results of § 154, it will require only 1.8×12, or 21 kg., to stretch the steel wire as far as the 12 kg. stretch the copper wire, it will be seen that the limits of perfect elasticity for steel are twice as wide as they are for copper.

There are some substances whose elasticity, measured by the method of § 154, is very small, but which nevertheless show *perfect elasticity* within very wide limits. India rubber is such a substance. When, in popular language, we speak of this substance as being very elastic, we have in mind the width of its elastic range rather than the numerical value of its elastic constant. In scientific discussion it is necessary to distinguish carefully between these two ideas. In this book a substance will be said to have a high elasticity only when it requires a large force to produce a small deformation.

156. Hooke's law. If we examine the stretches produced by the successive addition of kilogram weights in the experiment of § 153, Fig. 106, we shall find that these stretches are all equal, at least within the limits of observational error. Very carefully conducted experiments have shown that this law, namely that the successive application of equal forces produces a succession of equal stretches, holds very exactly for all sorts of elastic displacements, so long, and only so long, as the limits of perfect elasticity are not overstepped. This law is known as *Hooke's law*, after the Englishman, Robert Hooke (1635–1703). Another

way of stating this law is the following. *Within the limits of perfect elasticity elastic deformations of any sort, be they twists or bends or stretches, are directly proportional to the forces producing them.*

157. Molecular forces *vs.* **molecular motions.** The above experiments have shown that when the molecules of a solid are pulled farther apart than their natural distances, they tend to come back to these distances. Precisely similar experiments on compression show that if they are pushed closer together than their natural distances, they tend to spring apart. Thus, if one attempts to compress a rubber ball, a steel ball, an ivory ball, or almost any sort of a solid body, as soon as the force is removed the body will return to its natural size unless the compression has been carried too far.

At a given temperature, then, the molecules of any solid tend to remain a given distance apart and resist any attempt to increase or decrease this distance. The question which at once suggests itself is, Why do not the attractive forces existing between the molecules pull them into the most intimate contact possible, so that no spaces whatever are left between them, and no compressing forces can press them closer together. The answer is found in the effects of heat on solid bodies. The molecules do in fact come closer together as soon as we lower the temperature, i.e. as soon as we decrease the velocity with which the molecules are oscillating back and forth within their little intermolecular spaces, and they push out to greater distances as soon as we raise the temperature. The size which a given solid body possesses at any given temperature is then the result of a balance between two opposing tendencies, one a tendency to come as close together as possible on account of the *attractions* of the molecules, and the other a tendency to expand indefinitely like gases, because of the *motions* of the molecules. If we diminish the motions by lowering the temperature, we destroy the balance and the forces pull the molecules closer

together. If we increase the motions by raising the temperature, we render them more effective than the attractive forces, and the body expands. So long, however, as the temperature remains constant any attempt to press the molecules closer together or to push them farther apart is resisted, the one by the motions, the other by the attractive forces.

158. Cohesion and adhesion. The preceding experiments have brought out the fact that in the solid condition, at least, molecules of the same kind exert attractive forces upon one another. That molecules of unlike substances also exert mutually attractive forces is equally true, as is proved by the fact that glue sticks to wood with tremendous tenacity, mortar to bricks, nickel plating to iron, etc.

The forces which bind *like* kinds of molecules together are commonly called *cohesive forces;* those which bind together molecules of *unlike* kind are called *adhesive* forces. Thus we say that mucilage sticks to wood because of *adhesion,* while wood itself holds together because of *cohesion.* Again, adhesion holds the chalk to the blackboard, while cohesion holds together the particles of the crayon.

159. Properties of solids depending on cohesion. Many of the physical properties in which solid substances differ from one another depend on differences in the cohesive forces existing between their molecules. Thus we are accustomed to classify solids with relation to their hardness, brittleness, ductility, malleability, tenacity, elasticity, etc. The last two of these terms have been sufficiently explained in the preceding paragraphs; but since confusion sometimes arises from failure to understand the first four, the tests for these properties are here given.

We test the relative *hardness* of two bodies by seeing which will *scratch* the other. Thus the diamond is the hardest of all substances, since it scratches all others and is scratched by none.

We test the relative *brittleness* of two substances by seeing which will *break* most easily under a blow from a hammer.

Thus glass and ice are very brittle substances; lead and copper are not.

We test the relative *ductility* of two bodies by seeing which can be *drawn into the thinner wire.* Platinum is the most ductile of all substances. It has been drawn into wires but .00003 inch in diameter. Glass is also very ductile when sufficiently hot, as may be readily shown by heating it to softness in a Bunsen flame, when it may be drawn into threads which are so fine as to be almost invisible.

We test the relative *malleability* of two substances by seeing which can be *hammered into the thinner sheet.* Gold, the most malleable of all substances, has been hammered into sheets $\frac{1}{300000}$ inch in thickness.

QUESTIONS AND PROBLEMS

1. What must be the diameter of a wire of copper if it is to have the same tensile strength as a wire of iron 1 mm. in diameter?

2. How many times greater must the diameter of one wire be than that of another of the same material if it is to have 5 times the tensile strength?

3. If a force of 1 kg. stretches a wire of given length and diameter 1 mm., find the force required to stretch a wire of the same material and of the same length, but of twice the diameter, through 3 mm.

4. If 1 kg. stretches a wire 1 mm. in diameter .5 mm., how far will a wire of the same length but of twice the diameter be stretched by 7 kg.?

5. If the position of the pointer on a spring balance is marked when no load is on the spring, and again when the spring is stretched with a load of 10 g., and if the space between the two marks is then divided into ten equal parts, will each of these parts represent a gram? Why?

Molecular Forces in Liquids. Capillary Phenomena

160. Proof of the existence of molecular forces in liquids. The facility with which liquids change their shape might lead us to suspect that the molecules of such substances exert almost no forces upon one another, but a simple experiment will show that this is far from true.

By means of sealing wax and string let a glass plate be suspended horizontally from one arm of a balance, as in Fig. 107. After equilibrium is obtained let a surface of water be placed just beneath the plate and the beam pushed down until contact is made. It will be found necessary to add a considerable weight to the opposite pan in order to pull the plate away from the water. Since a layer of water will be found to cling to the glass it is evident that the added force applied to the pan has been expended in pulling water molecules away from water molecules, not in pulling glass away from water. Similar experiments may be performed with all liquids. In the case of mercury the glass will not be found to be wet, showing that the cohesion of mercury is greater than the adhesion of glass and mercury.

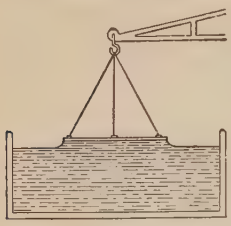


Fig. 107. Illustrating cohesion of water

161. Shape assumed by a free liquid. Since, then, every molecule of a liquid is pulling on every other molecule, any body of liquid which is free to take its natural shape, i.e. which is acted on only by its own cohesive forces, must draw itself together until it has the smallest possible surface compatible with its volume; for, since every molecule in the surface is drawn toward the interior by the attraction of the molecules within, it is clear that molecules must continually move toward the center of the mass until the whole has reached the most compact form possible. Now the geometrical figure which has the smallest area for a given volume is a sphere. We conclude, therefore, that if we could relieve a body of liquid from the action of gravity and other outside forces, it would at once take the form of a perfect sphere. This conclusion may be easily verified by the following experiment.

Let alcohol be added to water until a solution is obtained in which a drop of common lubricating oil will float at any depth. Then with a pipette insert a large globule of oil beneath the surface. The oil will be seen to float as a perfect sphere within the body of the liquid (Fig. 108). (Unless the drop is viewed from above, the vessel should

have flat rather than cylindrical sides, otherwise the curved surface of the water will act like a lens and make the drop *appear* flattened.)

The reason that liquids are not more commonly observed to take the spherical form is that ordinarily the force of gravity is so large as to be more influential in determining their shape than are the cohesive forces. As verification of this statement we have only to observe that as a body of liquid becomes smaller and smaller, — i.e. as the gravitational forces upon it become less and less, — it does indeed tend more and more to take the spherical form. Thus very small globules of mercury on a table will be found to be almost perfect spheres, and raindrops or minute *floating* particles of all liquids are quite accurately spherical.

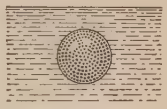


Fig. 108. Spherical globule of oil, freed from action of gravity

162. Contractility of liquid films. The tendency of liquids to assume the smallest possible surface furnishes a simple explanation of the contractility of liquid films.

Let a soap bubble two or three inches in diameter be blown on the bowl of a pipe and then allowed to stand. It will at once begin to shrink in size and in a few minutes will disappear within the bowl of

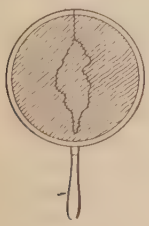


Fig. 109

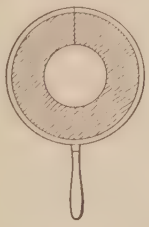


Fig. 110

Fig. 111

Illustrating the contractility of soap films

the pipe. The liquid of the bubble is simply obeying the tendency to reduce its surface to a minimum, a tendency which is due only to the mutual attractions which its molecules exert upon one another.

A candle flame held opposite the opening in the stem of the pipe will be deflected by the current of air which the contracting bubble is forcing out through the stem.

Again, let a loop of fine thread be tied to the edge of a wire ring, as in Fig. 109. Let the ring be dipped into a soap solution so as to form a film across it, and then let a hot wire be thrust through the film inside the loop. The tendency of the film outside of the loop to contract will instantly snap out the thread into a perfect circle (Fig. 110). The reason that the thread takes the circular form is that since the film outside the loop is striving to assume the smallest possible surface, the area inside the loop must of course become as large as possible. The circle is the figure which has the largest possible area for a given perimeter.

Let a soap film be formed across the mouth of a funnel, as in Fig. 111. The tendency of the film to contract will cause it to run quickly toward the small end of the funnel.

163. Ascension and depression of liquids in capillary tubes. It was shown in Chapter III that, in general, a liquid stands at the same level in any number of communicating vessels. The following experiments will show that this rule ceases to hold in the case of tubes of small diameter.

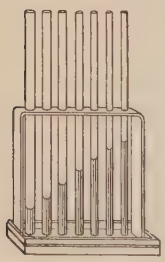


Fig. 112. Rise of liquids in capillary tubes

Let a series of capillary tubes of diameter varying from 2 mm. to .1 mm. be arranged as in Fig. 112.

When water is poured into the vessel it will be found to rise higher in the tubes than in the vessel, and it will be seen that the smaller the tube the greater the height to which it rises. If the water is replaced by mercury, however, the effects will be found to be just inverted. The mercury is depressed in all the tubes, the depression being greater in proportion as the tube is smaller (Fig. 113). This depression is most easily observed with a U-tube like that shown in Fig. 114.

Experiments of this sort have established the following laws.

1. *Liquids rise in capillary tubes when they are capable of wetting them, but are depressed in tubes which they do not wet.*

2. *The elevation in the one case, and the depression in the other, are inversely proportional to the diameters of the tubes.*

It will be noticed, too, that when a liquid rises, its surface within the tube is concave upward, and when it is depressed its surface is convex upward.

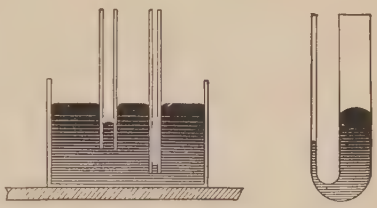


Fig. 113 Fig. 114

Depression of mercury in capillary tubes

164. Cause of curvature of a liquid surface in a capillary tube. All of the effects presented in the last paragraph can be explained by a consideration of cohesive and adhesive forces.

We shall take as the starting point of our reasoning the familiar fact that the surface of any large body of liquid is always horizontal, while the direction of the force acting upon its particles, i.e. gravity, is vertical. This shows that *when a liquid is at rest its surface is always at right angles to the direction of the final or resultant force which acts upon it.*

The second fact upon which the explanation will rest is one the truth of which was demonstrated by the spherical shape assumed by very small globules of liquid (see § 161). It is that the force of gravity acting on a *very small* body of liquid is negligible in comparison with its own cohesive force.

These two points established, consider a very small body of liquid close to the point o (Fig. 115), where water is in contact with the glass wall of the tube. Let the quantity of liquid considered be so minute that the force of gravity acting upon it may be disregarded. The force of *adhesion* of the wall will pull the liquid particles at o in the direction oE. The force of *cohesion* of the liquid will pull these same particles in the direction oF.

It was shown in Chapter II that if the lengths of the lines oE and oF are made proportional to the relative strengths of

these two forces, the actual direction and magnitude of the resultant force will be represented by the direction and magnitude of the diagonal oR of the parallelogram of which oE and oF are two adjacent sides (Fig. 115).

If, then, the adhesive force oE greatly exceeds the cohesive force oF, the direction oR of the resultant force will lie to the

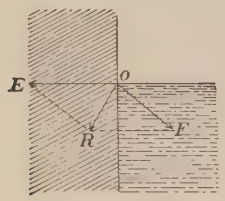


Fig. 115

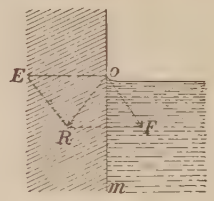


Fig. 116

Condition for elevation of a liquid near a wall

left of the vertical om, in which case, since a liquid always sets itself so that its surface is at right angles to the resultant force, the liquid about o must set itself in the position shown in Fig. 116; i.e. it must rise up against the wall as water does against glass.

If the cohesive force oF (Fig. 117) is strong in comparison with the adhesive force oE, the resultant oR will fall to the right of the vertical, in which case the liquid must be depressed about o.

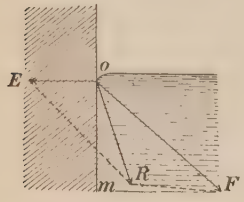


Fig. 117. Condition for depression of liquid near a wall

Whether, then, a liquid will rise against a solid wall or be depressed by it, will depend only on the relative strengths of the adhesion of the wall for the liquid and the cohesion of the liquid for itself. Since mercury does not wet glass we know that cohesion is here relatively strong, and we should expect, therefore, that the mercury would be depressed, as indeed we find it to be. The fact that water will wet glass

indicates that in this case adhesion is relatively strong, and hence we should expect water to rise against the walls of the containing vessel, as in fact it does.

It is clear that a liquid which is depressed near the edge of a vertical solid wall must assume within a tube a surface which is *convex upward*, while a liquid which rises against a wall must within such a tube be *concave upward*.

165. Explanation of ascension and depression in capillary tubes. The fact that liquids assume curved surfaces within tubes makes it easy to see why a liquid which is concave must rise and one which is convex must fall. For, consider first a

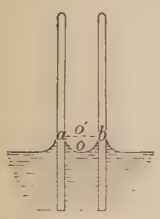


FIG. 118

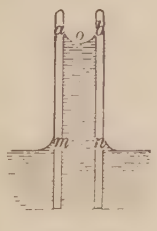


FIG. 119

A concave meniscus causes a rise in capillary tube

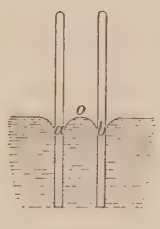


FIG. 120

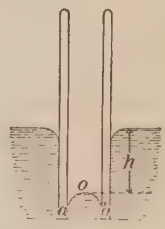


FIG. 121

A convex meniscus causes a fall

liquid which, because of the strength of the adhesion between it and the walls of the tube, assumes a *concave* surface within the tube (Fig. 118). It was shown in §§ 161 and 162 that the mutual attraction of the molecules of a liquid for one another always exhibits itself as a tendency to reduce the exposed surface of the liquid to a minimum. Hence this concave surface *aob* (Fig. 118) must tend to straighten out into the flat surface *ao'b*. But it no sooner thus begins to straighten out than adhesion again elevates it at the edges. It will be seen, therefore, that the liquid must continue to rise in the tube until the weight of the volume of liquid lifted, namely *amnb* (Fig. 119), balances the tendency of the surface *aob* to move up. That the

liquid will rise higher in a small tube than in a large one is to be expected, since the weight of the column of liquid to be supported in the small tube is less.

Precisely the same method of reasoning applied to the convex mercury surface *aob* (Fig. 120) shows why the mercury must fall in a capillary tube until the upward pressure at *o*, due to the depth *h* of mercury (Fig. 121), balances the tendency of the surface *aob* to flatten out.

166. Capillary phenomena in everyday life. Capillary phenomena play a very important part in the processes of nature and of everyday life. Thus the rise of oil in wicks of lamps, the complete wetting of a towel when one end of it is allowed to stand in a basin of water, the rapid absorption of liquid by a lump of sugar when one corner of it only is immersed, the taking up of ink by blotting paper, are all illustrations of precisely the same phenomena which we observe in the capillary tubes of Fig. 112.

167. Floating of small objects on water. Let a needle be laid very carefully on the surface of a dish of water. In spite of the fact that it is nearly eight times as dense as water it will be found to float. If the needle has been previously magnetized, it may be made to move about in any direction over the surface in obedience to the pull of the magnet.

To discover the cause of this apparently impossible phenomenon, examine closely the surface of the water in the immediate neighborhood of the needle. It will be found to be depressed in the manner shown in Fig. 122. This furnishes at once the explanation. So long as the needle is so small that its own weight is no greater than the upward force exerted upon it by the tendency of the depressed (and therefore concave) liquid surface to straighten out into a flat surface, the needle could not sink in the liquid, no matter how great its density. If the water had wetted the needle, i.e. if it had risen

Fig. 122. Cross section of a floating needle

about the needle instead of being depressed, the tendency of the liquid surface to flatten out would have pulled it down into the liquid instead of forcing it upward. Any body about which a liquid is depressed will therefore float on the surface of the liquid if its mass is not too great. Even if the body, when perfectly clean, causes the liquid to rise about it, an imperceptible film of oil on its surface will cause it to depress the liquid, and hence to float.

FIG. 123. Insect walking on water

The above experiment explains the familiar phenomenon of insects walking and running on the surface of water (Fig. 123) in apparent contradiction to the law of Archimedes, in accordance with which they should sink until they displace their own weight of the liquid.

QUESTIONS AND PROBLEMS

1. Why will blotting paper soak up ink so much more readily than glazed paper?

2. If water will rise 32 cm. in a tube .1 mm. in diameter, how high will it rise in a tube .01 mm. in diameter?

3. Candle grease may be removed from clothing by covering it with blotting paper and then passing a hot flatiron over the paper. Explain.

4. Why does a small stream of water break up into drops instead of falling as a continuous thread?

5. Why will a piece of sharp-cornered glass become rounded when heated to redness in a Bunsen flame?

6. The leads for pencils are made by subjecting powdered graphite to enormous pressures produced by hydraulic machines. Explain how the pressure changes the powder to a coherent mass.

SOLUTION AND CRYSTALLIZATION

168. Solution and molecular force. Let a speck of permanganate of potash, about as big as a pin head, be dropped into a quart flask full of water. The water will at once begin to be colored about the particle, and in a short time the particle itself will have completely disappeared. After a little shaking the whole body of water will have acquired a rich red tint.

This process of the solution of solids in liquids, so familiar to us from the use of salt and sugar in liquid foods, furnishes a good illustration of the differences in the attraction which the molecules of the same liquid exert on the molecules of different solids, or which the molecules of the same solid exert on those of different liquids. At ordinary temperatures water dissolves three times as much common table salt as does alcohol, and it dissolves gum arabic quite readily, whereas alcohol scarcely dissolves it at all. On the other hand, resin, shellac, etc., are readily soluble in alcohol, but quite insoluble in water. Benzine and gasoline are used for removing grease spots from clothing, because most forms of grease, although insoluble in water, are readily soluble in these liquids. Beeswax, which is not appreciably dissolved by water, alcohol, or benzine, is quite readily dissolved in turpentine.

From these facts it is clear that *adhesive forces* have much to do with the process of solution. On the other hand, the *motions of the molecules* must also be intimately concerned with this process, for we have seen that the facts of the evaporation of ice and of other solids prove that even where there are no adhesive forces pulling the molecules of a solid from one another, the motions alone cause some of them to escape from the surface and to pass off into the space above. This tendency to pass off must be present as well when the space is filled with a liquid as when it is empty.

169. Saturated solutions. The last conclusion is confirmed when we find that in many respects solution is analogous to evaporation. Just as at a given temperature only a certain amount of liquid will evaporate into a closed space, so also there is a definite limit to the amount of a solid which will dissolve at any temperature in a given body of liquid. This is proved by the familiar fact that after a certain amount of sugar has been added to a cup of coffee, further addition simply deposits so much more sugar in the bottom of the cup. At ordinary

temperatures the maximum amount of common salt which can be made to dissolve in 100 g. of water is about 36 g.

Now just as a vapor which has reached its highest possible density is called a saturated vapor, so a solution which contains as large an amount of a solid as it is capable of taking up is called a *saturated solution.*

170. Saturation and temperature. In the last chapter it was found that a liquid or a solid evaporates more readily at a high temperature than at a low one, — a fact which is readily explained by the theory that an increase in temperature means an increase in the average velocity of the molecules. It is to be expected from the same theory that increase in temperature will increase the ease with which a solid substance goes into solution in a liquid. For, as suggested above, the increased motions can be no less effective in causing molecules to leave the solid and pass off into the space above when that space is filled with a liquid than when it is empty. In the former case the adhesive force and the motion of the molecules *together* effect the disintegration of the solid, while in the latter the motions are the only agents at work.

As a matter of fact, experiment shows that it is true, in general, that solids are dissolved much more readily in hot liquids than in cold ones. It is for no other reason than this that hot water is so much more effective than cold for cleaning purposes. At 0° C. it requires but 13 g. of saltpeter (potassium nitrate) in 100 cc. of water to form a saturated solution. At 20° C. it requires 31 g.; at 40°, 64 g.; at 60°, 111 g.; at 80°, 172 g.; and at 100°, 247 g.

171. Effect of evaporating a solution. When a solution evaporates it is, in general, the liquid only which passes off into a vaporous condition, practically all of the dissolved substance remaining behind. This is proved by the fact that the rain which falls at sea is fresh water and not salt water, and by the fact that impurities are removed from water by distillation.

172. Effect of evaporating a saturated solution or of lowering its temperature. If a saturated solution is evaporating, it must soon become more than saturated, for the same amount of dissolved substance remains, while the volume of the solution continually diminishes. The result is exactly what would be expected from the analogy between solution and evaporation. It will be remembered that when the volume of a saturated vapor was diminished a part of the vapor condensed. So when the saturated solution evaporates the dissolved substance gradually separates out in the *solid* condition. This is illustrated by the fact that the evaporation of salt sea spray leaves the face and clothing covered with salt.

Again, just as there is a second way of causing a saturated vapor to condense, namely by lowering its temperature, so lowering the temperature of a saturated solution will also cause the molecules of the dissolved substance to pass out of solution and to collect in the solid form.

173. Crystallization. If the separation of a solid from a solution is made to take place slowly and quietly, by either of the above methods, the beautiful and striking phenomena of *crystallization* may be observed. The molecules of the separating solid group themselves in regular geometric forms. These forms vary greatly with the nature of the dissolved substance, thus indicating differences in the nature of the cohesive forces which act to bring the molecules together.

Thus if a saturated solution of common salt is filtered and then set aside, after twenty-four hours groups of crystals will be found floating on the surface. If one of these is carefully removed and examined with a magnifying glass, the crystals will be found to be perfect little cubes.

Again, if a thread be hung in a beaker or large test tube containing a saturated solution of alum, in a few days the thread will be covered with octahedral crystals (Fig. 124) about the size of a pea.

If copper sulphate be treated in the same way, large blue crystals of the form shown in Fig. 125 will collect on the thread.

If a hot saturated solution of saltpeter (potassium nitrate) be placed in a beaker and closely watched as it cools, it will be found possible to

Fig. 124. Alum crystal

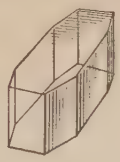


Fig. 125. Copper sulphate crystal

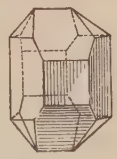


Fig. 126. Potassium nitrate crystal

actually see the process of the formation and growth of crystals of the form shown in Fig. 126.

Wherever the crystals are in contact with the sides of the vessel the free formation is interfered with and the resulting forms are very irregular.

Most minerals are found on microscopic study to have a crystalline structure, though in nature they have usually been formed under conditions which render it impossible for the crystals to be large and regular.

Diamond is carbon crystallized under conditions which once existed in nature but which man has been able to reproduce in the laboratory only upon a very diminutive scale.

Absorption of Gases by Solids and Liquids

174. Absorption of gases by solids. Let a large test tube be filled with ammonia gas by heating aqua ammonia and causing the evolved gas to displace mercury in the tube, as in Fig. 127. Let a piece of charcoal an inch long and nearly as wide as the tube be heated to redness and then plunged beneath the mercury. When it is cool let it be slipped underneath the mouth of the test tube and allowed to rise into the gas. The mercury will be seen to

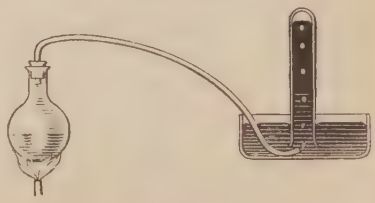


Fig. 127. Filling tube with ammonia

rise in the tube, as in Fig. 128, thus showing that the gas is being absorbed by the charcoal. If the gas is unmixed with air, the mercury will rise to the very top of the tube, thus showing that all the ammonia has been absorbed by the charcoal.

This property of absorbing gases is possessed to a notable degree by porous substances, such as charcoal, meerschaum, gypsum, silk, etc. It is not improbable that all solids hold, closely adhering to their surfaces, thin layers of the gases with which they are in contact, and that the prominence of the phenomena of absorption in porous substances is due to the great extent of surface possessed by such substances.

Fig. 128. Absorption of ammonia gas by charcoal

That the same substance exerts widely different attractions upon the molecules of different gases is shown by the fact that charcoal will absorb 90 times its own volume of ammonia gas, 35 times its volume of carbon dioxide, and but 1.7 times its volume of hydrogen. The usefulness of charcoal as a deodorizer is due to its enormous ability to absorb certain kinds of gases.

175. Absorption of gases in liquids. Let a beaker containing cold water be slowly heated. Small bubbles of air will be seen to collect in great numbers upon the walls and to rise through the liquid to the surface. That they are indeed bubbles of air and not of steam is proved first by the fact that they appear when the temperature is far below boiling, and second by the fact that they do not condense as they rise into the higher and cooler layers of the water.

The experiment shows two things,—first, that water ordinarily contains considerable quantities of air dissolved in it, and second, that the amount of air which water can hold decreases as the temperature rises. The first point is also proved by the existence of fish life, for fishes obtain the oxygen which they

need to support life, not immediately from the water but from the air which is dissolved in it.

The amount of gas which will be absorbed by water varies greatly with the nature of the gas. At 0° C. and 76 cm. barometer height 1 cc. of water will absorb 1050 cc. of ammonia, 1.8 cc. of carbon dioxide, and but .04 cc. of oxygen. Ammonia itself is a gas under ordinary conditions. The commercial aqua ammonia is simply ammonia gas dissolved in water.

The following experiment illustrates the absorption of ammonia by water.

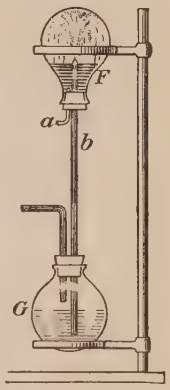


Fig. 129. Absorption of ammonia by water

Let the flask F (Fig. 129) and tube b be filled with ammonia by passing a current of the gas in at a and out through b. Then let a be corked up and b thrust into G, a flask nearly filled with water which has been colored slightly red by the addition of litmus and a drop or two of acid. As the ammonia is absorbed the water will slowly rise in b, and as soon as it reaches F it will rush up very rapidly until the upper flask is nearly full. At the same time the color will change from red to blue because of the action of the ammonia upon the litmus.

Experiment shows that *in every case of absorption of a gas by a liquid or solid, the quantity of gas absorbed decreases with an increase in temperature,* — a result which was to have been expected from the kinetic theory, since increasing the molecular velocity must of course increase the difficulty which the adhesive forces have in retaining the gaseous molecules.

It will be noticed that the effect of temperature upon solution (§ 170) is quite the opposite of its effect upon absorption.

176. Effect of pressure upon absorption. Soda water is ordinary water which has been made to absorb large quantities of carbon dioxide gas. This impregnation is accomplished by bringing the water into contact with the gas under high pressure.

As soon as the pressure is relieved the gas passes rapidly out of solution. This is the cause of the characteristic effervescence of soda water. These facts show clearly that the amount of carbon dioxide which can be absorbed by water is greater for high pressures than for low. As a matter of fact, careful experiments have shown that the amount of any gas absorbed is directly proportional to the pressure, so that if carbon dioxide under a pressure of 10 atmospheres is brought into contact with water, 10 times as much of the gas is absorbed as if it had been under a pressure of 1 atmosphere.

QUESTIONS AND PROBLEMS

1. Why do fishes in an aquarium die if the water is not frequently renewed?

2. Explain the apparent generation of ammonia gas when aqua ammonia is heated.

3. Why in the experiment illustrated in Fig. 129 was the flow so much more rapid after the water began to run over into *F*?

4. How can you tell whether bubbles which rise from the bottom of a vessel which is being heated are bubbles of air or bubbles of steam?

5. What is the density of a body which weighs 1200 g. in water and 1260 g. in another liquid of density .8?

6. The largest balloon ever made had a volume of 883,000 cu. ft. Assuming that air weighs 1 lb. for 12 cu. ft., and that hydrogen is one fourteenth as dense as air, find the total lifting power of this balloon when filled with hydrogen. If the balloon and attachments weighed 30,000 lb., how much cargo could the balloon carry?

7. Find the total force against a dam 30 ft. long, when the water stands 20 ft. higher on one side than on the other.

CHAPTER VII

THERMOMETRY–EXPANSION COEFFICIENTS[1]

THERMOMETRY

177. Meaning of temperature. When a body feels hot to the touch we are accustomed to say that it has a *high temperature*, and when it feels cold that it has a *low temperature*. Thus the word "temperature" is used to denote the condition of hotness or coldness of the body whose state is being described.

178. Measurement of temperature. So far as we know, up to the time of Galileo no one had ever used any special instrument for the measurement of temperature. People knew how hot or how cold it was from their feelings only. But under some conditions this temperature sense is a very unreliable guide. For example, if the hand has been in hot water, tepid water will feel cold; while if it has been in cold water, the same tepid water will feel warm; a room may feel hot to one who has been running, while it will feel cool to one who has been sitting still.

Difficulties of this sort have led to the introduction in modern times of mechanical devices for measuring temperature, called *thermometers*. These instruments depend for their operation upon the fact that practically all bodies expand as they grow hot.

179. Galileo's thermometer. It was in 1592 that Galileo, at the University of Padua in Italy, constructed the first thermometer. He was familiar with the facts of expansion of solids,

[1] It is recommended that this chapter be preceded by laboratory measurements on the expansions of a gas and a solid. See, for example, Experiments 13 and 14, authors' manual.

liquids, and gases; and, since gases expand more than solids or liquids, he chose a gas as his expanding substance. His device was that shown in Fig. 130.

Fig. 130. Galileo's thermometer

The relative hotness of two bodies was compared by observing which one of the two, when placed in contact with the air bulb, caused the liquid to descend farther in the stem *S*. As a matter of fact, barometric changes as well as temperature changes cause changes in the height of the liquid in the stem of such an instrument, but Galileo does not seem to have been aware of this fact.

It was not until about 1700 that mercury thermometers were invented. On account of their extreme convenience these have now replaced all others for practical purposes.

180. The construction of a Centigrade mercury thermometer. The meaning of a degree of temperature change is best understood from a description of the method of making and graduating a mercury thermometer.

A bulb is blown at one end of a piece of thick-walled glass tubing of small uniform bore. Bulb and tube are then filled with mercury, at a temperature slightly above the highest temperature for which the thermometer is to be used, and the tube is sealed off in a hot flame. As the mercury cools it contracts and falls away from the top of the tube, leaving a vacuum above it.

Fig. 131. Method of finding the 0° point of a thermometer

The bulb is next surrounded with melting snow or ice, as in Fig. 131, and the point at which the mercury stands in the tube is marked 0°. Then the bulb and tube are placed in the steam rising from boiling water, as in Fig. 132,

and the new position of the mercury is marked 100°. The space between these two marks on the stem is then divided into 100 equal parts, and divisions of the same length are extended above the 100° mark and below the 0° mark.

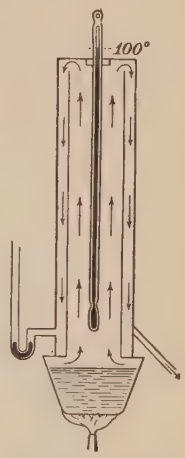


FIG. 132. Method of finding the 100° point of a thermometer

One degree of change in temperature, measured on such a thermometer, means, then, such a temperature change as will cause the mercury in the stem to move over one of these divisions; i.e. it is such a temperature change as will cause mercury contained in a glass bulb to expand $\frac{1}{100}$ of the amount which it expands in passing from the temperature of melting ice to that of boiling water. A thermometer in which the scale is divided in this way is called a Centigrade thermometer.

Thermometers graduated on the Centigrade scale are used almost exclusively in scientific work, and also for ordinary purposes in most countries which have adopted the metric system. This scale was first devised in 1742 by Celsius, of Upsala, Sweden. For this reason it is sometimes called the Celsius instead of the Centigrade scale.

181. Fahrenheit thermometers. The common household thermometer in England and the United States differs from the Centigrade only in the manner of its graduation. In its construction the temperature of melting ice is marked 32° instead of 0°, and that of boiling water 212° instead of 100°. The intervening stem is then divided into 180 parts. The zero of this scale is the temperature obtained by mixing equal weights of sal ammoniac (ammonium chloride) and snow. In 1714, when Fahrenheit, of Danzig, Germany, devised this scale, he chose this zero because he thought it represented the lowest possible temperature, i.e. the entire absence of heat.

182. Comparison of Centigrade and Fahrenheit thermometers. From the methods of graduation of the Fahrenheit and Centigrade thermometers it will be seen that 100° on the Centigrade scale denotes the same difference of temperature as 180° on the Fahrenheit scale (Fig. 133). Hence one Fahrenheit degree is equal to five ninths of a Centigrade degree, and one Centigrade degree is equal to nine fifths of a Fahrenheit degree. *Hence to reduce from the Fahrenheit to the Centigrade scale, first find how many Fahrenheit degrees the given temperature is above or below the freezing temperature, and then multiply by five ninths.*

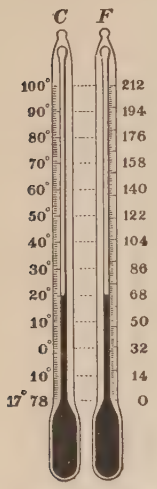


Fig. 133. The Centigrade and Fahrenheit scales

To reduce from Centigrade to Fahrenheit, first multiply by nine fifths in order to find how many Fahrenheit degrees the given temperature is above or below the freezing temperature. Knowing how far it is from the freezing point, it will be very easy to find how far it is from 0° F., which is 32° below the freezing point.

183. The range of the mercury thermometer. Since mercury freezes at − 39° C., temperatures lower than this are very often measured by means of *alcohol* thermometers, for the freezing point of alcohol is − 130° C. Similarly, since the boiling point of mercury is 360° C., mercury thermometers cannot be used for measuring very high temperatures. For both very high and very low temperatures, in fact for all temperatures, a *gas* thermometer is the standard instrument.

184. The standard hydrogen thermometer. The modern gas thermometer, however (Fig. 134), is widely different from that devised by Galileo (Fig. 130). It is not usually the increase in the volume of a gas kept under constant pressure which is

taken as the measure of temperature change, but rather the increase in pressure which the molecules of a confined gas exert against the walls of a vessel whose volume is kept constant. The essential features of the method of calibration and use of the standard hydrogen thermometer at the International Bureau of Weights and Measures at Paris are as follows.

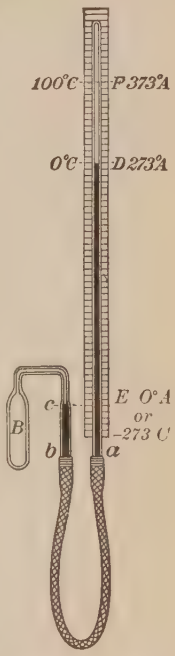


FIG. 134. The standard gas thermometer

The bulb B (Fig. 134) is first filled with hydrogen and the space above the mercury in the tube a made as nearly a perfect vacuum as possible. B is then surrounded with melting ice (as in Fig. 131) and the tube a raised or lowered until the mercury in the arm b stands exactly opposite the fixed mark c on the tube. Now, since the space above D is a vacuum, the pressure exerted by the hydrogen in B against the mercury surface at c just supports the mercury column ED. The point D is marked on a strip of metal behind the tube a. The bulb B is then placed in a steam bath like that shown in Fig. 132. The increased pressure of the gas in B at once begins to force the mercury down at c and up at D. But by raising the arm a the mercury in b is forced back again to c, the increased pressure of the gas on the surface of the mercury at c being balanced by the increased height of the mercury column supported, which is now EF instead of ED. When the gas in B is thoroughly heated to the temperature of the steam, the arm a is very carefully adjusted so that the mercury in b stands very exactly at c, its original level. A second mark is then placed on the metal strip exactly opposite the new level of the mercury, i.e. at F. D is then marked 0° C., and F is marked 100° C. The vertical distance between these marks is divided into 100 exactly equal parts. Divisions of exactly the same length are carried above the 100° mark and below the 0° mark. One degree of change in temperature is then defined as any change in temperature which will cause the pressure of the gas in B to change by the amount represented by the distance between any two of these divisions.

In other words, *one degree of change in temperature is such a temperature change as will cause the pressure exerted by a confined gas to change* $\frac{1}{100}$ *as much as it changes in passing between the temperatures of melting ice and boiling water.*

To find any unknown temperature, e.g. the temperature of liquid air, it is only necessary to immerse the bulb B in the liquid air, adjust the arm a until the mercury in b stands at c, and then take the reading of the top of the mercury on the scale behind a. If this reading is 180 divisions below the point D, the temperature of liquid air is $-180°$ C.

185. The pressure coefficient of expansion of a gas. The ratio between the increase in the pressure in B per degree rise in temperature and the total pressure which the gas in B exerts at 0° C. is called the pressure coefficient of expansion of the gas. Thus, for example, the pressure coefficient is $\frac{1}{100}$ of $\frac{DF}{ED}$. Now $\frac{DF}{ED}$ is found to be exactly $\frac{100}{273}$, or .367. Hence the pressure coefficient of hydrogen is $\frac{1}{273}$, or .00367. To state the definition in the form of an equation, let P_t be the pressure at $t°$ C. and P_0 that at 0° C.; then the increase in pressure has been $P_t - P_0$, the increase per degree has been $\frac{P_t - P_0}{t}$, and since the pressure coefficient c is this increase divided by P_0, we have

$$c = \frac{P_t - P_0}{P_0 t}. \qquad (1)$$

186. The law of Charles. In 1787 a Frenchman, Charles, discovered that *the pressure coefficients of all gases are the same;* i.e. they are all $\frac{1}{273}$. This is now called *the law of Charles.* It follows from this law that any gas thermometer may be considered as a standard thermometer, since all gas thermometers must agree with one another in their readings.

187. Comparison of gas and mercury thermometers. Since an international committee has chosen the hydrogen thermometer described in

§ 184 as the standard of temperature measurement, it is important to know whether mercury thermometers, graduated in the manner described in § 180, agree with gas thermometers at temperatures other than 0° and 100°, where, of course, they must agree, since these temperatures are in each case the starting points of the graduation. A careful comparison has shown that although they do not agree exactly, yet fortunately the disagreements at ordinary temperatures are small, not amounting to more than .2° anywhere between 0° and 100°. At 300° C., however, the difference amounts to about 4°.

Hence for all ordinary purposes mercury thermometers are sufficiently accurate, and no special standardization of them is necessary. But in all scientific work, if mercury thermometers are used at all, they must first be compared with a gas thermometer and a table of corrections obtained. The errors of an alcohol thermometer are considerably larger than those of a mercury thermometer.

188. Absolute temperature. It is clear from a description of the method of graduating and using the standard gas thermometer (§ 184) that we take as the measure of temperature the pressure which the body of air confined in B exerts, i.e. the height of the column of mercury above E. A certain increase in this height, namely an increase equal to $\frac{1}{100}$ of DF, means, by definition, 1° rise in temperature, and a decrease in this height equal to $\frac{1}{100}$ of DF means 1° fall in temperature. Now since this distance, $\frac{1}{100}$ of DF, is $\frac{1}{273}$ of ED, i.e. $\frac{1}{273}$ of the pressure which the gas exerts at 0°, it follows that if the temperature could be cooled 273° below 0° C., the level in a would be exactly the same as the level in b; i.e. the gas in the bulb would exert *no* pressure. But, from the standpoint of the kinetic theory, a gas must always exert pressure so long as its molecules are in motion. Hence the temperature at which its molecules cease to exert pressure, viz. −273° C., is the temperature at which its molecules cease to move. This temperature is called the *absolute zero*, because no lower temperature can possibly exist, since it is impossible to conceive that the gas in B can exert a pressure less than zero, i.e. that the level in a can ever fall below the level of c. A scale of temperature is now often used in which this point, namely

$-273°$ C., is taken as the zero. This is called the *absolute scale*, and temperatures expressed in terms of this scale are called *absolute temperatures*. Thus if A. is used to denote absolute temperatures and C. to denote Centigrade temperatures, we have $0°$ C. $= 273°$ A., $100°$ C. $= 373°$ A., $15°$ C. $= 288°$ A., etc. It is customary to indicate temperatures on the Centigrade scale by t and temperatures on the absolute scale by T. We have therefore

$$T = t + 273. \qquad (2)$$

189. Low temperatures. The absolute zero of temperature can, of course, never be attained, but in recent years rapid strides have been made toward it. Twenty-five years ago the lowest temperature which had ever been measured was $-110°$ C., the temperature attained by Faraday in 1845 by causing a mixture of ether and solid carbon dioxide to evaporate in a vacuum. But in 1880 air was first liquefied, and found, by means of a gas thermometer, to have a temperature of $-180°$ C. When liquid air evaporates into a space which is kept exhausted by means of an air pump, its temperature falls to about $-220°$ C. Recently hydrogen has been liquefied and found to have a temperature of $-243°$ C. All of these temperatures have been measured by means of hydrogen thermometers. By allowing liquid hydrogen to evaporate into a space kept exhausted by an air pump, Dewar in 1900 attained a temperature as low as $-260°$ C., only 13° C. above the absolute zero.

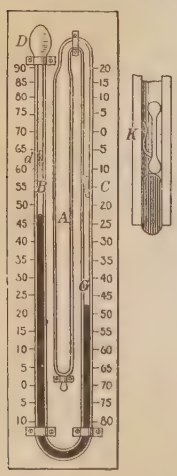
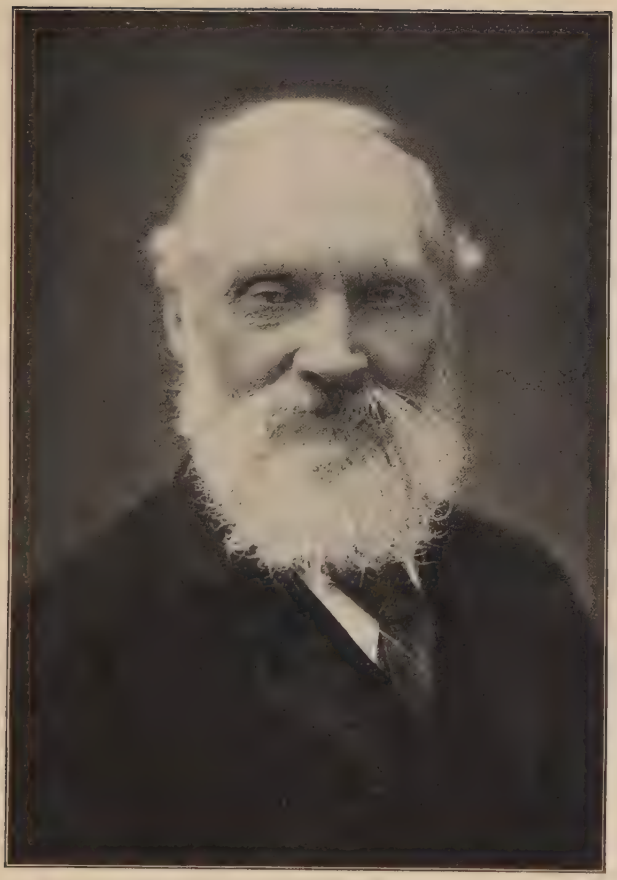


Fig. 135. The maximum and minimum thermometer

190. Maximum and minimum thermometers. In all weather bureaus the lowest temperature reached during the night, and the highest temperature reached during the day, are automatically recorded by a special device called a maximum and minimum thermometer. The construction of one form of this instrument is shown in Fig. 135.



SIR WILLIAM THOMSON, LORD KELVIN (1824-)

Best known and most prolific of living physicists; born in Belfast, Ireland; professor of physics in Glasgow University, Scotland, for more than fifty years; especially renowned for his investigations in heat and electricity; originator of the absolute thermodynamic scale of temperature; formulator of the second law of thermodynamics; inventor of the electrometer, the mirror galvanometer, and many other important electrical devices; knighted in 1866; made Baron Kelvin in 1892.

The bulb A and the stem down to the point G are filled with alcohol, from G to B the stem is filled with mercury, while the liquid above B is again alcohol. The bulb D contains only alcohol and its vapor. The two indices d and C move with slight friction in the stem. As the temperature falls the alcohol in A contracts and the mercury pushes up the index on the right and leaves it opposite the mark corresponding to the lowest temperature reached. As the temperature rises the alcohol in A expands and the mercury pushes up the index on the left and leaves it opposite the mark corresponding to the highest temperature reached. In order to obtain the right amount of friction a small steel spring is attached to the indices, as in K. After each observation the observer pulls the index back to contact with the mercury by means of a small magnet.

QUESTIONS AND PROBLEMS

1. When a Centigrade thermometer reads 15°, what is the temperature Fahrenheit?

2. When the temperature is 100° F., what is it Centigrade?

3. What temperature Centigrade corresponds to 0° F.? What to −40 F.?

4. The temperature of liquid air is −180° C. What is it Fahrenheit?

5. The lowest temperature yet attained experimentally is −260° C. What is it Fahrenheit?

6. What is the absolute zero of temperature on the Fahrenheit scale?

7. Why is a fever thermometer made with a very long cylindrical bulb, instead of a spherical one?

8. When the bulb of a thermometer is placed in hot water, it at first falls a trifle and then rises. Why?

9. How does the distance between the 0° mark and the 100° mark vary with the size of the bore, the size of the bulb remaining the same?

10. What is meant by the absolute zero of temperature?

11. Why is the temperature of liquid air lowered if it is placed under the receiver of an air pump and the air exhausted.

Expansion Coefficient of Gases

191. Volume coefficient of expansion of a gas. When we measure the pressure coefficient of expansion of a gas we keep the volume constant (Fig. 134) and observe the rate at which the pressure increases with a rise in temperature. If, however, we arrange the experiment so that the gas can expand as the

temperature rises, the pressure remaining constant, we obtain what is called *the volume coefficient of expansion.* It is defined as *the ratio between the increase in volume per degree and the total volume of the gas at zero.*

It was a Frenchman, Gay-Lussac, who in 1802 first made measurements upon this quantity and found that *all gases have the same volume coefficient of expansion,* this coefficient being the same as the pressure coefficient, namely $\frac{1}{273}$.

192. Volume proportional to absolute temperature if pressure is constant. It will be seen from the discussion of the gas thermometer (Fig. 134) that if the volume of a gas is kept constant, *the pressure is proportional to the absolute temperature;* for by definition absolute temperature is measured by the difference in the heights of the two mercury columns in a and b; but this same difference is also a measure of the pressure in B. To state this relation algebraically, let P_1 and P_2 be the pressures at the two absolute temperatures T_1 and T_2 respectively, then

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}. \tag{3}$$

Now since, by § 191, the rate of change of volume at constant pressure is the same as the rate of change of pressure at constant volume, we have also, when the pressure is constant and the volume changes from V_1 to V_2,

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}; \tag{4}$$

i.e. *the volume of a gas at constant pressure is directly proportional to the absolute temperature.*

Or, again, since volume is always inversely proportional to density, we have at once, from (4), that *at constant pressure the density of a gas is inversely proportional to its absolute temperature;* i.e.

$$\frac{D_1}{D_2} = \frac{T_2}{T_1}. \tag{5}$$

QUESTIONS AND PROBLEMS

1. To what temperature must a cubic foot of gas initially at 0° C. be raised in order to double its volume, the pressure remaining constant?

2. What fractional part of the air in a room passes out when the air in it is heated from −10° C. to 20° C.? (−10° C. = 263° A. 20° C. = 293° A.)

3. If the air within a bicycle tire is under a pressure of two atmospheres, i.e. 150 cm. of mercury, when the temperature is 10° C., what pressure will exist within the tube when the temperature changes to 30° C.?

4. Compare the amount of oxygen taken into the lungs at one inhalation in summer when the temperature is 30° C. with that inhaled in winter when the temperature is −20° C.

5. If the pressure to which 10 cc. of air is subjected changes from 76 cm. to 40 cm., the temperature remaining constant, what does its volume become? (See Boyle's law, p. 67.) If, then, the temperature of the same gas changes from 15° C. to 100° C., the pressure remaining constant, what will be the final volume?

6. If the volume of a gas at 10° C. and 76 cm. pressure is 500 cc., what is its volume at 50° C. and 70 cm. pressure? (First find the change in volume due to change in pressure alone; then, starting with this new volume, find the effect of the change in temperature.)

7. If a diver descends to a depth of 100 feet, what is the pressure to which he is subjected? What is the density of the air in his suit, the density at the surface where the pressure is 75 cm. being .00123? (Assume the temperature to remain unchanged.)

8. A bubble escapes from a diver's suit at a depth of 100 feet where the temperature is 4° C. To how many times its original volume has the bubble grown by the time it reaches the surface, where the temperature is 30° C. and the barometric height 75 cm.?

9. Find the density of the air in a furnace whose temperature is 1000° C., the density at 0° C. being .001293.

10. The gas within a half-inflated balloon occupies a volume of 100,000 liters. The temperature is 15° C. and the barometric height 75 cm. What will be its volume after the balloon has risen to the height of Mt. Blanc, where the pressure is 37 cm. and the temperature −10° C.?

11. If the volume of a quantity of air at 30° C. is 200 cc., at what temperature will its volume be 300 cc., the pressure remaining the same?

12. It was found by the noted French physicist Regnault that when the barometric height is 76 cm. and the temperature 0° C., the density of air is .001293. Find the density of air on a summer day on which the temperature is 38° C. and the barometric height is 73 cm. Find the density of air when the temperature is −40° C. and the barometric height 74 cm.

Coefficients of Expansion of Liquids and Solids

193. Peculiarities in the expansion of liquids. The expansion of liquids differs from that of gases in that:

1. The coefficients of liquids are all considerably smaller than those of gases.

2. Different liquids expand at wholly different rates, e.g. the coefficient of alcohol between 0° and 10° C. is .0011; of ether it is .0015; of petroleum, .0009.

3. The same liquid often has different coefficients at different temperatures; i.e. the expansion is irregular. Thus, if the coefficient of alcohol is obtained between 0° and 60° C., instead of between 0° and 10° C., it is .0013 instead of .0011.

The coefficient of mercury, however, is very nearly constant through a wide range of temperature, which indeed might have been inferred from the fact that mercury thermometers agree so well with gas thermometers.

Fig. 136. Bulb for investigating expansions of liquids

194. Method of measuring the expansion coefficients of liquids. One of the most convenient and common methods of measuring the coefficients of liquids is to place them in bulbs of known volume, provided with capillary necks of known diameter, like that shown in Fig. 136, and then to watch the rise of the liquid in the neck for a given rise in temperature. A certain allowance must be made for the expansion of the bulb, but this can readily be done if the coefficient of expansion of the substance of which the bulb is made is known.

195. Maximum density of water. When water is treated in the way described in the preceding paragraph, it reaches its lowest position in the stem at 4° C. As the temperature falls from that point down to 0° C., water exhibits the peculiar property of *expanding* with a *decrease* in temperature.

An experiment usually known as Hope's experiment demonstrates very clearly that the maximum density of water is at 4° C. A tall cylinder of water at say 15° C. is surrounded in the middle with a mixture of ice and salt, and two thermometers are placed, one in the bottom and one in the top, as in Fig. 137. At first the lower thermometer is seen to fall very much faster than the upper one, until it reaches a temperature of 4° C., where it becomes stationary. This indicates that water at 4° C. is more dense than at *higher* temperatures, for in being cooled to this temperature it became heavier and sank to the bottom. After the lower thermometer has become stationary at 4° C., the upper thermometer begins to fall; but instead of stopping at 4° C. it continues to fall to 0° C., the lower one remaining all the time at 4° C. This indicates that water at 4° C. is heavier than at any *lower* temperature, for, since the cooling is done in the middle, the only way in which the upper temperature could fall to 0° C. was through the rising to the surface of water both lighter and colder than the water at 4° C. (The experiment may require an hour or more and several renewals of the freezing mixture.)

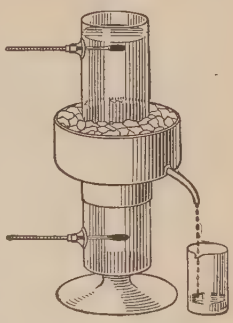


Fig. 137. Maximum density of water

196. The cooling of a lake in winter. The last experiment makes it easy to understand the cooling of any large body of water with the approach of winter. The surface layers are first cooled and contract. Hence, being then heavier than the lower layers, they sink and are replaced by the warmer water from beneath. This process of cooling at the surface, and sinking, goes on until the whole body of water has reached a temperature of 4° C. After this condition has been reached, further cooling of the surface layers makes them *lighter* than the water beneath, and they now remain on top until they freeze. Thus, before any ice whatever can form on the surface of a lake, the whole mass of water to the very bottom must be cooled to 4° C. This is why it requires a much longer and more severe period of cold to freeze deep bodies of water than shallow ones. Further,

since the circulation described above ceases at 4° C., practically all of the unfrozen water will be at 4° C. even in the coldest weather. Only the water which is in the immediate neighborhood of the ice will be lower than 4° C. This fact is of vital importance in the preservation of aquatic life.

197. Linear coefficients of expansion of solids. It is often more convenient to measure the increase in *length* of one edge of an expanding solid than to measure its increase in *volume. The ratio between the increase in length per degree rise in temperature, and the total length, is called the linear coefficient of expansion of the solid.* Thus if l_1 represent the length of a bar at $t_1°$, and l_2 its length at $t_2°$, the equation which defines the linear coefficient k is

$$k = \frac{l_2 - l_1}{l_1(t_2 - t_1)}. \tag{7}$$

Fig. 138 illustrates the method now in use at the International Bureau of Weights and Measures for obtaining these coefficients.

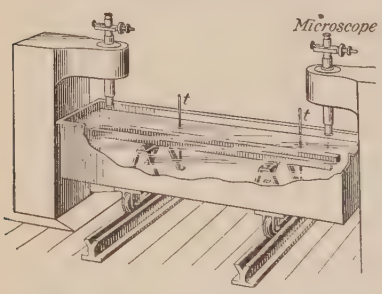


Fig. 138. Apparatus for determination of linear coefficients of expansion

The two microscopes which are mounted in fixed positions upon heavy piers are focused upon scratches near the ends of the bar whose coefficient is to be obtained. The temperature of the water is then changed from say 0° C. to 10° C., and the amount of elongation of the bar is determined from the observed amounts of motion of its ends as seen through the microscopes.

The linear coefficients of a few common substances are given in the following table.

Aluminium	.000023	Glass	.000009	Silver	.000019
Brass	.000018	Iron	.000012	Steel	.000011
Copper	.000017	Lead	.000029	Tin	.000022
Gold	.000014	Platinum	.000009	Zinc	.000029

Applications of Expansion

198. Compensated pendulum. Since a long pendulum vibrates more slowly than a short one, the expansion of the rod which carries the pendulum bob causes an ordinary clock to run too slowly in summer, and its contraction causes it to run too fast in winter. For this reason very accurate clocks are provided with *compensated* pendulums, which are so constructed that the distance of the bob beneath the point of support is independent of the temperature. This is accomplished by suspending the bob by means of two sets of rods of different material, in such a way that the expansion of one set raises the bob, while the expansion of the other set lowers it. Such a pendulum is shown in Fig. 139. The expansion of the iron rods *b*, *d*, *e*, and *i* tends to lower the bob, while that of the copper rods *c* tends to raise it. In order to produce complete compensation it is only necessary to make the total lengths of iron and copper rods inversely proportional to the coefficients of expansion of iron and copper.

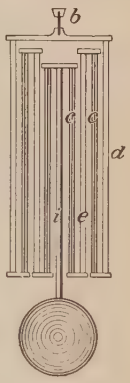


Fig. 139. The compensated pendulum

199. Compensated balance wheel. In the balance wheel of an accurate watch (Fig. 140) another application of the unequal expansion of metals is made. Increase in temperature both increases the radius of the wheel and weakens the elasticity of the spring which controls it. Both of these effects tend to make the watch lose time. This tendency may be counteracted by bringing the mass of the rotating parts in toward the center of the wheel. This is accomplished by making the arcs *bc* of metals of different expansion coefficients, the inner metal, shown in black in the figure, having the smaller coefficient.

Fig. 140. The compensated balance wheel

The weighted ends of the arcs are then sufficiently pulled in by a rise in temperature to counteract the retarding effects.

The principle is precisely the same as that which finds simple illustration in the compound bar shown in Fig. 141. This bar consists of

Fig. 141

Fig. 142

Unequal expansion of metals

two strips, one of brass and one of iron, riveted together. When the bar is placed edgewise in a Bunsen flame, so that both metals are heated equally, it will be found to bend in such a way that the more expansible metal, namely the brass, is on the outside of the curve, as shown in Fig. 142. When it is cooled with snow or ice it bends in the opposite direction.

200. The dial thermometer. The dial thermometer furnishes another illustration of the unequal expansion of metals. It consists of a compound metallic ribbon wound in helical form. One end *a* of the helix (Fig. 143) is fixed, while the other end is attached to a lever arm *b*, the motion of which rotates the pointer *d* over the dial (Fig. 144), which is graduated by comparison with a mercury thermometer. The more expansible metal is on the outside. Hence rise in temperature causes the helix to wind up closer, the index then moving to the right; while a decrease in temperature causes it to unwind, in which case the pointer moves to the left.

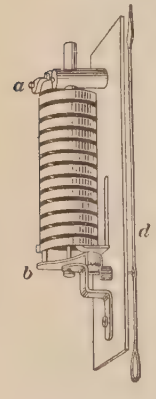


Fig. 143

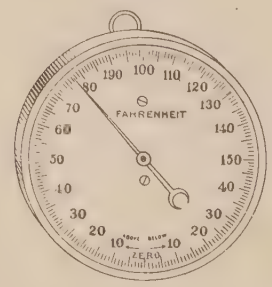


Fig. 144

The dial thermometer

201. Iron bridge supports. Since the coefficient of iron is .000012, an iron bridge 100 feet long would change appreciably (about three fourths of an inch) in length between say $-25°$ C.

in winter and 25° C. in summer. In order to avoid the strain which would thus be placed on the piers if the girders were rigidly attached at both ends, a rolling support R, like that shown in Fig. 145, is sometimes provided at one end.

FIG. 145. Roller providing for expansion of iron bridge

QUESTIONS AND PROBLEMS

1. Why may a glass stopper sometimes be loosened by pouring hot water on the neck of a bottle?

2. If an iron steam pipe is 60 ft. long at 0° C., what is its length when steam passes through it at 100° C.?

3. The steel cable from which Brooklyn Bridge hangs is more than a mile long. By how many feet does a mile of its length vary between a winter day when the temperature is −20° C. and a summer day when it is 30° C.?

4. The changes in temperature to which long lines of steam pipes are subjected makes it necessary to introduce "expansion joints." These joints consist of brass collars fitted tightly by means of packing over the separated ends of two adjacent lengths of pipe. If the pipe is of iron, and such a joint is inserted every 200 ft., and if the range of temperature which must be allowed for is from −30° C. to 125° C., what is the minimum play which must be allowed for at each expansion joint?

5. A metal rod 230 cm. long expanded 2.75 mm. in being raised from 0° C. to 100° C. Find its coefficient of linear expansion.

6. If iron rails are 30 ft. long, and if the variation of temperature throughout the year is 50° C., what space must be left between their ends?

7. If the total length of the iron rods b, d, e, and i, in a compensated pendulum (Fig. 139), is 2 m., what must be the total length of the copper rods c if the period of the pendulum is independent of temperature?

8. Decide from the table of expansion coefficients given on page 142 why the wires which lead the current through the walls of incandescent electric light bulbs are always made of platinum, i.e. why it is impossible to seal any other metal into glass.

CHAPTER VIII

WORK AND MECHANICAL ENERGY[1]

Definition and Measurement of Work

202. Definition of work. Whenever a force *moves* a body on which it acts, it is said to do work upon that body; and the amount of the work accomplished is measured by the product of the force acting and the distance through which it moves the body. Thus if 1 g. of mass is lifted 1 cm. in a vertical direction, 1 g. of force has acted, and the distance through which it has moved the body is 1 cm. We say, therefore, that the lifting force has accomplished 1 *gram centimeter* of work. If the gram of force had lifted the body upon which it acted through 2 cm., the work done would have been 2 g. cm. If a force of 3 g. had acted and the body had been lifted through 3 cm., the work done would have been 9 g. cm., etc. Or, in general, if W represent the work accomplished, F the value of the acting force, and s the distance through which its point of application moves, then the definition of work is given by the equation

$$W = F \times s. \qquad (1)$$

In the scientific sense, no work is ever done unless the force succeeds in *producing motion* in the body on which it acts. A pillar supporting a building does no work; a man tugging at a stone, but failing to move it, does no work. In the popular sense we sometimes say that we are doing work when we are

[1] It is recommended that this chapter be preceded by an experiment in which the student discovers for himself the law of the lever, i.e. the principle of moments (see, for example, Experiment 15, authors' manual), and that it be accompanied by a study of the principle of work as exemplified in at least one of the other simple machines (see, for example, Experiment 16, authors' manual).

simply holding a weight, or doing anything else which results in fatigue; but in physics the word "work" is used to describe, not the effort put forth, but the *effect accomplished,* as represented in equation (1).

203. Units of work. Corresponding to the two metric units of force, the gram of force and the dyne (see § 53), there are the two metric units of work, the gram centimeter and the dyne centimeter, the latter of which is usually called the *erg.*

The *gram centimeter* is the amount of work done by 1 gram of force when it moves the point upon which it acts 1 cm.

The *erg* is the amount of work done by 1 dyne of force when it moves the point upon which it acts 1 cm.

The erg is called an absolute unit of work for the reason that it involves in its definition the absolute unit of force, namely the dyne. To raise 1 l. of water from the floor to a table 1 m. high would require the expenditure of $1000 \times 980 \times 100 = 98{,}000{,}000$ ergs. It will be seen, therefore, that the erg is an exceedingly small unit. For this reason it is customary to employ for practical purposes a unit which is equal to 10,000,000 ergs. It is called the *joule,* in honor of the great English physicist, James Prescott Joule (1818–1889). The work done in lifting a liter of water one meter is therefore 9.8 joules.

Corresponding to the English unit of force, the pound, we have the English unit of work, the foot pound, which is the amount of work done by a "pound of force" when it moves the point on which it acts through a distance of one foot.

QUESTIONS AND PROBLEMS

1. How many foot pounds of work does a 150-lb. man do in climbing to the top of Mt. Washington, which is 6300 ft. high?

2. A horse pulls a metric ton of coal to the top of a hill 30 m. high. Express the work accomplished, first in kilogram meters, then in gram centimeters, then in ergs, and then in joules.

3. If the 20,000 inhabitants of a city use an average of 20 l. of water per day per capita, how many kilogram meters of work must the engines do per day, if the water has to be raised to a height of 75 m.?

Work Expended upon and Accomplished by Systems of Pulleys

204. The single fixed pulley. Let the force of the earth's attraction upon a mass W be overcome by pulling upon a spring balance S, in the manner shown in Fig. 146, until W moves slowly upward. If W is 100 g., the spring balance will also be found to register a force of 100 g.

Fig. 146. Single fixed pulley

Experiment therefore shows that in the use of the single fixed pulley the acting force F which is producing the motion is equal to the resisting force W which is opposing the motion.

Again, since the length of the string is always constant, the distance s through which the point A, at which F is applied, must move, is always equal to the distance s' through which the weight W is lifted. Hence, if we consider the work put into the system at A, viz. $F \times s$, and the work accomplished by the system at W, viz. $W \times s'$, we find obviously, since $W = F$ and $s = s'$, that

$$Fs = Ws'; \tag{2}$$

i.e. in the case of the single fixed pulley, *the work done by the acting force F is equal to the work done against the resisting force W;* or the work *put into* the machine at A is equal to the work *accomplished by* the machine at W.

205. The single movable pulley. Let now the force of the earth's attraction upon the mass W be overcome by a single movable pulley, as shown in Fig. 147. Since the weight of W (W representing in this case the weight of both the pulley and the suspended mass) is now supported half by the strand C and half by the strand B, the force F which must act at A to hold the weight in place, or to move it slowly upward if there is no friction, should be only one half of W. A reading of the balance will show that this is indeed the case.

Experiment thus shows that in the case of the single movable pulley the acting force F is just one half as great as the resisting force W.

But when again we consider the *work* which the force F must do to lift the weight W a distance s', we see that A must move upward 2 in. in order to raise W 1 in. For when W moves up 1 in. both of the strands B and C must be shortened one inch. As before, therefore, since $W = 2F$, and $s' = \frac{1}{2}s$,

$$F \times s = W \times s';$$

i.e. in the case of the single movable pulley, as in the case of the fixed pulley, *the work put into the machine at F is equal to the work accomplished by the machine at W.*

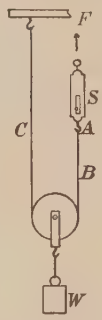


FIG. 147. Single movable pulley

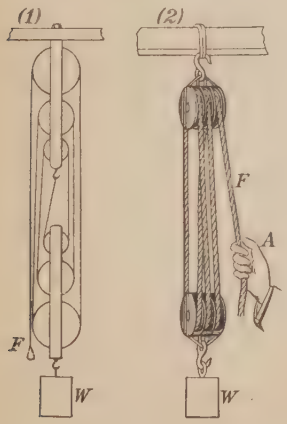


FIG. 148. Combinations of pulleys

206. Combinations of pulleys. Let a weight W be lifted by means of such a system of pulleys as is shown in Fig. 148, either (1) or (2). Here, since W is supported by 6 strands of the cord, it is clear that the force which must be applied at A in order to hold W in place, or to make it move slowly upward if there is no friction, should be but $\frac{1}{6}$ of W.

The experiment will show this to be the case if the effects of friction, which are often very considerable, are eliminated by taking the mean of the forces which must be applied at F to cause it to move first slowly upward and then slowly downward. The law of any combination of movable pulleys may then be stated thus: *If n represent the number of strands between which the weight is divided,*

$$F = W/n. \qquad (3)$$

But when again we consider the work which the force F must do in order to lift the weight W through a distance s', we see that, in order that the weight W may be moved up through 1 in., each of the strands must be shortened 1 in., and hence the point A must move through n in.; i.e. $s' = s/n$. Hence, ignoring friction, in this case also we have

$$F \times s = W \times s';$$

i.e. although the acting force F is only $\frac{1}{n}$ of the resisting force W, *the work put into the machine at F is equal to the work accomplished by the machine at W.*

207. Mechanical advantage. The above experiments show that it is sometimes possible, by applying a small force F, to overcome a much larger resisting force W. *The number of times that the resisting force W contains the applied force F is called the mechanical advantage of the machine.* Thus the mechanical advantage of the single fixed pulley is 1, that of the single movable pulley is 2, that of the systems of pulleys shown in Fig. 148 is 6, etc.

If the acting force is applied at W instead of at F, the mechanical advantage of the systems of pulleys of Fig. 148 is $\frac{1}{6}$; for it requires an application of 6 lb. at W to lift 1 lb. at F. But it will be observed that the resisting force at F now moves six times as fast and six times as far as the acting force at W. We can thus either sacrifice speed to gain force, or sacrifice force to gain speed; but in every case, whatever we gain in the one we lose in the other. Thus in the hydraulic elevator shown in Fig. 40, page 48, the cage moves only as fast as the piston; but in that shown in Fig. 41 it moves four times as fast. Hence the force applied to the piston in the latter case must be four times as great as in the former if the same load is to be lifted. This means that the diameter of the latter cylinder must be twice as great.

QUESTIONS AND PROBLEMS

1. If the hydraulic elevator of Fig. 41, page 48, is to carry a total load of 10,000 lb., what force must be applied to the piston? If the water pressure is 70 lb. per square inch, what must be the diameter of the piston?

2. Draw a diagram of a set of pulleys by which a force of 50 lb. can support a load of 200 lb.

3. Draw a diagram of a set of pulleys by which a force of 50 lb. can support 250 lb. What would be the mechanical advantage of this arrangement?

Work and the Lever

208. The law of the lever. The lever is a rigid rod free to turn about some point P called the fulcrum (Fig. 149).

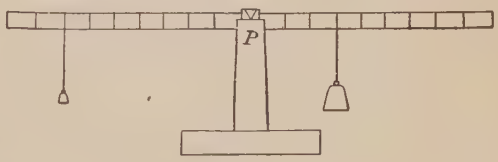


Fig. 149. The simple lever

Let a meter stick be first balanced as in the figure, and then let a mass, of say 300 g., be hung by a thread from a point 15 cm. from the fulcrum. Then let a point be found on the other side of the fulcrum at which a weight of 100 g. will just support the 300 g. This point will be found to be 45 cm. from the fulcrum. It will be seen at once that the product of 300 × 15 is equal to the product of 100 × 45.

Next let the point be found at which 150 g. just balance the 300 g. This will be found to be 30 cm. from the fulcrum. Again the products 300 × 15 and 150 × 30 are equal.

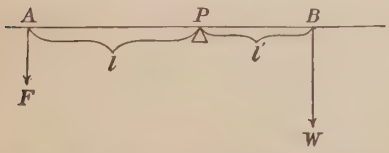


Fig. 150. Illustrating law of moments, viz. $Fl = Wl'$

No matter where the weights are placed, or what weights are used on either side of the fulcrum, the product of the acting force F by its distance l from the fulcrum (Fig. 150) will be found to be equal to the product of the resisting force W by its distance

F from the fulcrum. Now the distances l and l' are called the lever arms of the forces F and W, and the product of a force by its lever arm is called the moment of that force. The above experiments on the lever may then be generalized in the following law. *The moment of the acting force is equal to the moment of the resisting force.* Algebraically stated, it is

$$Fl = Wl'. \tag{4}$$

It will be seen that the mechanical advantage of the lever, namely W/F, is equal to l/l', i.e. *to the lever arm of the acting force divided by the lever arm of the resisting force.*

209. Addition of moments. Let 200 g., for example, be placed 60 cm. from F (Fig. 151), and on the other side 100 g., 20 cm. from F, and let the point be found at which another 100 g. weight must be placed in order to produce equilibrium.

Fig. 151 Fig. 152

Conditions of equilibrium of a bar acted upon by several forces

This point will be found to be 40 cm. from F, and it will be seen that $200 \times 60 = 100 \times 20 + 100 \times 40$, i.e. that the moments on the left is equal to the sum of the moments on the right.

Next, let the bar be arranged as in Fig. 152, and let 100 g. be hung at A, 20 cm. from the fulcrum; 100 g. at B, 15 cm. from the fulcrum; and 200 g. hung over the pulley b, the thread being produced at a distance 40 cm. from the fulcrum. Then let the point be found at which a weight of 200 g. will produce equilibrium. This point will be 32.5 cm. from the fulcrum.

It will be seen that

$$200 \times 20 + 100 \times 40 = 100 \times 15 + 200 \times 32.5;$$

i.e. *the sum of all the moments which are trying to make the beam rotate in one direction is equal to the sum of all the*

moments tending to make it rotate in the opposite direction. This is the general statement of the law of the lever.

210. Work expended upon and accomplished by the lever. We have just seen that when the lever is in equilibrium — that is, when it is *at rest* or is *moving uniformly* — the relation between the acting force F and the resisting force W is expressed in the equation of moments, viz. $Fl = Wl'$. Let us now suppose, precisely as in the case of the pulleys, that the force F raises the weight W through a small distance s'. To accomplish this, the point A to which F is attached must move through a distance s (Fig. 153). From the similarity of the triangles APn and BPm it will be seen that l/l' is equal to s/s'. Hence equation (4), which represents the law of the lever, and which may be written $F/W = l'/l$, may also be written in the form $F/W = s'/s$, or

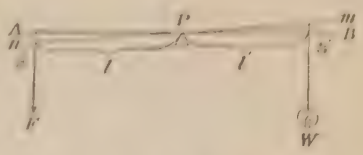


FIG. 153. Showing that the equation of moments, $Fl = Wl'$, is equivalent to $Fs = Ws'$

$$Fs = Ws'.$$

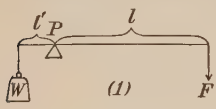
Now Fs represents the work done by the acting force F, and Ws' the work done against the resisting force W. Hence the law of moments, which has just been found by experiment to be the law of the lever, is equivalent to the statement that *whenever work is accomplished by the use of the lever, the work expended upon the lever by the acting force F is equal to the work accomplished by the lever against the resisting force W.*

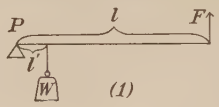
211. The three classes of levers. It is customary to divide levers into three classes, as follows.

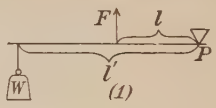
1. In levers of the first class the fulcrum P is between the acting force F and the resisting force W (Fig. 154). The mechanical advantage of levers of this class is greater or less

than unity, according as the lever arm l of the acting force is greater or less than the lever arm l' of the resisting force.

2. In levers of the second class the resisting force W is between the acting force F and the fulcrum P (Fig. 155).







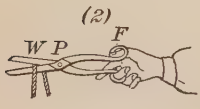


Fig. 154. Levers of first class

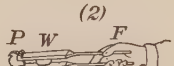


Fig. 155. Levers of second class

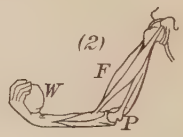


Fig. 156. Levers of third class

Here the lever arm of the acting force, i.e. the distance from F to P, is necessarily greater than the lever arm of the resisting force, i.e. the distance from W to P. Hence the mechanical advantage is always greater than one.

3. In levers of the third class the acting force is between the resisting force and the fulcrum (Fig. 156). The mechanical advantage is then obviously less than one, i.e. in this type of lever force is always sacrificed for the sake of gaining speed.

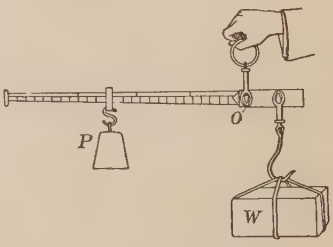


Fig. 157. Steelyards

QUESTIONS AND PROBLEMS

1. Explain the principle of weighing by the steelyards (Fig. 157). What must be the weight of the bob P if, at a distance of 30 cm. from the fulcrum O, it balances a weight of 10 kg. placed at a distance of 2 cm. from O?

2. In which of the three classes of levers does the wheelbarrow belong? grocer's scales? pliers? sugar tongs? a claw hammer?

3. How would you arrange a crowbar to use it as a lever of the first class in overturning a heavy object? as a lever of the second class?

4. A lever is 3 ft. long. Where must the fulcrum be placed so that a weight of 300 lb. at one end shall be balanced by 50 lb. at the other?

5. Two boys weighing 75 lb. and 50 lb. respectively are balancing on opposite sides of a teeter board. How far from the fulcrum must the smaller boy sit if the larger one is 5 ft. from it?

6. Two boys carry a load of 50 lb. on a pole between them. If the load is 4 ft. from one boy and 6 ft. from the other, how many pounds does each boy carry? (Consider the force exerted by one of the boys as the acting force, the load as the resisting force, and the second boy as the fulcrum.)

7. Where must a load of 100 lb. be placed on a stick 10 ft. long, if the man who holds one end is to support 30 lb., while the man at the other end supports 70 lb.?

The Principle of Work

212. Statement of the principle of work. The study of pulleys led us to the conclusion that in all cases where such machines are used the work done by the acting force is equal to the work done against the resisting force, provided always that the motions are uniform, and that friction may be neglected. The study of levers led to precisely the same result. In Chapter II the study of the hydraulic press showed that the same law applied in this case also, for it was shown that the force on the small piston times the distance through which it moved was equal to the force on the large piston times the distance through which it moved. Similar experiments upon all sorts of machines have shown that in all cases where friction may be neglected the following is an absolutely general law: *In all mechanical devices of whatever sort the work expended upon the machine is equal to the work accomplished by it.*

This important generalization is called "the principle of work," and was first enunciated by Sir Isaac Newton in 1687, in a scholium to the third law of motion. It has proved to be one of the most fruitful principles ever put forward in the history of physics. By its application it is easy to deduce the relation between the force applied and the force overcome in any sort of machine, provided only that friction is negligible, and that the

motions take place slowly. It is only necessary to produce, or imagine, a displacement at one end of the machine, and then to measure or calculate the corresponding displacement at the other end. The ratio of the second displacement to the first is the ratio of the force acting to the force overcome.

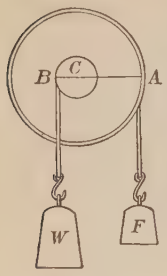


FIG. 158. The wheel and axle

213. The wheel and axle. Let us apply the work principle to discover the law of the wheel and axle (Fig. 158). When the large wheel has made one revolution the point A moves down a distance equal to the circumference of this wheel. During this time the weight W is lifted a distance equal to the circumference of the axle. Hence the equation $Fs = Ws'$ becomes $F \times 2\pi R = W \times 2\pi r$, where R and r are the radii of the wheel and axle respectively. This equation may be written in the form

$$W/F = R/r; \qquad (5)$$

i.e. *the weight lifted on the axle is as many times the force applied to the wheel as the radius of the wheel is times the radius of the axle.* Otherwise stated, *the mechanical advantage of the wheel and axle is equal to the radius of the wheel divided by the radius of the axle.*

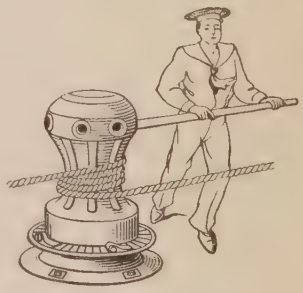


FIG. 159. The capstan

The *capstan* (Fig. 159) is a special case of the wheel and axle, the length of the lever arm taking the place of the radius of the wheel, and the radius of the barrel corresponding to the radius of the axle.

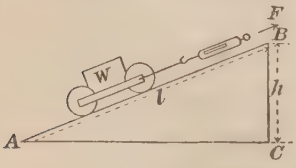


FIG. 160. The inclined plane

214. The work principle applied to the inclined plane. The work done against gravity in lifting

a weight W (Fig. 160) from the bottom to the top of a plane is evidently equal to W times the height h of the plane. But the work done by the acting force F, while the carriage of weight W is being pulled from the bottom to the top of the plane, is equal to F times the length l of the plane. Hence the principle of work gives

$$Fl = Wh, \text{ or } W/F = l/h; \tag{6}$$

i.e. *the mechanical advantage of the inclined plane, or the ratio of the weight lifted to the force acting parallel to the plane, is the ratio of the length of the plane to the height of the plane.* This is precisely the conclusion at which we arrived in another way in Chapter II (p. 19).

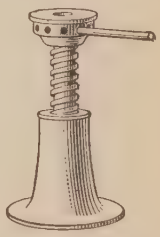


Fig. 161. The jackscrew

215. The screw. The screw (Fig. 161) is a combination of the inclined plane and the lever. Its law is easily obtained from the principle of work. When the force which acts on the end of the lever has moved this point through one complete revolution, the weight W, which rests on top of the screw, has evidently been lifted through a vertical distance equal to the distance between two adjoining threads. This distance d is called *the pitch* of the screw. Hence, if we represent by l the length of the lever, the work principle gives

$$F \times 2\pi l = Wd; \tag{7}$$

i.e. *the mechanical advantage of the screw, or ratio of the weight lifted to the force applied, is equal to the ratio of the circumference of the circle moved over by the end of the lever, to the distance between the threads of the screw.* In actual practice the friction in such an arrangement is always very great, so that the mechanical advantage is considerably less than its full theoretical value. The common jackscrew just described, and used

chiefly for raising buildings, the letter press (Fig. 162), and the vise (Fig. 163) are all familiar forms of the screw.

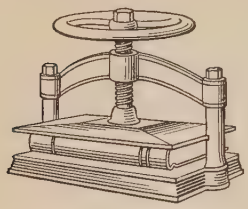


Fig. 162. The letter press

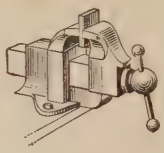


Fig. 163. The vise

216. A train of gear wheels. A form of machine capable of very high mechanical advantage is the train of gear wheels shown in Fig. 164. Let the student show from the principle of work, namely $Fs = Ws'$, that the mechanical advantage, i.e. $\frac{W}{F}$, of such a device is

$$\frac{\text{circum. of } a \times \dfrac{\text{no. cogs in } d}{\text{no. cogs in } c} \times \dfrac{\text{no. cogs in } f}{\text{no. cogs in } b}}{\text{circum. of } e}. \qquad (8)$$

217. The worm wheel. Another device of high mechanical advantage is the worm wheel (Fig. 165). Show that if l is the length of the crank arm C, n the number of teeth in the cog wheel W, and r the radius of the axle, the mechanical advantage is given by

$$\frac{2\,\pi l n}{2\,\pi r} = n\frac{l}{r}. \qquad (9)$$

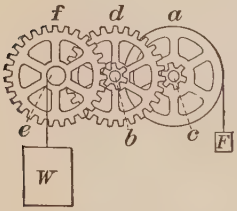


Fig. 164
Train of gear wheels

This device is used most frequently when the primary object is to decrease speed rather than to multiply force.

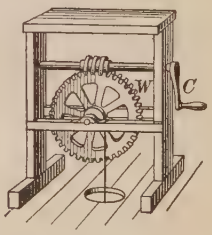


Fig. 165
The worm gear

It will be seen that the crank handle must make n turns while the cog wheel is making one.

218. The differential pulley. In the differential pulley (Fig. 166) an endless chain passes first over the fixed pulley A, then down over the movable pulley C, then up again over the fixed pulley B, which is rigidly attached to A, but differs slightly from it in diameter. On the circumference of all the pulleys are projections which fit between the links, and thus keep the chains from slipping. When the chain is

pulled down at F, as in Fig. 166 (2), until the upper rigid system of pulleys has made one complete revolution, the chain between the upper and lower pulleys has been shortened by the difference between the circumferences of the pulleys A and B, for the chain has been pulled up a distance equal to the circumference of the larger pulley and let down a distance equal to the circumference of the smaller pulley. Hence the load W has been lifted by half the difference between the circumferences of A and B. The mechanical advantage is therefore equal to the circumference of A divided by one half the difference between the circumferences of A and B.

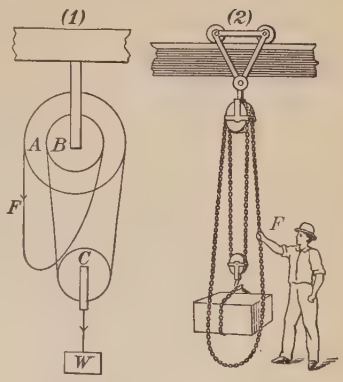


Fig. 166. The differential pulley

QUESTIONS AND PROBLEMS

1. In the differential wheel and axle (Fig. 167) the rope is wound in opposite directions on two axles of different diameter. For a complete revolution of the axle the weight is lifted by a distance equal to $\frac{1}{2}$ the difference between the circumferences of the two axles. If the crank has a radius of 2 ft., the larger axle a diameter of 6 in., and the smaller one a diameter of 5 in., find the mechanical advantage of the arrangement.

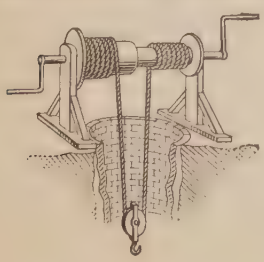


Fig. 167. Differential windlass

2. A barrel is being rolled up a plank which is 12 ft. long, into a doorway which is 4 ft. high. If the barrel weighs 300 lb., with what force must a man push on it parallel to the plank in order to keep it from rolling back?

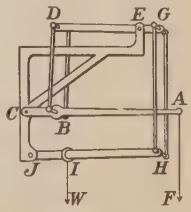


Fig. 168. The compound lever

3. A 1500-lb. safe must be raised 5 ft. The force which can be applied is 250 lb. What is the shortest inclined plane which can be used for the purpose?

4. If, in the compound lever of Fig. 168, $AC = 5$ ft., $BC = 1$ ft., $DE = 4$ ft., $EG = 8$ in., $HJ = 4$ ft., and $IJ = 2$ ft., what force applied at F will support a weight of 2000 lb. at W?

5. The hay scales shown in Fig. 169 consist of a compound lever with fulcrums at F, F', F'', and F'''. If Fo and $F'o'$ are lengths of 6 in., FE and

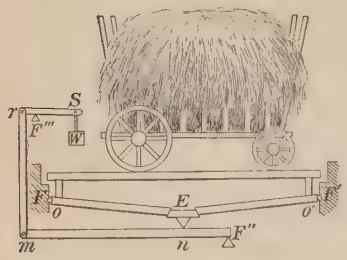


Fig. 169. Hay scales

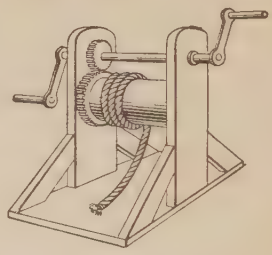


Fig. 170. Windlass with gears

$F'E$ 5 ft., $F''n$ 1 ft., $F''m$ 6 ft., rF''' 2 in., and $F'''S$ 20 in., how many pounds at W will be required to balance a weight of a ton on the platform?

6. If the capstan of a ship is 12 in. in diameter and the levers are 5 ft. long, what force must be exerted by each of 4 men in order to raise an anchor weighing 2000 lb.?

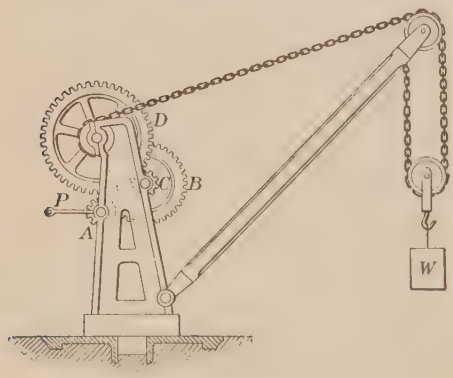


Fig. 171. The crane

7. In the windlass of Fig. 170 the crank handle has a length of 2 ft., and the barrel a diameter of 8 in. There are 20 cogs in the small cog wheel and 50 in the large one. What is the mechanical advantage of the arrangement?

8. A force of 70 kg. on a wheel whose diameter is 3 m. balances a weight of 150 kg. on the axle. Find the diameter of the axle.

9. Ten jackscrews each of which has a pitch of 1/4 in. and a lever arm of 18 in. are being worked simultaneously to raise a building weighing 100,000 lb. What force would have to be exerted at the end of each lever if there were no friction?

10. If a worm wheel (Fig. 165) has 40 teeth, and the crank is 30 cm. long, while the radius of the axle is 3 cm., what is the mechanical advantage of the arrangement?

11. If in the crane of Fig. 171 the crank arm has a length of 1/2 m., and the gear wheels A, B, C, and D have 12, 48, 12, and 60 cogs respectively, while the axle over which the chain runs has a radius of 10 cm., what is the mechanical advantage of the crane?

Power and Energy

219. Definition of power. When a given load has been raised a given distance a given amount of work has been done, whether the time consumed in doing it is small or great. Time is therefore not a factor which enters into the determination of work; but it is often as important to know the *rate* at which work is done as to know the *amount* of work accomplished. *The rate of doing work is called power, or activity.* Thus, if P represent power, W the work done, and t the time required to do it,

$$P = \frac{W}{t}. \tag{10}$$

220. Horse power. James Watt (1736–1819), the inventor of the steam engine, considered that an average horse could do 33,000 ft. lb. of work per minute, or 550 ft. lb. per second. The metric equivalent is 76.05 kg. m. per second. This number is probably considerably too high, but it has been taken ever since, in English-speaking countries, as the unit of power, and named the *horse power* (H.P.). The power of steam engines has usually been rated in horse power. The horse power of an ordinary railroad locomotive is from 500 to 1000. Stationary engines and steamboat engines of the largest size often run from 5000 to 20,000 H.P. The power of an average horse is about 3/4 H.P., and that of an ordinary man about 1/7 H.P.

221. The kilowatt. In the metric system the erg has been taken as the absolute unit of work. The corresponding unit of power is an erg per second. This is, however, so small that it

is customary to take as the practical unit 10,000,000 ergs per second, i.e. one joule per second (see § 203, p. 147). This unit is called the *watt*, in honor of James Watt. The power of dynamos and electric motors is almost always expressed in kilowatts, a kilowatt representing 1000 watts, and in modern practice even steam engines are being increasingly rated in kilowatts rather than in horse power. A horse power is equivalent to 746 watts; it may therefore in general be considered to be 3/4 of a kilowatt.

222. Definition of energy. The *energy* of a body is defined as its *capacity for doing work*. In general, inanimate bodies possess energy only because of work which has been done upon them at some previous time. Thus, suppose a kilogram weight is lifted from the first position in Fig. 172 through a height of one meter, and placed upon the hook H at the end of a cord which passes over a frictionless pulley p and is attached at the other end to a second kilogram weight B. The operation of lifting A from position 1 to position 2 has required an expenditure upon it of 1 kg. m. (100,000 gr. cm., or 98,000,000 ergs) of work. But in position 2, A is itself possessed of a certain capacity for doing work which it did not have before. For if it is now started downward by the application of the slightest conceivable force, it will, of its own accord, return to position 1, and will in so doing raise the kilogram weight B through a height of 1 m. In other words, it will do upon B exactly the same amount of work which was originally done upon it.

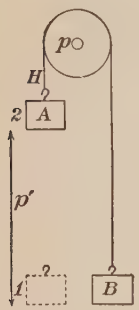


Fig. 172. Illustration of potential energy

223. Potential and kinetic energy. A body may have a capacity for doing work not only because it has been given an elevated position, but also because it has in some way acquired velocity: e.g. a heavy fly wheel will keep machinery running for some time after the power has been shut off; a bullet shot

upward will lift itself a great distance against gravity because of the velocity which has been imparted to it. Similarly, any body which is in motion is able to rise against gravity, or to set other bodies in motion by colliding with them, or to overcome resistances of any conceivable sort. Hence, in order to distinguish between the energy which a body may have because of an *advantageous position*, and the energy which it may have because it is in *motion*, the two terms "potential" and "kinetic" energy are used. Potential energy includes the energy of lifted weights, of coiled or stretched springs, of bent bows, etc.; in a word, it is *energy of position*, while kinetic energy is *energy of motion*.

224. Transformations of potential and kinetic energy. The swinging of a pendulum, and the oscillation of a weight attached to a spring, illustrate well the way in which energy which has once been put into a body may be transformed back and forth between the potential and kinetic varieties. When the pendulum bob is at rest at the bottom of its arc it possesses no energy of either type, since, on the one hand, it is as low as it can be, and on the other, it has no velocity. When we pull it up the arc to the position A (Fig. 173), we do an amount of work upon it which is equal in gram centimeters to its weight in grams times the distance AD in centimeters; i.e. we store up in it this amount of potential energy. As now the bob falls to C this potential energy is completely transformed into kinetic. That this kinetic energy at C is exactly equal to the potential energy at A is proved by the fact that if friction is completely eliminated, the bob rises to a point B such that BE is equal to AD. We see, therefore, that at the ends of its swing the energy

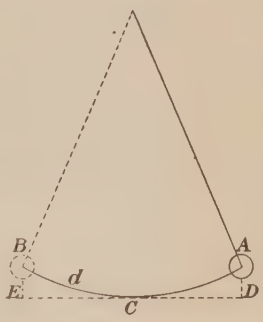


Fig. 173. Transformation of potential and kinetic energy

of the pendulum is all potential, while in the middle of the swing its energy is all kinetic. In intermediate positions the energy is part potential and part kinetic, but the sum of the two is equal to the original potential energy.[1]

225. General statement of the law of frictionless machines. In our development of the law of machines, which led us to the conclusion that the work of the acting force is always equal to the work of the resisting force, we were careful to make two important assumptions, — first, that friction was negligible, and second that the motions were all either uniform or so slow that no appreciable velocities were imparted. In other words, we assumed that the work of the acting force was expended simply in lifting weights or compressing springs, i.e. in storing up potential energy. If now we drop the second assumption, a very simple experiment will show that our conclusion must be somewhat modified. Suppose, for instance, that instead of lifting a 500-g. weight slowly by means of a balance, we jerk it up suddenly. We shall now find that the initial pull indicated by the balance, instead of being 500 g., will be considerably more, — perhaps as much as several thousand grams if the pull is sufficiently sudden. This is obviously because the acting force is now overcoming not merely the 500 g. which represents the resistance of gravity, but also the inertia of the body, since velocity is being imparted to it. Now work done in imparting velocity to a body, i.e. in overcoming its inertia, always appears as *kinetic* energy, while work done in overcoming gravity appears as the *potential* energy of a lifted weight. Hence, whether the motions produced by machines are slow or fast, if friction is negligible, the law for all devices for transforming work may be stated thus: *The work of the acting force is equal to the sum of the potential and kinetic energies stored up in the mass acted upon.* In machines which work against gravity

[1] It is recommended that a laboratory exercise on the laws of the pendulum precede this discussion. See, for example, Experiment 17, authors' manual.

the body usually starts from rest and is left at rest, so that the kinetic energy resulting from the whole operation is zero. Hence in such cases the work done is the weight lifted times the height through which it is lifted, whether the motion is slow or fast. The kinetic energy imparted to the body in starting is all given up by it in stopping.

226. The measure of potential and kinetic energy. The measure of the potential energy of any lifted body, such as a lifted pile driver, is equal to the work which has been spent in lifting the body. Thus if h is the height in centimeters and M the weight in grams, then the potential energy P.E. of the lifted mass is

$$\text{P.E.} = Mh \text{ gram centimeters.} \tag{11}$$

Since the force of the earth's attraction for M grams is Mg dynes, if we wish to express the potential energy in ergs instead of in gram centimeters, we have

$$\text{P.E.} = Mgh \text{ ergs.} \tag{12}$$

Since this energy is all transformed into kinetic energy when the mass falls the distance h, the product Mgh also represents the number of ergs of kinetic energy which the moving weight has when it strikes the pile.

If we wish to express this kinetic energy in terms of the velocity with which the weight strikes the pile, instead of the height from which it has fallen, we have only to substitute for h its value in terms of g and the velocity acquired (see equation (5), p. 31), namely $h = v^2/2g$. This gives the kinetic energy K.E. in the form

$$\text{K.E.} = \tfrac{1}{2} Mv^2 \text{ ergs.} \tag{13}$$

Since it makes no difference how a body has acquired its velocity, this represents the general formula for the kinetic energy *in ergs* of any moving body, in terms of its mass and its velocity.

Thus the kinetic energy of a 100-g. bullet moving with a velocity of 10,000 cm. per second is

$$\text{K.E.} = \tfrac{1}{2} \times 100 \times (10{,}000)^2 = 5{,}000{,}000{,}000 \text{ ergs.}$$

Since 1 gram centimeter is equivalent to 980 ergs, the energy of this bullet is $\frac{5000000000}{980} = 5{,}102{,}000$ g. cm., or 51.02 kg. m.

We know, therefore, that the powder pushing on the bullet as it moved through the rifle barrel did 51.02 kilogram meters of work upon the bullet in giving it the velocity of 100 m. per second.

QUESTIONS AND PROBLEMS

1. What must be the power in kilowatts of the engines supplying the city water in problem 3, p. 147? Express the power also in horse power. (Assume a 24-hour day.)

2. What must be the horse power of an engine which is to pump 10,000 l. of water per second from a mine 100 m. deep?

3. A water motor discharges 100 l. of water per minute when fed from a reservoir in which the water surface stands 100 ft. above the level of the motor. If all of the potential energy of the water were transformed into work in the motor, what would be the horse power of the motor? (The potential energy of the water is the amount of work which would be required to carry it back to the top of the reservoir.)

4. A pile driver weighing 3000 lb. is raised to a height of 20 ft. and allowed to fall on the head of a pile which it drives 4 in. What transformations of energy take place in the whole operation, and what is the average resistance offered by the earth? (Remember that work is equal to force × distance.)

5. The falls of Niagara are about 160 ft. high. It is estimated that 700,000 tons of water pass over them per minute. If this energy could all be utilized, what horse power could be obtained from the falls?

6. A 200-g. ball leaves a bat with a velocity of 20 m. per second. Find its K.E. in g. cm.

7. A train weighing 200 metric tons is moving at a rate of 80 km. per hour. Find its K.E. in kilogram meters. (Find the energy first in ergs, then reduce to kilogram meters.)

CHAPTER IX

WORK AND HEAT ENERGY

Friction

227. Friction always results in wasted work. All of the experiments mentioned in the last chapter were so arranged that *friction* could be neglected or eliminated. So long as this condition was fulfilled it was found that the result of universal experience could be stated in the law, *The work done by the acting force is equal to the sum of the kinetic and potential energies stored up.* In other words, if there were no friction, no work would ever be wasted. We should be able to obtain from every machine exactly as much work as we put into it, no more and no less.

But wherever friction is present this law is found to be inexact, for the work of the acting force is then always somewhat greater than the sum of the kinetic and potential energies stored up. If, for example, a block is pulled over the horizontal surface of a table, at the end of the motion no velocity has been imparted to the block, and hence no kinetic energy has been stored up. Further, the block has not been lifted nor put into a condition of elastic strain, and hence no potential energy has been communicated to it. We cannot in any way obtain from the block more work after the motion than we could have obtained before it was moved. It is clear, therefore, that all of the work which was done in moving the block against the friction of the table was *wasted work.* Experience shows that, in general, where work is done against friction it can never be regained. Before considering what becomes of this wasted work

we shall consider some of the factors on which friction depends, and some of the laws which are found by experiment to hold in cases in which friction occurs.

228. Laws of sliding friction. The unavoidable irregularities in all surfaces make accurate experiments upon friction impossible. The following laws are therefore to be regarded as rough approximations. They may easily be verified in a general way by pulling a block over a smooth board with the aid of a spring balance, or by means of a pulley and weights arranged as in Fig. 174.

1. The friction between two solid surfaces is greater at starting than after the motion has begun. This is doubtless because the inequalities in the upper surface sink into those in the lower more completely at rest than in motion.

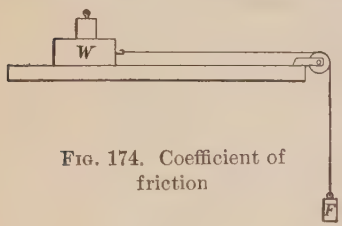


Fig. 174. Coefficient of friction

2. After the motion has started the friction between solid surfaces is independent of the speed of the motion.

3. Friction between two surfaces is proportional to pressure; i.e. doubling the weight W of block and load together (Fig. 174) makes it necessary to double the force F required to maintain uniform motion.

4. Friction is independent of extent of surface so long as the total force pressing the two surfaces together is constant; i.e. it requires the same force to keep a brick sliding on its end as on its side. This result follows necessarily from 3.

229. Coefficient of friction. From 3 and 4 of the preceding section it follows that if F (Fig. 174) is the force necessary to maintain uniform motion in the weight W, then the ratio F/W depends only on the nature of the two surfaces in contact. It is called the *coefficient of friction* for the given materials. Thus if F is 300 g. and W 800 g., then the coefficient of friction

between the table and the block is $\frac{300}{800} = .375$. The coefficient of iron upon iron is about .2, of oak on oak about .4.

230. Rolling friction. The chief cause of sliding friction is the interlocking of minute projections (shown greatly magnified at *a*, *b*, *c*, and *d* in Fig. 175). When a round solid *rolls* over a smooth surface the frictional resistance is generally much less than when it slides; e.g. the coefficient of friction of cast-iron wheels rolling on iron rails may be as low as .002, i.e. $\frac{1}{100}$ of the sliding friction of iron on iron. This means that a pull of 1 lb. will keep a 500-lb. car in motion. Sliding friction is not, however, entirely dispensed with in ordinary wheels, for although the rim of the wheel rolls on the track, the axle slides continuously at some point *c* (Fig. 176) upon the surface of the journal.

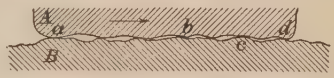


Fig. 175. Illustrating friction of rubbing surfaces

The great advantage of the ball bearing (Fig. 177) is that the sliding friction in the hub is almost completely replaced by rolling friction.

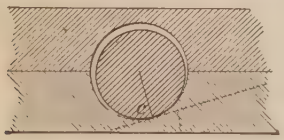


Fig. 176. Common bearing

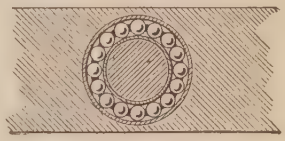


Fig. 177. Ball bearing

231. Fluid friction. When a solid moves through a fluid, as when a bullet moves through the air or a ship through the water, the resistance encountered is not at all independent of velocity, as in the case of solid friction, but increases for slow speeds nearly as the square of the velocity, and for high speeds at a rate considerably greater. This explains why it is so expensive to run a fast train; for the resistance of the air, which is a small part of the total resistance so long as the train is moving slowly, becomes the predominant factor at high speeds. The resistance offered to steamboats running at high speeds is usually considered to increase as the cube of the velocity. Thus the *Cedric*, of the White Star Line, having a speed of 17 knots, has a horse power of 14,000, and a total weight when loaded of about 38,000 tons, while the *Kaiser Wilhelm II*, of the North German Lloyd Line, having a speed of 24 knots, has engines of 40,000 horse power, although the total weight when loaded is only 26,000 tons.

232. Internal friction. When a lead bullet strikes against a target the layers of lead slip over one another in the process of flattening, and thus the total kinetic energy of the bullet is wasted in internal friction within the lead. This waste of energy because of internal friction takes place, to some extent, whenever a body is distorted, but in elastic bodies the waste is much less rapid than in inelastic ones. Thus when a rubber ball is dropped upon a stone sidewalk, the kinetic energy of the ball just before impact is largely transformed into potential energy of strain, and this is again transformed into kinetic energy in the rebound. But since the ball will never rebound quite to its original height, we know that there is in this case also a certain amount of energy wasted in internal friction within the ball. In general, the greater the amount of *permanent deformation* the greater the waste in internal friction.

QUESTIONS AND PROBLEMS

1. Why is it harder for a team of horses to start a heavy load on a hard road than to keep it going after it is started? Give two reasons.

2. Why is a stream swifter at the center than at the banks?

3. A smooth block is $10 \times 8 \times 3$ inches. Compare the distances which it will slide when given a certain initial velocity on smooth ice, if resting first on a 10×8 face; second, on a 10×3 face; and third, on an 8×3 face.

4. What is the coefficient of friction of brass on brass if a force of 20 lb. is required to maintain uniform motion in a brass block weighing 200 lb., when it slides horizontally on a brass bed?

5. The coefficient of friction between a block and a table is .3. What force will be required to keep a block weighing 500 g. in uniform motion?

6. In what way is friction an advantage in lifting buildings with a jack-screw? In what way is it a disadvantage?

Efficiency

233. Definition of efficiency. Since it is only in an ideal machine that there is no friction, in all actual machines the work done by the acting force always exceeds, by the amount

of the work done against friction, the amount of potential and kinetic energy stored up. We have seen that the latter is wasted work in the sense that it can never be regained. Since the energy stored up represents work which can be regained, it is termed *useful work*. In most machines an effort is made to have the useful work as large a fraction of the total work expended as possible. *The ratio of the useful work to the total work done by the acting force is called the* EFFICIENCY *of the machine.* Thus

$$\text{Efficiency} = \frac{\text{Useful work accomplished}}{\text{Total work expended}}. \qquad (1)$$

Thus, if in the system of pulleys shown in Fig. 148 it is necessary to add a weight of 50 g. at F in order to pull up slowly an added weight of 240 g. at W, the work done by the 50 g. while F is moving over 1 cm. will be 50×1 g. cm. The useful work accomplished in the same time is $240 \times \frac{1}{6}$ g. cm. Hence the efficiency is equal to $\frac{240 \times \frac{1}{6}}{50 \times 1} = \frac{4}{5} = 80\%$.

234. Efficiencies of some simple machines. In simple levers the friction is generally so small as to be negligible; hence the efficiency of such machines is approximately 100%. When inclined planes are used as machines the friction is also small, so that the efficiency generally lies between 90% and 100%. The efficiency of the commercial block and tackle (Fig. 148), with several movable pulleys, is usually considerably less, varying between 40% and 60%. In the jackscrew there is necessarily a very large amount of friction, so that although the mechanical advantage is enormous, the efficiency is often as low as 25%. The differential pulley of Fig. 166 has also a very high mechanical advantage with a very small efficiency. Gear wheels such as those shown in Fig. 164, or chain gears such as those used in bicycles, are machines of comparatively high efficiency, often utilizing between 90% and 100% of the energy expended upon them.

235. Efficiency of overshot water wheels. The overshot water wheel (Fig. 178) utilizes chiefly the potential energy of the water at S; for the wheel is turned by the weight of the water in the buckets. The work expended on the wheel per second, in ft. lb. or g. cm., is the product of the weight of the water which passes over it per second by the distance through which it falls. The efficiency is the work which the wheel can accomplish in a second, divided by this quantity. Such wheels are very common in mountainous regions, where it is easy to obtain considerable fall, but where the streams carry a small *volume* of water. The efficiency is high, being often between 80% and 90%. The loss is due not only to the friction in the bearings and gears (see C), but also to the fact that some of the water is spilled from the buckets, or passes over without entering them at all. This may still be regarded as a frictional loss, since the energy disappears in internal friction when the water strikes the ground.

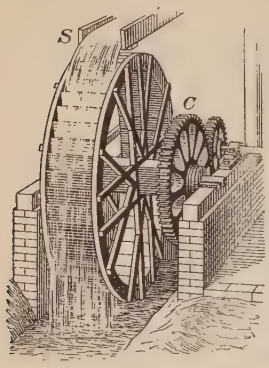


Fig. 178. Overshot water wheel

236. Efficiency of undershot water wheels. The old-style undershot wheel (Fig. 179), so common in flat countries where there is little fall but abundance of water, utilizes only the kinetic energy of the water running through the race from A. It seldom transforms into useful work more than 25% or 30% of the potential energy of the water above the dam. There are, however, certain modern forms of undershot wheel which are extremely efficient. For example, the *Pelton wheel* (Fig. 180), developed since 1880, and now very commonly used for small-power purposes in cities supplied with waterworks, sometimes has an efficiency as high as 83%. The water is delivered from a nozzle O against cup-shaped buckets arranged as in the figure.

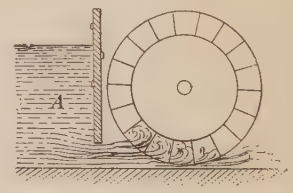


Fig. 179. The undershot wheel

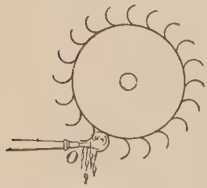


Fig. 180. The Pelton water motor

237. Efficiency of water turbines. The turbine wheel was invented in France in 1833, and is now used more than any other form of water wheel. It stands completely under water in a case at the bottom of a *turbine pit*, rotating in a horizontal plane. Fig. 181 shows one of several methods of installing such a wheel. AB is the turbine pit and C the outer case into which the water enters from the pit. Fig. 182 shows the outer case with contained turbine; Fig. 183 is the inner case in which are the fixed guides G, which direct the water at the most

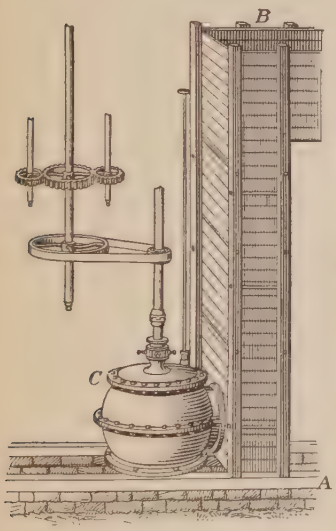


Fig. 181. A turbine installed

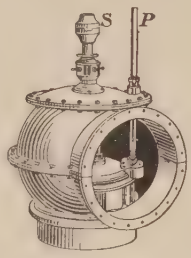


Fig. 182. Outer case of turbine

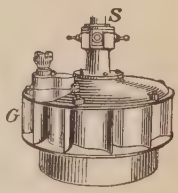


Fig. 183. The inner case

Fig. 184. A turbine wheel

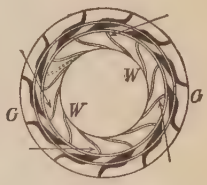


Fig. 185. Section of turbine wheel

advantageous angle against the blades of the wheel inside; Fig. 184 is the wheel itself; and Fig. 185 is a section of wheel and inner case, showing how the water enters through the guides and impinges upon the blades W. The spent water simply falls down from the blades into the tailrace (Fig. 181). The amount of water which passes through the turbine can be controlled by means of the rod P (Fig. 182), which can be turned so as to increase or decrease the size of the openings between the guides G (Fig. 183). The energy expended upon the turbine per second is the product of the mass of water which passes through it by the

height of the turbine pit. Efficiencies as high as 90% have been attained with such wheels. The most powerful turbine in existence is at Shawenegan Falls, Quebec, Canada. The pit is 135 ft. deep, the wheel 10 ft. in diameter, and the horse power developed 10,500.

QUESTIONS AND PROBLEMS

1. If it is necessary to pull on a block and tackle with a force of 100 lb. in order to lift a weight of 400 lb., and if the force must move 6 ft. to raise the weight 1 ft., what is the efficiency of the system?

2. How many strands of rope were supporting the weight in the previous problem?

3. The largest overshot water wheel in existence is at Laxey, on the Isle of Man. It has a horse power of 150, a diameter of 72.5 ft., and an efficiency of 85%. How many cubic feet of water pass over it per second? (1 cu. ft. weighs 62.3 lb.)

4. The Niagara turbine pits are 136 ft. deep and their average horse power is 5000. Their efficiency is 85%. How much water does each turbine discharge per minute?

5. There is a Pelton wheel at the Sutro tunnel in Nevada which is driven by water supplied from a reservoir 2100 ft. above the level of the motor. The diameter of the nozzle is but 1/2 in., and that of the wheel but 3 ft., yet 100 H.P. is developed. If the efficiency is 80%, how many cubic feet of water are discharged per second?

Mechanical Equivalent of Heat[1]

238. What becomes of wasted work? In all of the devices for transforming work which we have considered we have found that on account of frictional resistances a certain per cent of the work expended upon the machine is wasted. The question which at once suggests itself is, "What becomes of this wasted work?" The following familiar facts suggest an answer. When two sticks are vigorously rubbed together they become hot; augers and drills often become too hot to hold; matches are

[1] This subject should be preceded by a laboratory experiment upon the "law of mixtures," and either preceded or accompanied by experiments upon specific heat and mechanical equivalent. See, for example, Experiments 18, 19, and 20, authors' manual.

ignited by friction; if a strip of lead be struck a few sharp blows with a hammer, it is appreciably warmed. Now since we learned in Chapter V that, according to modern notions, increasing the temperature of a body means simply increasing the average velocity of its molecules, and therefore their average kinetic energy, the above facts point strongly to the conclusion that in each case the mechanical energy expended has been simply transformed into the energy of molecular motion. This view was first brought into prominence in 1798 by Benjamin Thompson, Count Rumford, an American by birth. It was first carefully tested by the English physicist, James Prescott Joule (1818–1889), in a series of epoch-making experiments extending from 1842 to 1870. In order to understand these experiments we must first learn how heat quantities are measured.

239. Unit of heat, — the calorie. A unit of heat is defined as *the amount of heat which is required to raise the temperature of 1 g. of water through 1° C.* This unit is called *the calorie.* Thus, for example, when a hundred grams of water has its temperature raised four degrees, we say that four hundred calories of heat have entered the water. Similarly, when a hundred grams of water has its temperature lowered ten degrees, we say that a thousand calories have passed out of the water. If, then, we wish to measure, for instance, the amount of heat developed in a lead bullet when it strikes against a target, we have only to let the spent bullet fall into a known weight of water and to measure the number of degrees through which the temperature of the water rises. The product of the number of grams of water by its rise in temperature is then, by definition, the number of calories of heat which have passed into the water.

It will be noticed that in the above definition we make no assumption whatever as to what *heat* is. Previous to the nineteenth century physicists generally held it to be an invisible, weightless fluid, the passage of which into or out of a body caused it to grow hot or cold. This view accounts well enough

for the heating which a body experiences when it is held in contact with a flame or other hot body, but it has difficulty in explaining the heating produced by rubbing or pounding. Rumford's view accounts easily for this, as we have seen, while it accounts no less easily for the heating of cold bodies by contact with hot ones; for we have only to think of the hotter and therefore more energetic molecules of the hot body as communicating their energy to the molecules of the colder body in much the same way in which a rapidly moving billiard ball transfers part of its kinetic energy to a more slowly moving ball against which it strikes.

240. Joule's experiment on the heat developed by friction. Joule argued that if the heat produced by friction, etc., is indeed merely mechanical energy which has been transferred to the molecules of the heated body, then the same number of calories must always be produced by the disappearance of a given amount of mechanical energy. And this must be true no matter whether the work is expended in overcoming the friction of wood on wood, of iron on iron, in percussion, in compression, or in any other conceivable way. To see whether or not this were so he caused mechanical energy to disappear in as many ways as possible and measured in every case the amount of heat developed.

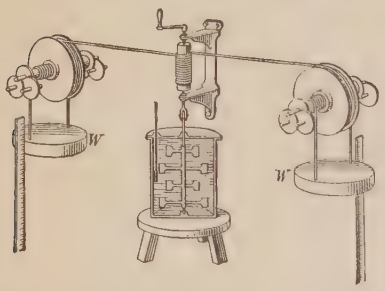


Fig. 186. Joule's first experiment on the mechanical equivalent of heat

In his first experiment he caused paddle wheels to rotate in a vessel of water by means of falling weights W (Fig. 186). The amount of work done by gravity upon the weights in causing them to descend through any distance d was equal to their weight W times this distance. If the weights descended slowly and uniformly, this work was all

JAMES PRESCOTT JOULE (1818–1889)

English physicist, born at Manchester; most prominent figure in the establishment of the doctrine of the conservation of energy; studied chemistry as a boy under John Dalton and became so interested that his father, a prosperous Manchester brewer, fitted out a laboratory for him at home; conducted most of his researches either in a basement of his own house or in a yard adjoining his brewery; discovered the law of heating a conductor by an electric current (see p. [illegible]); carried out, in connection with Lord Kelvin, epoch-making researches upon the thermal properties of gases; did important work in magnetism; first proved experimentally the identity of various forms of energy.

expended in overcoming the resistance of the water to the motion of the paddle wheels through it; i.e. it was wasted in eddy currents in the water. Joule measured the rise in the temperature of the water and found that the mean of his three best trials gave 427 gram meters as the amount of work required to develop enough heat to raise a gram of water one degree. He then repeated the experiment, substituting mercury for water, and obtained 425 gram meters as the work necessary to produce a calorie of heat. The difference between these numbers is less than was to have been expected from the unavoidable errors in the observations. He then devised an arrangement in which the heat was developed by the friction of iron on iron, and again obtained 425.

241. Heat produced by collision. A Frenchman by the name of Hirn was the first to make a careful determination of the relation between the heat developed by *collision* and the kinetic energy which disappears. He allowed a steel cylinder to fall through a known height and crush a lead ball by its impact upon it. The amount of heat developed in the lead was measured by observing the rise in temperature of a small amount of water into which the lead was quickly plunged. As the mean of a large number of trials he, also, found that 425 gram meters of energy disappeared for each calorie of heat which appeared.

242. Heat produced by the compression of a gas. Another way in which Joule measured the relation between heat and work was by compressing a gas and comparing the amount of work done in the compression with the amount of heat developed.

Every bicyclist is aware of the fact that when he inflates his tires the pump grows hot. This is due partly to the friction of the piston against the walls, but chiefly to the fact that the downward motion of the piston is transferred to the molecules which come in contact with it, so that the velocity of these molecules is increased. The principle is precisely the same as that involved in the velocity communicated to a ball by a bat. If the bat is held rigidly fixed and a ball thrown against it, the ball rebounds with a certain velocity; but if the bat is moving rapidly forward to meet the ball, the latter rebounds with a

much greater velocity. So the molecules which in their natural motions collide with an advancing piston, rebound with greater velocity than they would if they had impinged upon a fixed wall. This increase in the molecular velocity of a gas on compression is so great that when a mass of gas at 0° C. is compressed to one half its volume the temperature rises to 87° C.

Fig. 187. The fire syringe

The effect may be strikingly illustrated by the fire syringe (Fig. 187). Let a few drops of carbon bisulphide be placed on a small bit of cotton, dropped to the bottom of the tube *A*, and then removed; then let the piston *B* be inserted and very suddenly depressed. Sufficient heat will be developed to ignite the vapor and a flash will result. (If the flash does not result from the first stroke, withdraw the piston completely, then reinsert, and compress again.)

To measure the heat of compression Joule surrounded a small compression pump with water, took 300 strokes on the pump, and measured the rise in temperature of the water. As the result of these measurements he obtained 444 gram meters as the "mechanical equivalent" of the calorie. The experiment, however, could not be performed with great exactness.

243. Cooling by expansion. Joule also obtained the relation between heat and work from experiments on the cooling produced by expansion. This process is exactly the converse of heating by compression. If a compressed gas is allowed to expand and force out a piston, or merely to expand against atmospheric pressure, it is always found to be cooled by the process. This is because the kinetic energy of the molecules is transferred to the piston, so that the former rebound from the latter with less velocity than they had before the blow. The refrigerators used on shipboard are good illustrations of this principle. Air is compressed by an engine to perhaps one fourth its natural volume. The heat generated by the compression is

then removed by causing the air to circulate about pipes kept cool by the flow of cold water through them. This compressed air is then allowed to expand into the refrigerating chamber, the temperature of which is thus lowered many degrees.

Joule's determination of the mechanical equivalent of heat from the amount of work done by an expanding gas and the amount of heat lost in expansion gave 437 gram meters. This experiment also was one for which no great amount of exactness could be claimed.

244. Significance of Joule's experiments. Joule made three other determinations of the relation between heat and work by methods involving electrical measurements. He published as the mean of all his determinations 426.4 gram meters as the mechanical equivalent of the calorie. But the value of his experiments does not lie primarily in the accuracy of the final results, but rather in the proof which they for the first time furnished that whenever a given amount of work is wasted, no matter in what particular way this waste may occur, there is always an appearance of the same definite invariable amount of heat.

The most accurate determination of the mechanical equivalent of heat was made by Rowland (1848–1901) in 1880 at Johns Hopkins University. He obtained 427 g. m., or 4.19×10^7 ergs.

245. The conservation of energy. We are now in a position to state the law of all machines in its most general form, i.e. in such a way as to include even the cases where friction is present. It is: *The work done by the acting force is equal to the sum of the kinetic and potential energies stored up plus the mechanical equivalent of the heat developed.*

In other words, *whenever energy is expended on a machine or device of any kind, an exactly equal amount of energy always appears either as useful work or as heat.* The useful work may be represented in the potential energy of a lifted mass, as when water is pumped up to a reservoir; or in the kinetic energy of

a moving mass, as when a stone is thrown from a sling; or in the potential energies of molecules whose positions with reference to one another have been changed, as when a spring has been bent; or in the molecular potential energy of chemically separated atoms, as when an electric current separates a compound substance. The *wasted* work always appears in the form of increased molecular motion, i.e. in the form of heat. This important generalization has received the name of the *Principle of the Conservation of Energy.* It may be stated thus: *Energy may be transformed, but it can never be created or destroyed.*

246. Perpetual motion. In all ages there have been men who have spent their lives in trying to invent a machine out of which work could be continually obtained, without the expenditure of an equivalent amount of work upon it. Such devices are called perpetual-motion machines. Even to this day the United States patent office annually receives scores of applications for patents on such devices. The possibility of the existence of such a device is absolutely denied by the statement of the principle of the conservation of energy. For only in case there is no heat developed, i.e. in case there are no frictional losses, can the work taken out be equal to the work put in, and in no case can it be greater. Since, in fact, there are always some frictional losses, the principle of the conservation of energy asserts that it is impossible to make a machine which will keep itself running forever, even though it does no useful work; for no matter how much kinetic or potential energy is imparted to the machine to begin with, there must always be a continual drain upon this energy to overcome frictional resistances; so that, as soon as the wasted work has become equal to the initial energy, the machine must stop.

The first man to make a formal and complete statement of the principle of the conservation of energy was the German physician, Robert Mayer, whose work was published in 1842. Twenty years later, partly through the theoretical writings of

Helmholtz and Clausius in Germany, and of Kelvin and Rankine in England, but more especially through the experimental work of Joule, the principle had gained universal recognition and had taken the place which it now holds as the corner stone of all physical science.

247. Examples of transformation of energy. When a bullet is fired vertically from a rifle the energy which projects it upward first exists in the form of molecular potential energy in the separated chemical constituents of the gunpowder. Gunpowder is a mixture of from 70% to 80% potassium nitrate (niter) and 10% to 15% each of sulphur and charcoal. These elements combine in the explosion so as to form a mixture of the gases nitrogen and carbon dioxide. These gases occupy at atmospheric pressure about 1500 times the volume of the gunpowder from which they are formed. At the instant of formation they therefore possess the potential energy of highly compressed elastic bodies. In the process of expanding, this energy is transformed into the kinetic energy of the rising bullet. In the ascent this kinetic energy is transformed into the potential energy of a lifted mass, and in the descent this potential energy is again transformed into kinetic energy. Finally, as the bullet strikes the earth this kinetic energy is all changed into heat, i.e. into molecular kinetic energy.

The transformations of energy which take place in any power plant, such as that at Niagara, are as follows. The energy first exists as the potential energy of the water at the top of the falls. This is transformed in the turbine pits into the kinetic energy of the rotating wheels. These turbines drive dynamos in which there is a transformation into the energy of electric currents. These currents are carried by wires to Buffalo and other cities, where they run street cars and other forms of motors. The principle of conservation of energy asserts that the work which gravity did upon the water in causing it to descend from the top to the bottom of the turbine pits is exactly equal to the work done by all of the motors, plus the heat developed in all the wires and bearings, and in the eddy currents in the water.

248. Energy derived from the sun. Let us next consider where the water at the top of the falls obtained its potential energy. Water is being continually evaporated at the surface of the ocean by the sun's heat. This heat imparts sufficient kinetic energy to the molecules to enable them to break away from the attractions of their fellows and to rise above the surface in the form of vapor. The lifted vapor is carried

by winds over the continents and precipitated in the form of rain or snow. Thus the potential energy of the water above the falls at Niagara is simply transformed heat energy of the sun. If, in this way, we analyze any available source of energy at man's disposal, we find in practically every case that it is directly traceable to the sun's heat as its source. Thus the energy contained in coal is simply the energy of separation of the oxygen and carbon which were separated in the processes of growth. This separation was effected by the sun's rays.

We can form some conception of the enormous amount of energy which the sun radiates in the form of heat by reflecting that of this heat the earth receives not more than $\frac{1}{2,000,000,000}$ part. Of the amount received by the earth not more than $\frac{1}{1000}$ part is stored up in animal and vegetable life and lifted water. This is practically all of the energy which is available on the earth for man's use.

QUESTIONS AND PROBLEMS

1. How many calories of heat are generated by the impact of a 200-kilo bowlder when it falls vertically through 100 meters?

2. Thousands of meteorites are falling into the sun with enormous velocities every minute. From a consideration of the preceding example account for a portion, at least, of the sun's heat.

3. The Niagara Falls are 160 ft. high. How much warmer is the water at the bottom of the falls than at the top?

4. A car weighing 60,000 kilos slides down a grade which is 2 m. lower at the bottom than at the top, and is brought to rest at the bottom by the brakes. How many calories of heat are developed by the friction?

5. A body weighing 10 kilos is pushed 10 m. along a level plane. If the coefficient of friction between the block and the plane is .25, how many gram centimeters of work have been done? How many ergs? How many calories of heat have been developed?

6. Meteorites are small cold bodies moving about in space. Why do they become luminous when they enter the earth's atmosphere?

Specific Heat

249. Two ways of heating a body. In the preceding paragraphs we have called attention chiefly to the heating which bodies may experience because mechanical energy is expended upon them. But common experience teaches us that a body

may also be heated by bringing it into contact with a hotter body. In this case the increased velocity of the molecules is due to energy received by collisions with the more energetically moving molecules of the hotter body. Whether the energy is received by the first method or by the second, the amount of energy which must be imparted to the molecules of one gram of water to raise it through 1° C. must evidently always be the same, viz. 427 gram meters, or 42,000,000 ergs. This energy is called *heat energy* as soon and only as soon as it exists in the form of molecular vibrations. If it exists in the form of a moving or a lifted mass or a compressed spring, it is called simply mechanical energy. Hence, in the first method of heating, the heat energy is *created* at the expense of mechanical energy; in the second method the heat energy is simply *transferred* from one body to another. *A calorie may now be defined, without reference to water or any other particular substance, simply as 42,000,000 ergs of heat energy.*

250. Definition of specific heat. When we experiment upon different substances we find that it requires wholly different amounts of heat energy to produce in one gram of mass one degree of change in temperature.

Let 100 g. of lead shot be placed in one test tube, 100 g. of bits of iron wire in another, and 100 g. of aluminium wire in a third. Let them all be placed in a pail of boiling water for ten or fifteen minutes, care being taken not to allow any of the water to enter any of the tubes. Let three small vessels be provided, each of which contains 100 g. of water at the temperature of the room. Let the heated shot be poured into the first beaker, and after thorough stirring let the rise in the temperature of the water be noted. Let the same be done with the other metals. The aluminium will be found to raise the temperature about twice as much as the iron, and the iron about three times as much as the lead. Hence, since the three metals have cooled through approximately the same number of degrees, we must conclude that about six times as much heat has passed out of the aluminium as out of the lead; i.e. each gram of aluminium in cooling 1° C. gives out about six times as many calories as a gram of lead.

The number of calories taken up by one gram of a substance when its temperature rises through 1° C., or given up when it falls through 1° C., is called the specific heat of that substance.

It will be seen from this definition, and the definition of the calorie, that the specific heat of water is 1.

251. Determination of specific heat by the method of mixtures. The preceding experiments illustrate a method for measuring accurately the specific heats of different substances. For, in accordance with the principle of the conservation of energy, when hot and cold bodies are mixed, as in these experiments, so that heat energy passes from one to the other, *the gain in the heat energy of one must be just equal to the loss in the heat energy of the other.*

This method is by far the most common one for determining the specific heats of substances. It is known as the *method of mixtures.*

Suppose, to take an actual case, that the initial temperature of the shot used in § 250 was 95° C., and that of the water 19.7°, and that, after mixing, the temperature of the water and shot was 22°. Then, since 100 g. of water has had its temperature raised through $22° - 19.7° = 2.3°$, we know that 230 calories of heat have entered the water. Since the temperature of the shot fell through $95° - 22° = 73°$, the number of calories given up by the 100 g. of shot in falling 1° was $\frac{230}{73} = 3.15$. Hence the specific heat of lead, i.e. *the number of calories of heat given up by 1 g. of lead when its temperature falls 1°* C., is $\frac{3.15}{100} = .0315$.

Or again, we may work out the problem algebraically as follows. Let x equal the specific heat of lead. Then the number of calories which come out of the shot is $100\,(95 - 22)\,x$, and the number which enter the water is $100\,(22 - 19.7)$. Since, then, the heat lost by the shot is equal to the heat gained by the water, we have

$$100\,(95 - 22)\,x = 100\,(22 - 19.7) \text{ or } x = .0315.$$

By experiments of this sort the specific heats of some of the common substances have been found to be as follows.

Table of Specific Heats

Aluminium	.218	Iron	.113
Brass	.094	Lead	.0315
Copper	.095	Mercury	.0333
Glass	.2	Platinum	.032
Gold	.0316	Silver	.0568
Ice	.504	Zinc	.0935

QUESTIONS AND PROBLEMS

1. If 100 g. of mercury at 100° C. are mixed with 100 g. of water at 20° C., and if the resulting temperature is 22.6° C., what is the specific heat of mercury?

2. A 10 g. bullet of lead is shot from a gun with a velocity of 300 meters per second. Find its K. E. in ergs. Through how many degrees C. is its temperature raised when it strikes a target? (Assume that all of the heat stays in the bullet.)

3. From what height must a block of lead fall in order to have its temperature raised through 10° C.?

4. If 200 g. of water at 80° C. are mixed with 100 g. of water at 10° C., what will be the temperature of the mixture? (Let x equal the final temperature; then $100(x - 10)$ calories are gained by the cold water, while $200(80 - x)$ calories are lost by the hot water.)

5. What temperature will result if 300 g. of copper at 100° C. are placed in 200 g. of water at 10° C.?

6. The specific heat of water is much greater than that of any other liquid, or of any solid. Explain how this accounts for the fact that an island in mid-ocean undergoes less extremes of temperature than an inland region.

7. How many grams of ice-cold water must be poured into a tumbler weighing 300 g. to cool it from 60° C. to 20° C., the specific heat of glass being .2?

8. If a block of lead is pounded a given number of blows with a hammer, it will become quite hot. Why? If a block of iron of the same weight is given the same number of blows, will it be heated more or less than the lead? (In the answer take into account both the fact that the hammer rebounds from the iron with much greater velocity than from the lead, and the fact that the specific heat of iron is greater than that of lead.)

9. If the moon were to collide with the earth while moving at the velocity which it now has in its orbit about the earth, viz. about 10 km. per second, how many calories of heat would be developed per gram of the moon's mass.

If because of the mixture of the matter of the earth and moon in the collision this heat were distributed over a mass twice as great as that of the moon, what would be the mean rise in temperature in the region of the collision if the mean specific heat of the earth and moon be assumed to be .5?

10. A piece of platinum weighing 10 g. is taken from a furnace and plunged instantly into 40 g. of water at 10° C. The temperature of the water rises to 24° C. What was the temperature of the furnace? (This method is used for estimating the temperature of the electric arc (about 3500° C.) and other temperatures which are above the points of vaporization of most substances, a carbon button being used instead of platinum. The results, however, are uncertain since we do not know that the specific heat of carbon is the same at very high as at low temperatures.)

Heat Engines

252. The modern steam engine. Thus far we have considered only cases in which mechanical energy was transformed into heat energy. In all heat engines we have examples of exactly the reverse operation, namely the transformation of heat energy back into mechanical energy. How this is done may best be understood from a study of various modern forms of heat engines. The invention of the form of the steam engine which is now in use is due to James Watt, who, at the time of the invention (1768), was an instrument maker in the University of Glasgow.

The operation of such a machine can best be understood from the ideal diagram shown in Fig. 188. Steam generated by the fire F in the boiler B passes through the pipe S into the steam chest V, and thence through the passage N into the cylinder C, where its pressure forces the piston P to the left. It will be seen from the figure that, as the driving rod R moves toward the left, the so-called eccentric rod R', which controls the valve V, moves toward the right. Hence, when the piston has reached the left end of its stroke the passage N will have been closed, while the passage M will have been opened, thus throwing the pressure from the left to the right side of the piston, and at the

same time putting the right end of the cylinder, which is full of spent steam, into connection with the exhaust pipe E. This operation goes on continually, the rod R' opening and closing the passages M and N at just the proper moments to keep the piston moving back and forth throughout the length of the cylinder. The shaft carries a heavy fly wheel W, the great inertia of which insures constancy in speed. The motion of

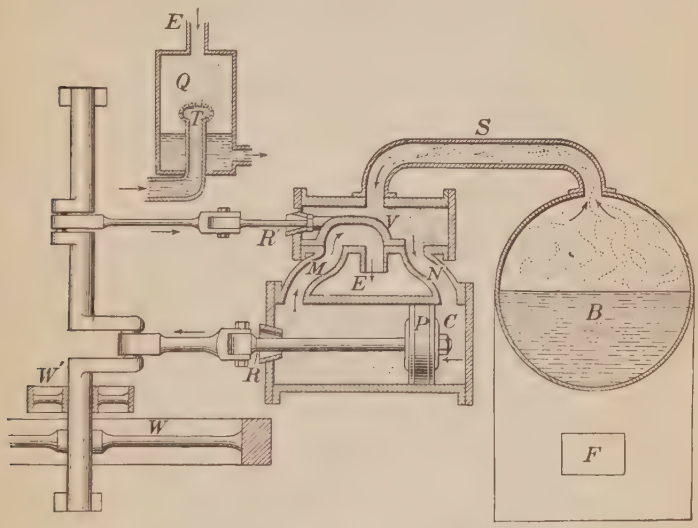


FIG. 188. Ideal diagram of a steam engine

the shaft is communicated to any desired machinery by means of a belt which passes over the pulley W'.

253. Condensing and noncondensing engines. In most stationary engines the exhaust E leads to a condenser which consists of a chamber Q, into which plays a jet of cold water T, and in which a partial vacuum is maintained by means of an air pump. In the best engines the pressure within Q is not more than from 3 to 5 cm. of mercury, i.e. not more than a pound to the square inch. Hence the condenser reduces the

back pressure against that end of the piston which is open to the atmosphere from 15 lb. down to 1 lb., and thus increases the effective pressure which the steam on the other side of the piston can exert. Since, however, the addition of the condenser makes the engine more expensive, more heavy, and more complicated, it is generally omitted on locomotives, and on other engines in which simplicity, compactness, and stability are of more importance than economy of fuel. It is obvious that if a noncondensing engine is to have the same effective pressure on the piston head as a condensing engine, the pressure maintained within the boiler must be about 15 lb. higher. For this reason noncondensing engines are often called high-pressure engines. Such engines can easily be recognized by the puffs of exhaust steam which they send out into the atmosphere at each stroke of the piston.

254. The eccentric. In practice the valve rod R' is not attached as in the ideal engine indicated in Fig. 188, but motion is communicated to it by a so-called *eccentric*. This consists of a circular disk K (Fig. 189) rigidly attached to the axle, but so set that its center does not coincide with the center of the axle A. The disk K rotates inside the collar C and thus communicates to the eccentric rod R' a back-and-forth motion which operates the valve V in such a way as to admit steam through M and N at the proper time.

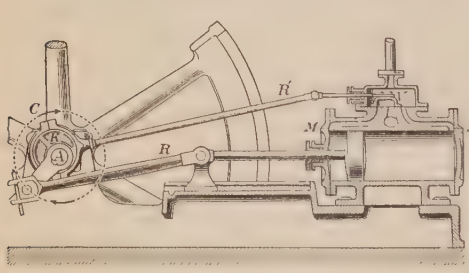


Fig. 189. The eccentric

255. The boiler. When an engine is at work steam is being removed very rapidly from the boiler; e.g. a railway locomotive consumes from 3 to 6 tons of water per hour. It is therefore necessary to have the fire in contact with as large a surface as possible. In the tubular boiler this end is accomplished by causing the flames to pass through

a large number of metal tubes immersed in water. The arrangement of the furnace and the boiler may be seen from the diagram of a locomotive shown in Fig. 190.

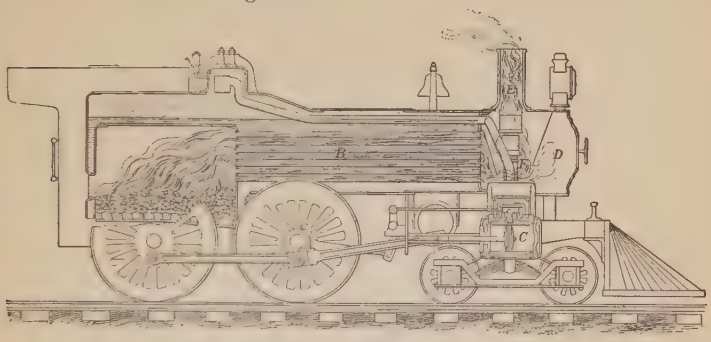


Fig. 190. Diagram of locomotive

256. The draft. In order to suck the flames through the tubes B of the boiler a powerful draft is required. In locomotives this is obtained by running the exhaust steam from the cylinder C (Fig. 190) into the smokestack E through the blower F. The strong current through F draws with it a portion of the air from the smoke box D, thus producing within D a partial vacuum into which a powerful draft rushes from the furnace through the tubes B. The coal consumption of an ordinary locomotive is from one-fourth ton to one ton per hour.

In stationary engines a draft is obtained by making the smokestack very high. Since in this case the pressure which is forcing the air through the furnace is equal to the difference in the weights of columns of air of unit cross section inside and outside the chimney, it is evident that this pressure will be greater the greater the height of the smokestack. This is the reason for the immense heights given to chimneys in large power plants.

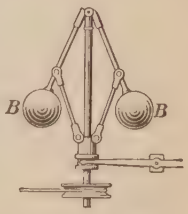


Fig. 191. The governor

257. The governor. Fig. 191 shows an ingenious device of Watt's, called a *governor*, for regulating automatically the speed with which a stationary engine runs. If it runs too fast, the heavy rotating balls B move apart and upward, and in so doing operate a valve which partially shuts off the supply of steam from the cylinder.

258. The reversing and speed-regulating device of a locomotive. In order to control the speed of a locomotive and reverse it when desired, two eccentrics, A, A, (Fig. 192), are provided. These are set exactly opposite on the shaft, so that the two points B and B' are always moving in opposite directions, while the point midway between them remains stationary. In the position shown in the figure the motion of the valve rod T is controlled entirely by the motion of the point B, but when the lever is thrown to the left the point B' moves up and assumes complete control of T. Since the two eccentrics are set oppositely, throwing the lever reverses instantly the direction in which steam acts against the piston head and thus reverses the locomotive. If the lever is set in the middle, all communication between the valve chest and the steam chest is cut off. The speed may be controlled at will by setting the lever at intermediate positions, for in this way the steam passages may be opened as much or as little as desired.

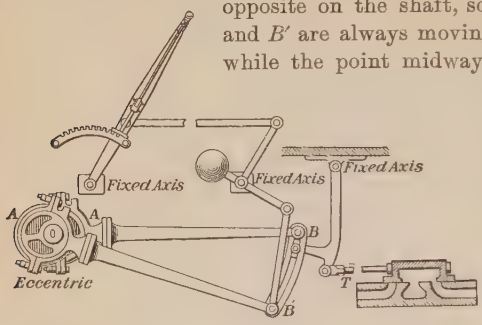


Fig. 192. Reversing mechanism of a locomotive

259. Compound engines. In an engine which has but a single cylinder the full force of the steam has not been spent when the cylinder is opened to the exhaust. The waste of energy which this entails is obviated in the compound engine by allowing the partially spent steam to pass into a second cylinder of larger area than the first. The most efficient of modern engines have three and sometimes four cylinders of this sort, and the engines are accordingly called *triple* or *quadruple expansion engines*. Fig. 193 shows the relation between any two successive cylinders of a compound engine. By automatic devices not differing in principle from the eccentric, valves

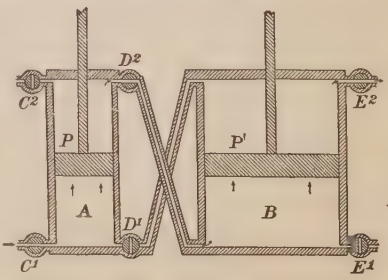


Fig. 193. Compound engine cylinders

C^1, D^2, and E^2 open simultaneously and thus permit steam from the boiler to enter the small cylinder A, while the partially spent steam in the other end of the same cylinder passes through D^2 into B, and the more fully exhausted steam in the upper end of B passes out through E^2. At the upper end of the stroke of the pistons P and P', C^1, D^2, and E^2 automatically close, while C^2, D^1, and E^1 simultaneously open and thus reverse the direction of motion of both pistons. These pistons are attached to the same shaft.

260. Efficiency of a steam engine. We have seen that it is possible to transform completely a given amount of mechanical energy into heat energy. This is done whenever a moving body is brought to rest by means of a frictional resistance. But the inverse operation, namely that of transforming heat energy into mechanical energy, differs in this respect, that it is only a comparatively small fraction of the heat developed by combustion which can be transformed into work. For it is not difficult to see that in every steam engine at least a part of the heat must of necessity pass over with the exhaust steam into the condenser or out into the atmosphere. This loss is so great that even in an ideal engine not more than about 23% of the heat of combustion could be transformed into work. In practice the very best condensing engines of the quadruple expansion type transform into mechanical work not more than 17% of the heat of combustion. Ordinary locomotives utilize at most not more than 8%. *The efficiency of a heat engine is defined as the ratio between the heat utilized, or transformed into work, and the total heat expended.* The efficiency of the best steam engines is therefore about $\frac{17}{23}$ or 75% of that of an ideal heat engine, while that of the ordinary locomotive is only about $\frac{6}{23}$ or 26% of the ideal limit.

261. The principle of the gas engine. Within the last decade gas engines have begun to replace steam engines to a very great extent, especially for small power purposes. These engines are driven by properly timed explosions of a mixture of gas and air occurring within the cylinder.

Fig. 194 is a diagram illustrating the four stages into which it is convenient to divide the complete cycle of operations which goes on within such an engine. Suppose that the heavy fly wheels W have already been set in motion. As the piston p moves to the right in the first stroke (see 1) the valve E opens and an explosive mixture of gas and air is drawn into the cylinder through E. As the piston returns to the left (see 2) valve E closes, and the mixture of gas and air is compressed into a small space in the left end of the cylinder. An electric spark ignites the explosive mixture, and the force of the explosion drives the piston violently to the right (see 3). At the beginning of the return stroke (see 4) the exhaust valve D opens, and as the piston moves to the left the spent gaseous products of the explosion are forced out of the cylinder. The initial condition is thus restored and the cycle begins over again.

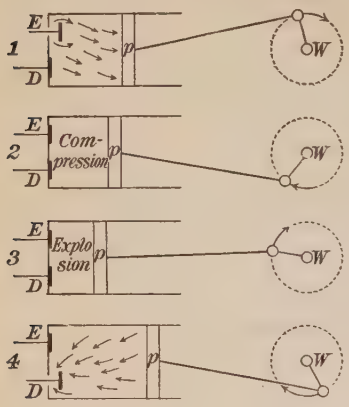


Fig. 194. Principle of the gas engine

Since it is only during the third stroke that the engine is receiving energy from the exploding gas, the fly wheel is always made very heavy so that the energy stored up in it in the third stroke may keep the machine running with little loss of speed during the other three parts of the cycle.

262. Mechanism of the gas engine. The mechanism by which the above operations are carried out in one type of modern gas engine (the Foos) may be seen from a study of Figs. 195 *a* and 195 *b*. 195 *b* is a section of the left end of the engine shown in perspective in 195 *a*. Suppose that the fly wheels W are first set in motion by hand. When the cam or eccentric c_1 (195 *a*) drives the rod R to the left it opens a

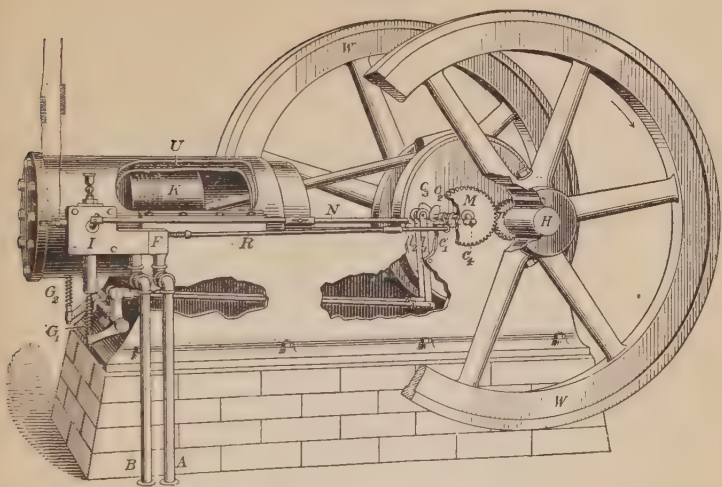


FIG. 195 *a*. The gas engine

valve in F through which gas passes from the inlet pipe A into the mixing chamber I (195 *a* and *b*). Here it mixes with air which entered through the pipe B. As soon as the cam c_2 has moved about to the position in which it throws the lever arm l_1 to the left, the rod G_1 is forced upward and the inlet valve E (195 *b*) is therefore opened. This happens at the beginning of stage No. 1 (§ 261) when the piston K is beginning to move to the right. Hence the explosive mixture is at once drawn into C (195 *b*). At the beginning of stage No. 3 a third eccentric rod N operated by an eccentric c_4 (195 *a*) breaks an electric contact at i (195 *b*), and thus produces a spark which explodes the gas. At the beginning of stage No. 4 the cam c_3 drives the lever arm l_2 (195 *a*) to the left, and thus with the aid of G_2 (195 *a* and *b*) opens the exhaust valve D (195 *b*) and thus permits the spent gases to escape. This completes the cycle.

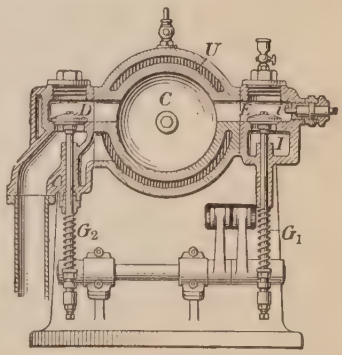


FIG. 195 *b*. Section through end of gas engine

Since each of the four cams, c_1, c_2, c_3, c_4, must open its valve once in two revolutions of the fly wheel, all four of these cams are placed not on the main shaft H, but on the axle of the gear wheel, M, which has twice as many teeth as has the gear wheel n on the main shaft. M therefore revolves but once while the main shaft is revolving twice. In order that the cylinder may be kept cool it is surrounded by a jacket U through which water is kept continually circulating.

The efficiency of the gas engine is often as high as 25%, or nearly double that of the best steam engines. Furthermore, it is free from smoke, is very compact, and may be started at a moment's notice. On the other hand, the fuel, gas or gasoline, is comparatively expensive. Most automobiles are run by gasoline engines, chiefly because the lightness of the engine and of the fuel to be carried are here considerations of great importance.

263. The steam turbine. The steam turbine represents the latest development of the heat engine. In principle it is very much like the

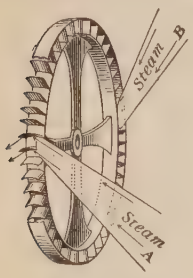


Fig. 196 *a*

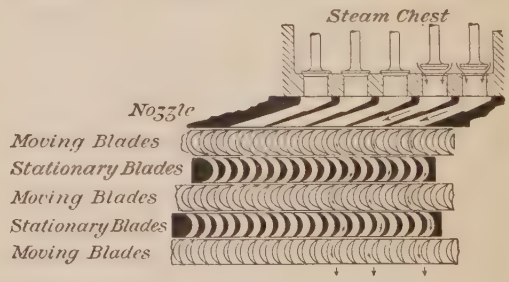


Fig. 196 *b*

Illustrating the principle of the steam turbine

common windmill, the chief difference being that it is steam instead of air which is driven at a high velocity against a series of blades which are arranged radially about the circumference of the wheel which is to be set into rotation. The steam however, unlike the wind, is always directed by nozzles (A and B, Fig. 196 *a*), at the angle of greatest efficiency against the blades. Furthermore, since the energy of the steam is not nearly spent after it has passed through one set of blades, such as that shown in Fig. 196 *a*, it is in practice always passed through a whole series of such sets (Fig. 196 *b*), every alternate row of which is rigidly attached to the rotating shaft, while the intermediate rows are fastened to the immovable outer jacket of the engine, and only serve

as guides to redirect the steam at the most favorable angle against the next row of movable blades. In this way the steam is kept alternately bounding from fixed to movable blades till its energy is expended. The number of rows of blades is often as high as sixteen.

Turbines are at present coming rapidly into use, chiefly for large power purposes. Their advantages over the reciprocating steam engine lie first in the fact that they run with almost no jarring and therefore require much lighter and less expensive foundations, and second in the fact that they occupy less than one tenth the floor space of ordinary engines of the same capacity. Their efficiency is fully as high as that of the best reciprocating engines. The highest speeds attained by vessels at sea, namely about 40 miles per hour, have been made with the aid of steam turbines. The largest vessel which has thus far ever been launched, the thirty-thousand-ton Cunard steamer *Carmania*, which made her maiden trip in December, 1905, is driven by three steam turbines, which have a total of no less than 1,250,000 blades.

QUESTIONS AND PROBLEMS

1. If the average pressure in the cylinder of a steam engine is 10 kilos to the square centimeter and the area of the piston is 300 sq. cm., how much work is done by the piston in a stroke of length 50 cm.? How many calories did the steam lose in this operation?

2. The total efficiency of a certain 700 H.P. locomotive is 6%; 8000 calories of heat are produced by the burning of 1 g. of the best anthracite coal; how many kilos of such coal are consumed per hour by this engine?

3. It requires a force of 300 kilos to drive a given boat at a speed of 15 knots (25 km.). How much coal will be required to run this boat at this speed across a lake 200 km. wide, the efficiency of the engines being 7% and the coal being of a grade to furnish 6000 calories per gram?

4. What total pushing force do the propellors of the *Kaiser Wilhelm der Zweite* exert when she is using her maximum horse power (40,000) and is running at 24 knots (40 km.) per hour?

5. The efficiency of good condensing engines is about 16%. How much coal is consumed per hour by 40,000 H.P. condensing engines like those mentioned in Problem 4, each gram of coal being assumed to produce 4000 calories?

6. The average locomotive has an efficiency of about 6%. What horse power does it develop when it is consuming 1 ton of coal per hour? (See 5.)

7. What pull does a 1000 H.P. locomotive exert when it is running at 25 miles per hour and exerting its full horse power?

CHAPTER X

CHANGE OF STATE

Fusion[1]

264. Heat of fusion. If on a cold day in winter a quantity of snow is brought in from out of doors, where the temperature is below 0° C., and placed over a source of heat, a thermometer plunged into the snow will be found to rise slowly until the temperature reaches 0° C, when it will become stationary and remain so during all the time that the snow is melting, provided only that the contents of the vessel are continuously and vigorously stirred. As soon as the snow is all melted the temperature will begin to rise again.

Since the temperature of ice at 0° C. is the same as the temperature of water at 0° C., it is evident from this experiment that when ice is being changed to water the entrance of heat energy into it does not produce any change in the average kinetic energy of its molecules. This energy must therefore all be expended in pulling apart the molecules of the crystals of which the ice is composed and thus reducing it to a form in which the molecules are held together less intimately, i.e. to the liquid form. In other words, the energy which existed in the flame as the kinetic energy of molecular motion has been transformed, upon passage into the melting solid, into the potential energy of molecules which have been pulled apart against the force of their mutual attraction. *The number of calories of heat energy required to melt one gram of any substance without producing any change in its temperature is called the heat of fusion of that substance.*

[1] This subject should be preceded by a laboratory exercise on the curve of cooling through the point of fusion, and followed by a determination of the heat of fusion of ice. See, for example, Experiments 21 and 22 of the authors' manual.

265. Numerical value of heat of fusion of ice. Since it is found to require about 80 times as long for a given flame to melt a quantity of snow as to raise the melted snow through 1° C., we conclude that it requires about 80 calories of heat to melt 1 g. of snow or ice. This constant is, however, much more accurately determined by the method of mixtures. Thus suppose that a piece of ice weighing, for example, 131 g. is dropped into 500 g. of water at 40° C., and suppose that after the ice is all melted the temperature of the mixture is found to be 15° C. The number of calories which have come out of the water is $500 \times (40 - 15) = 12{,}500$. But $131 \times 15 = 1965$ calories of this heat must have been used in raising the ice from 0° C. to 15° C. after it, by melting, became water at 0°. The remainder of the heat, namely $12{,}500 - 1965 = 10{,}535$, must have been used in melting the 131 g. of ice. Hence the number of calories required to melt 1 g. of ice is $\frac{10535}{131} = 80.4$.

To state the problem algebraically, let $x =$ the heat of fusion of ice. Then we have

$$131\,x + 1965 = 12{,}500\,; \text{ i.e. } x = 80.4.$$

According to the most careful determinations the heat of fusion of ice is 80.0 calories.

266. Heat given out when water freezes. Let snow and salt be added to a beaker of water until the temperature of the liquid mixture is as low as −10° C. or −12° C. Then let a test tube containing a thermometer and a quantity of pure water be thrust into the cold solution. If the thermometer is kept very quiet, the temperature of the water in the test tube will fall four or five, or even ten, degrees below 0° C. without producing solidification. But as soon as the thermometer is stirred, or a small crystal of ice is dropped into the neck of the tube, the ice crystals will form with great suddenness and at the same time the thermometer will rise to 0° C. where it will remain until all the water is frozen.

The experiment shows in a very striking way that the process of freezing is a heat-evolving process. This was to have been

expected from the principle of the conservation of energy; for since it takes 80 calories of heat energy to turn a gram of ice at 0° C. into water at 0° C., this amount of energy must reappear when the water turns back to ice.

267. Utilization of heat evolved in freezing. The heat given off by the freezing of water is often turned to practical account; e.g. tubs of water are sometimes placed in vegetable cellars to prevent the vegetables from freezing. The effectiveness of this procedure is due to the fact that the temperature at which the vegetables freeze is slightly lower than 0° C. As the temperature of the cellar falls the water therefore begins to freeze first, and in so doing evolves enough heat to prevent the temperature of the room from falling as far below 0° C. as it otherwise would.

It is partly because of the heat evolved by the freezing of large bodies of water that the temperature never falls so low in the vicinity of large lakes as it does in inland localities.

268. Latent heat. Before the time of Joule, when heat was supposed to be a weightless fluid, the heat which disappears in a substance when it melts and reappears again when it solidifies was called *latent* or *hidden* heat. Thus water was said to have a latent heat of 80 calories. This expression is still in common use, although, with the change which has taken place in our views of the nature of heat, its appropriateness is entirely gone. For the heat energy which is required to change a substance from a solid to a liquid does not exist within the liquid as concealed or hidden heat energy, but has instead *ceased to exist as heat energy at all,* having been transformed into the potential energy of partially separated molecules, i.e. it represents the work which has been done in effecting the change of state.

269. Melting points of crystalline substances. If a piece of ice is placed in a vessel of boiling water for an instant and then removed and wiped, it will not be found to be in the slightest degree warmer than a piece of ice which has not been exposed to the heat of the warm water. The melting point of ice is,

therefore, a perfectly fixed, definite temperature, above which the ice can never be raised so long as it remains ice, no matter how fast heat is applied to it. All crystalline substances are found to behave exactly like ice in this respect, each substance of this class having its characteristic melting point. The following table gives the melting points of some of the commoner crystalline substances.

Mercury .	– 39.5° C.	Sulphur . .	114° C.	Silver . .	954° C.
Ice . . .	0 "	Tin	233 "	Copper . .	1100 "
Benzine .	7 "	Lead . . .	330 "	Cast iron .	1200 "
Acetic acid	17 "	Zinc . . .	433 "	Platinum .	1775 "
Paraffin . .	54 "	Aluminium .	650 "	Iridium . .	1950 "

We may summarize the experiments upon melting points of crystalline substances in the two following laws.

1. *The temperatures of solidification and of fusion are the same.*

2. *The temperature of the melting or solidifying substance remains constant from the moment at which melting or solidification begins until the process is completed.*

270. Fusion of noncrystalline or amorphous substances. Let the end of a glass rod be held in a Bunsen flame. Instead of changing suddenly from the solid to the liquid state, it will gradually grow softer and softer until, if the flame is sufficiently hot, a drop of molten glass will finally fall from the end of the rod.

If the temperature of the rod had been measured during this process, it would have been found to be continually rising. This behavior, so completely unlike that of crystalline substances, is characteristic of tar, wax, resin, glue, gutta-percha, alcohol, carbon, and in general of all amorphous substances. Such substances cannot be said to have any definite melting points at all, for they pass through all stages of viscosity both in melting and in solidifying. It is in virtue of this property that glass and other similar substances can be heated to softness and then molded or rolled into any desired shapes.

271. Change of volume on solidifying. One has only to reflect that ice floats, or that bottles or crocks of water burst when they freeze, in order to know that water expands upon solidifying. In fact, 1 cu. ft. of water becomes 1.09 cu. ft. of ice, thus expanding more than one twelfth of its initial volume when it freezes. This may seem strange in view of the fact that the molecules are certainly more closely knit together in the solid than in the liquid state; but the strangeness disappears when we reflect that in freezing the molecules of water group themselves into crystals, and that this operation presumably leaves comparatively large free spaces between different crystals, so that, although groups of individual molecules are more closely joined than before, the total volume occupied by the whole assemblage of molecules is greater.

But the great majority of crystalline substances are unlike water in this respect, for, with the exception of antimony and bismuth, they all contract upon solidifying and expand on liquefying. It is only from substances which expand, or which, like cast iron, change in volume very little on solidifying, that sharp castings can be made. For it is clear that contracting substances cannot retain the shape of the mold. It is for this reason that gold and silver coins must be stamped rather than cast. Any metal from which type is to be cast must be one which expands upon solidifying, for it need scarcely be said that perfectly sharp outlines are indispensable to good type. Ordinary type metal is an alloy of lead, antimony, and copper which fulfills these requirements.

272. Effect of the expansion which water undergoes on freezing. If water were not unlike most substances in that it expands on freezing, many, if not all, of the forms of life which now exist on the earth would be impossible. For in winter the ice would sink on ponds and lakes as fast as it froze, and soon our rivers, lakes, and perhaps our oceans also would become solid ice.

The force exerted by the expansion of freezing water is very great. Steel bombs have been burst by filling them with water and exposing them on cold winter nights. One of the chief agents in the disintegration of rocks is the freezing and consequent expansion of water which has percolated into them.

273. Pressure lowers the melting point of substances which expand on solidifying. Since the outside pressure acting on the surface of a body tends to prevent its expansion, we should expect that any increase in the outside pressure would tend to prevent the solidification of substances which expand upon freezing. It ought, therefore, to require a lower temperature to freeze ice under a pressure of two atmospheres than under a pressure of one. Careful experiments have verified this conclusion, and have shown that the melting point of ice is lowered .0075° C. for an increase of one atmosphere in the outside pressure. Although this lowering is so small a quantity, its existence may be shown as follows.

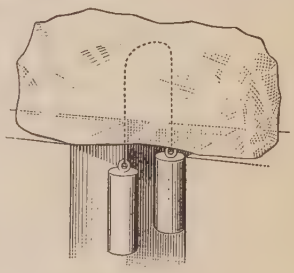


Fig. 197. Regelation

Let two pieces of ice be pressed firmly together beneath the surface of a vessel full of warm water. When taken out they will be found to be frozen firmly together in spite of the fact that they have been immersed in a medium much warmer than the freezing point of water. The explanation is as follows.

At the points of contact the pressure reduces the freezing point of the ice below 0° C., and hence it melts and gives rise to a thin film of water the temperature of which is slightly below 0° C. When this pressure is released the film of water at once freezes, for its temperature is below the freezing point corresponding to ordinary atmospheric pressure. The same phenomenon may be even more strikingly illustrated by the following experiment.

Let two weights of from 5 to 10 kilos be hung by a wire over a block of ice as in Fig. 197. In half an hour or less the wire will be found to

have cut completely through the block, leaving the ice, however, as solid as at first. The explanation is as follows. Just below the wire the ice melts because of the pressure; as the wire sinks through the layer of water thus formed, the pressure on the water is relieved and it immediately freezes again above the wire.

This process of melting under pressure and freezing again as soon as the pressure is relieved is known as *regelation.*

274. Pressure raises the freezing point of substances which contract on solidifying. Substances like paraffin, zinc, and lead which contract on solidifying have their melting points raised by an increase in pressure, for in this case the outside pressure tends to assist the molecular forces which are pulling the body out of the larger liquid form into the smaller solid form; hence this result can be accomplished at a higher temperature with pressure than without it.

We may therefore summarize the effects of pressure on the melting points of crystalline substances in the following law.

Substances which expand on solidifying have their melting points lowered by pressure, and those which contract on solidifying have their melting points raised by pressure.

QUESTIONS AND PROBLEMS

1. Which is the more effective as a cooling agent, 100 lb. of ice at 0° C. or 100 lb. of water at the same temperature? Why?

2. Equal weights of hot water and ice are mixed and the result is water at 0° C. What was the temperature of the hot water?

3. How many grams of ice must be put into 200 g. of water at 40° C. to lower the temperature to 10° C.?

4. From what height must a gram of ice at 0° C. fall in order to melt itself by the heat generated in the impact?

5. If water were like gold in contracting on solidification, what would happen to lakes and rivers during a cold winter?

6. Why will a snowball "pack" if the snow is melting, but not if it is much below 0° C.?

7. What temperature will result from mixing 10 g. of ice at 0° C. with 200 g. of water at 25° C.?

Vaporization[1]

275. Heat of vaporization defined. The experiments performed in Chapter V, on molecular motions, led us to the conclusion that, at the free surface of any liquid, molecules frequently acquire velocities sufficiently high to enable them to lift themselves beyond the range of attraction of the molecules of the liquid, and to pass off as free gaseous molecules into the space above. They taught us further that since it is only such molecules as have unusually high velocities which are able thus to escape, the *average kinetic energy* of the molecules left behind is continually diminished by this loss from the liquid of the most rapidly moving molecules, and consequently the temperature of an evaporating liquid constantly falls until the rate at which it is losing heat is equal to the rate at which it receives heat from outside sources. Evaporation, therefore, always takes place at the expense of the heat energy of the liquid. *The number of calories of heat which disappear in the formation of one gram of vapor is called the heat of vaporization of the liquid.* Since it requires about five times as long to boil away a vessel of water as to raise it from 0° to 100° C., we conclude that the heat of vaporization of water at 100° is in the neighborhood of 500 calories.

276. Heat due to condensation. When molecules pass off from the surface of a liquid they rise against the downward forces exerted upon them by the liquid, and, in so doing, exchange a part of their kinetic energy for the potential energy of separated molecules in precisely the same way in which a ball thrown upward from the earth exchanges its kinetic energy in rising for the potential energy which is represented by the

[1] It is recommended that this subject be accompanied by a laboratory determination of the boiling point of alcohol by the direct method, and by the vapor-pressure method, and that it be followed by an experiment upon the fixed points of a thermometer and the change of boiling point with pressure. See, for example, Experiments 23 and 24 of the authors' manual.

separation of the ball from the earth. Similarly, just as when the ball falls back, it regains in the descent all of the kinetic energy lost in the ascent, so when the molecules of the vapor reënter the liquid they must regain all of the kinetic energy which they lost when they passed out of the liquid. We may expect, therefore, that *every gram of steam which condenses will generate in this process the same number of calories which was required to vaporize it.*

277. Measurement of heat of vaporization. To find accurately the number of calories expended in the vaporization, or released in the condensation, of a gram of water at 100° C., we pass steam rapidly for two or three minutes from an arrangement like that shown in Fig. 198 into a vessel containing say, 500 grams of water. We observe the initial and final temperatures and the initial and final weights of the water. If, for example, the gain in weight of the water is 16.5 g., we know that 16.5 g. of steam have been condensed. If the rise in temperature of the water is from 10° C. to 30° C., we know that $500 \times (30 - 10) = 10{,}000$ calories of heat have entered the water. If x represents the number of calories given up by one gram of steam in condensing, then the total heat imparted to the water by the condensation of the steam is $16.5x$ calories. This condensed steam is at first water at 100° C., which is then cooled to 30° C. In this cooling process it gives up $16.5 \times (100 - 30) = 1155$ calories. Therefore, equating the heat gained by the water to the heat lost by the steam, we have

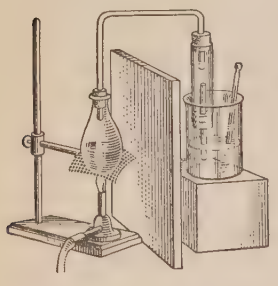


Fig. 198. Heat of vaporization of water

$$10{,}000 = 16.5x + 1155, \text{ or } x = 536.1.$$

This is the method usually employed for finding the heat of vaporization. The now accepted value of this constant is 536.

278. Boiling temperature defined. If a liquid is heated by means of a flame, it will be found that there is a certain temperature above which it cannot be raised, no matter how rapidly the heat is applied. This is the temperature which exists when

bubbles of vapor form at the bottom of the vessel and rise to the surface, growing in size as they rise. This temperature is commonly called the *boiling temperature.*

But a second and more exact definition of the boiling point may be given. It is clear that a bubble of vapor can exist within the liquid only when the pressure exerted by the vapor within the bubble is at least equal to the atmospheric pressure pushing down on the surface of the liquid. For if the pressure within the bubble were less than the outside pressure, the bubble would immediately collapse. But the pressure exerted by a vapor in a closed space in contact with its liquid is what was called in Chapter V the pressure of the saturated vapor. This pressure was found to increase rapidly as the temperature was raised. Since now the boiling point is the temperature at which bubbles of vapor first begin to exist within the body of the liquid, it is clear that it is also the temperature at which the pressure of the saturated vapor first becomes equal to the pressure existing outside. The boiling point of a liquid is therefore defined as *that temperature at which the pressure of the saturated vapor of the liquid is equal to the pressure acting upon the surface of the liquid.* On account of the fact that the temperature of the liquid itself is under some circumstances slightly different from the temperature of the vapor which rises from it, in measuring the boiling points of liquids thermometers should always be placed in the vapor rising from the liquid rather than in the liquid itself (Fig. 132, p. 131).

279. Variation of the boiling point with pressure. Since the pressure of a saturated vapor varies rapidly with the temperature, and since the boiling point has been defined as the temperature at which the pressure of the saturated vapor is equal to the outside pressure, it follows that the boiling point must vary as the outside pressure varies.

Thus let a round-bottomed flask be half filled with water and boiled. After the boiling has continued for a few minutes, so that the steam

has driven out most of the air from the flask, let a rubber stopper be inserted, and the flask removed from the flame and inverted as shown in Fig. 199. The temperature will fall rapidly below the boiling point. But if cold water is poured over the flask, the water will again begin to boil vigorously, for the cold water, by condensing the steam, lowers the pressure within the flask, and thus enables the water to boil at a temperature lower than 100° C. The boiling will cease, however, as soon as enough vapor is formed to restore the pressure. The operation may be repeated many times without reheating.

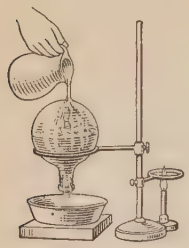


FIG. 199. Lowering the boiling point by diminishing the pressure

At the city of Quito, Ecuador, water boils at 90° C., and on the top of Mt. Blanc it boils at 84° C. On the other hand, in the boiler of a steam engine in which the pressure is 100 lb. to the square inch, the boiling point of the water is 155° C.

280. Evaporation and boiling. The only essential difference between evaporation and boiling is that the former consists in the passage of molecules into the vaporous condition *from the free surface only*, while the latter consists in the passage of the molecules into the vaporous condition both at the free surface and at the surface of bubbles which exist within the body of the liquid. The only reason that vaporization takes place so much more rapidly at the boiling temperature than just below it is that the evaporating *surface* is enormously increased as soon as the bubbles form. The reason the temperature cannot be raised above the boiling point is that the surface always increases, on account of the bubbles, to just such an extent that the loss of heat because of evaporation is always exactly equal to the heat received from the fire.

QUESTIONS AND PROBLEMS

1. The hot water which leaves a steam radiator may be as hot as the steam which entered it. How then has the room been warmed?

2. How much heat is given up by 20 g. of steam in condensing?

3. How many calories are given up by 20 g. of steam at 100° C. in condensing and then cooling to 20° C.? How much water will this steam raise from 10° C. to 20° C.?

4. A vessel contains 200 g. of water at 0° C. and 130 g. of ice. If 25 g. of steam are condensed in it, what will be the resulting temperature?

5. Why do fine bubbles rise in a vessel of water which is being heated long before the boiling point is reached? How can you distinguish between this phenomenon and boiling?

6. Why does steam produce so much more severe burns than hot water of the same temperature?

7. When water is boiled in a deep vessel it will be noticed that the bubbles rapidly increase in size as they approach the surface. Give two reasons for this.

8. Why in winter does not all the snow melt at once as soon as the temperature of the air rises above 0° C.?

9. In the fall we expect frost on clear nights when the dew-point is low, but not on cloudy nights when the dew-point is high. Can you see any reason why a large deposit of dew will prevent the temperature of the air from falling very low?

10. Water is contained in a closed vessel and is heated slowly. Will it ever boil?

11. When a teakettle is heated rapidly the lower layers of water become considerably hotter than the upper layers. In view of this fact, explain why a teakettle sings before it begins to boil.

12. How will the boiling point of water be affected if it is taken down into a deep mine?

13. A fall of 1° C. in the boiling point is caused by rising about 960 ft. above the earth's surface. How hot is boiling water at Denver, 5000 ft. above sea level?

14. At the top of a mountain water boils 2.5° C. lower than it does at the base. What is the height of the mountain?

Artificial Cooling

281. Cooling by solution. Let a handful of common salt be placed in a small beaker of water at the temperature of the room and stirred with a thermometer. The temperature will fall several degrees. If equal weights of ammonium nitrate and water at 15° C. are mixed, the temperature will fall as low as −10° C. If the water is nearly at 0° C. when the ammonium nitrate is added, and if the stirring is done with a test tube partly filled with ice-cold water, the water in the tube will be frozen.

These experiments show that the breaking up of the crystals of a solid requires an expenditure of heat energy, as well when this operation is effected by solution as by fusion. The reason for this will appear at once if we consider the analogy between solution and evaporation. For just as the molecules of a liquid tend to escape from its surface into the air, so the molecules at the surface of the salt are tending, because of their velocities, to pass off, and are only held back by the attractions of the other molecules in the crystal to which they belong. If, however, the salt is placed in water, the attraction of the water molecules for the salt molecules aids the natural velocities of the latter to carry them beyond the attraction of their fellows. As the molecules escape from the salt crystals two forces are acting on them, the attraction of the water molecules tending to increase their velocities, and the attraction of the remaining salt molecules tending to diminish these velocities. If the latter force has a greater resultant effect than the former, the mean velocity of the molecules after they have escaped will be diminished, and the solution will be cooled. But if the attraction of the water molecules amounts to more than the backward pull of the dissolving molecules, as when caustic potash or sulphuric acid is dissolved, the mean molecular velocity is increased and the solution is heated.

When a liquid will not dissolve a solid we infer that the pull of the liquid molecules added to the natural velocities of the molecules of the solid is not sufficient to detach the latter from their fellows. It sometimes happens, however, that a liquid which is unable to dissolve a solid at a low temperature will dissolve it at a higher temperature. Explain.

282. Freezing points of solutions. If a solution of one part of common salt to ten of water is placed in a test tube and immersed in a "freezing mixture" of water, ice, and salt, the temperature indicated by a thermometer in the tube will not be zero when ice begins to form, but several degrees below zero.

The ice which does form, however, will be found to be free from salt, and it is this fact which furnishes a key to the explanation of why the freezing point of the salt solution is lower than that of pure water. For cooling a substance to its freezing point simply means reducing its temperature, and therefore the mean velocity of its molecules, sufficiently to enable the cohesive forces of the liquid to pull the molecules together into the crystalline form. Since in the freezing of a salt solution the cohesive forces of the water are obliged to overcome the attractions of the salt molecules as well as the molecular motions, the motions must be rendered less, i.e. the temperature must be made lower, than in the case of pure water in order that crystallization may occur. We should expect from this reasoning that the larger the amount of salt in solution the lower would be the freezing point. This is indeed the case. The lowest freezing point obtainable with common salt in water is $-22°$ C. This is the freezing point of a saturated solution.

283. Freezing mixtures. If snow or ice is placed in a vessel of water, the water melts it, and in so doing its temperature is reduced to the freezing point of pure water. Similarly, if ice is placed in salt water it melts and reduces the temperature of the salt water to the freezing point of the solution. This may be one, or two, or twenty-two degrees below zero, according to the concentration of the solution. Whether then we put the ice in pure water or in salt water, enough of it always melts to reduce the whole mass to the freezing point of the liquid, and each gram of ice which melts uses up 80 calories of heat. The efficiency of a mixture of salt and ice in producing cold is therefore due simply to the fact that the freezing point of a salt solution is lower than that of pure water.

The best proportions are three parts of snow or finely shaved ice to one part of common salt. If three parts of calcium chloride are mixed with two parts of snow, a temperature of $-55°$ C. may be produced. This is sufficient to freeze mercury.

284. Intense cold by evaporation. If, instead of utilizing as above the heats of fusion, we utilize the larger heats of vaporization, still lower temperatures may be produced.

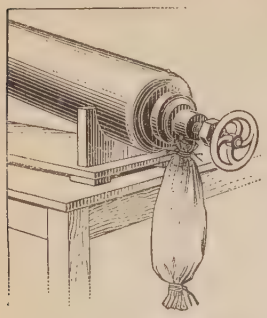


Fig. 200. Cold by rapid evaporation of carbon dioxide

Thus if a cylinder of liquid carbon dioxide is placed as in Fig. 200 and the stopcock opened, such intense cold is produced by the rapid evaporation of the liquid which rushes out into the bag that the liquid freezes to a snowy solid. The solid itself evaporates so rapidly that it maintains, as long as it lasts, a temperature of $-80°$ C. If a little of this solid is placed in a beaker containing ether, and the mixture is stirred with a test tube filled with mercury, the mercury will be frozen solid. The chief function of the ether is to insure intimate contact between the cold solid and the test tube.

Industrial Applications of Change of State

285. Distillation. In general when solids are dissolved in liquids the vapor which rises from the solution, like the ice which freezes out of it, contains none of the dissolved substance. In order to obtain pure water, therefore, from water containing solid matter in solution, it is only necessary to cause the solution to evaporate and to condense the vapor. This is done ordinarily by means of an arrangement essentially like that shown in Fig. 201.

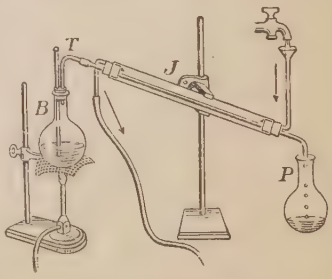


Fig. 201. Distillation

The solution is boiled in B and the pure vapor of the liquid passes into the tube T, where it is condensed by the cold water which is kept in continual circulation through the jacket J. The condensed liquid is

collected in a receiver P. Sometimes it is the pure liquid in P which is desired, and sometimes the solid which remains in B. In the manufacture of white sugar it is necessary that the evaporation take place at a low temperature, so that the sugar may not be scorched. Hence the boiler is kept partially exhausted by means of an air pump, thus enabling the solution to boil at considerably reduced temperatures.

286. Fractional distillation. When both of the constituents of a solution are volatile, as in the case of a mixture of alcohol and water, the vapor of both will rise from the liquid. But the one which has the lower boiling point, i.e. the higher vapor pressure, will predominate. Hence if we have in B, Fig. 201, a solution consisting of 50% alcohol and 50% water, it is clear that we can obtain in P, by evaporating and condensing, a solution containing a much larger percentage of alcohol. By repeating this operation a number of times we can increase the purity of the alcohol. This process is called *fractional distillation.* The boiling point of the mixture lies between the boiling points of alcohol and water, being higher the greater the percentage of water in the solution.

287. Critical temperatures. We saw in Chapter V that when a vapor is in a closed vessel in contact with its liquid there is a limit to its possible density, namely the density corresponding to saturation at the given temperature. We found also that if we either diminished the volume or lowered the temperature of such a vapor it began to condense. Now if a vapor is not in contact with its liquid, there are in general two ways by which we may proceed to condense it. First, we may compress it, i.e. reduce its volume, until it reaches a density corresponding to saturation; or second, we may cool it down to the dew-point, i.e. to the temperature at which its existing density is sufficient to produce saturation. It is obvious that if we both cool and compress the vapor, neither cooling nor compression need be so excessive as though only one process had been performed. Experiment shows, however, that there is a

temperature characteristic of every substance above which pressure alone, no matter how great, cannot produce condensation. This temperature is called the *critical temperature.* The pressure necessary to produce condensation at the critical temperature is called the *critical pressure.* The following table gives the critical temperatures, the critical pressures, and the boiling points at atmospheric pressure of some substances.

Substance	Critical Temperature	Critical Pressure in Atmospheres	Boiling Point at Atmospheric Pressure
Hydrogen	− 240° C.	14	− 252° C.
Air	− 140° C.	39	− 182° C.
Carbon dioxide	31° C.	73	− 80° C.
Ammonia	130° C.	115	− 33° C.
Ether	190° C.	37	38.5° C.
Alcohol	243° C.	63	78° C.
Water	365° C.	200	100° C.

The table shows that such substances as water, alcohol, ether, ammonia, and carbon dioxide can be liquefied at ordinary temperatures by pressure alone, since their critical temperatures are above the ordinary temperature of the atmosphere. But air, for example, cannot possibly be liquefied until its temperature is reduced below −140° C. If it is to be a liquid at this temperature, it must be under a pressure of at least 39 atmospheres. If it is to be a liquid at a pressure of 1 atmosphere, its temperature must of course be lower still, namely, −182° C. (see last column of the table). This is the temperature which liquid air assumes in an open vessel.

288. The liquid-air machine. In the actual manufacture of liquid air a pressure pump P (Fig. 202) forces the air into a spiral coil C under a pressure of about 200 atmospheres. The heat produced by this compression is carried off by running water which circulates through the tank R. The cock c is then opened and the air expands from 200

atmospheres down to 1 atmosphere. In this expansion the temperature falls. This cooled air returns through a larger spiral S which incloses the high-pressure spiral s and thus cools off the air which is coming down to the expansion valve through the inner spiral. In this process the temperature of the air issuing from the valve c continuously falls until it reaches the temperature of liquefaction. Liquid air can then be drawn off through the stopcock R. The air which escapes liquefaction returns to the compressor, where it is again forced into the inner spiral s, together with a certain amount of air which enters from outside at o.

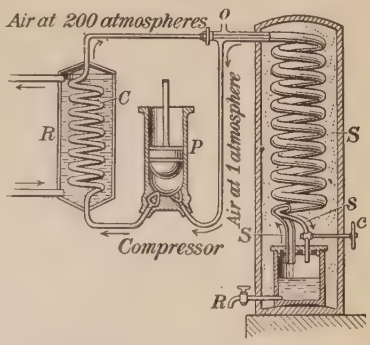


Fig. 202. The liquefaction of air

289. Manufactured ice. In the great majority of modern ice plants the low temperature required for the manufacture of the ice is produced

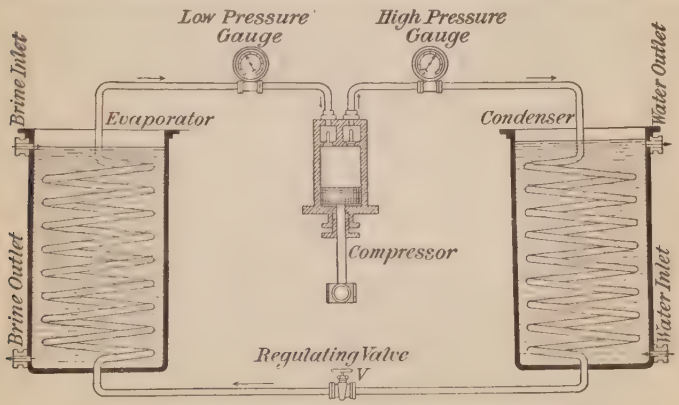


Fig. 203. Compression system of ice manufacture

by the rapid evaporation of liquid ammonia. At ordinary temperatures ammonia is a gas, but since its critical temperature is 130° C., it may be liquefied by pressure alone. At 80° F. a pressure of 155 lb. per square inch, or about 10 atmospheres, is required to produce its liquefaction. Fig. 203 shows the essential parts of an ice plant. The

compressor, which is usually run by a steam engine, forces the gaseous ammonia under a pressure of 155 lb. into the condenser coils shown on the right, and there liquefies it. The heat of condensation of the ammonia is carried off by the running water which constantly circulates about the condenser coils. From the condenser the liquid ammonia is allowed to pass very slowly through the regulating valve V into the coils of the evaporator, from which the evaporated ammonia is pumped out so rapidly that the pressure within the coils does not rise above 34 lb. It will be noted from the figure that the same pump which is there labeled the compressor exhausts the ammonia from the evaporating coils and compresses it in the condensing coils; for, just as in Fig. 202, the valves are so arranged that the pump acts as an exhaust pump on one side and as a compression pump on the other. The rapid evaporation of the liquid ammonia under the reduced pressure existing within the evaporator cools these coils to a temperature of about 5° F. The brine with which these coils are surrounded has its temperature thus reduced to about 16° or 18° F. This brine is made to circulate about the cans containing the water to be frozen.

290. Cold storage. The artificial cooling of factories and cold-storage rooms is accomplished in a manner exactly similar to that employed in the manufacture of ice. The brine is cooled exactly as described above, and is then pumped through coils placed in the rooms to be cooled. Fig. 204 is a sketch of such a refrigerating plant in a packing house. The ammonia is liquified in the *condenser* and evaporated in the coils of the *brine tank*.

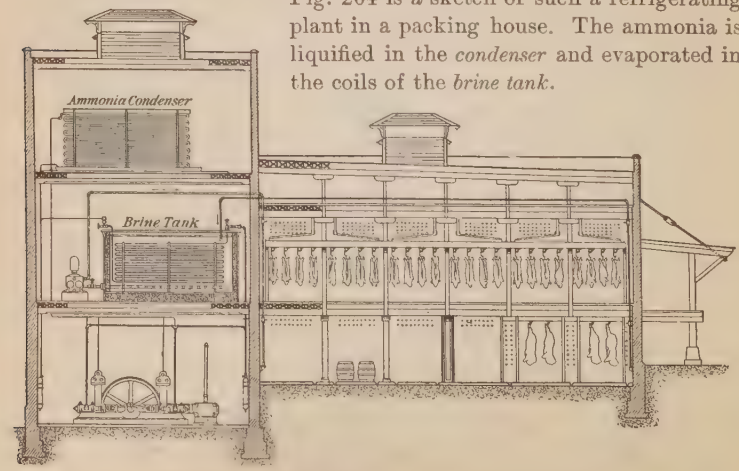


Fig. 204. Packing house with the brine system of refrigeration

QUESTIONS AND PROBLEMS

1. How may we obtain pure drinking water from sea water?

2. Explain why salt is sometimes thrown on icy sidewalks on cold winter days.

3. When salt water freezes the ice formed is practically free from salt. What effect, then, does freezing have on the concentration of a salt solution?

4. A partially concentrated salt solution which has a freezing point of $-5°$ C. is placed in a room which is kept at $-10°$ C. Will it all freeze?

5. A French physicist, Amagat, subjected air to a pressure of 3000 atmospheres. Under these conditions it became as dense as water, but showed no signs of liquefaction. Why not?

6. If liquid air is placed in an open vessel its temperature will not rise above $-182°$ C. Why not? Suggest a way in which its temperature could be made to rise above $-182°$ C., and a way in which it could be made to fall below that temperature.

7. If there were no water on the earth, would the difference in temperature between winter and summer be greater or less than it is now? Why?

8. Why is not the boiling point of water in the boiler of a steam engine 100° C.?

9. Why does the distillation of a mixture of alcohol and water always result to some extent in a mixture of alcohol and water?

10. Give two reasons why the ocean freezes less easily than the lakes.

CHAPTER XI

THE TRANSFERENCE OF HEAT

Conduction

291. Conduction in solids. If one end of a short metal bar be held in the fire the other end soon becomes too hot to hold. But if the metal rod is replaced by one of wood or glass, the end away from the flame will not be appreciably heated.

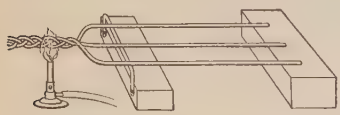


Fig. 205. Differences in heat conductivities of metals

This experiment and others like it show that nonmetallic substances possess a much smaller ability to conduct heat than do metallic substances. But although all metals are good conductors as compared with nonmetals, they differ widely among themselves in their conducting powers.

Let copper, iron, and German silver wires 50 cm. long and about 3 mm. in diameter be twisted together at one end as in Fig. 205, and let a Bunsen flame be applied to the twisted ends. After the heating has continued for three or four minutes, let a match be slid slowly from the cool end of each wire toward the hot end, until the heat from the wire ignites it. The copper will be found to have carried the heat farther from the source than the iron, and the iron farther than the German silver.

In the following table some common substances are arranged in the order of their heat conductivities. The measurements have been made by a method not differing in principle from that just described. For the sake of comparison silver is taken as 100.

Silver . .	100	Tin	15	Mercury . . .	1.35
Copper . .	74	Iron	12	Ice	.21
Gold . .	53	Lead	8.5	Glass	.046
Brass . .	27	German silver . .	6.3	Hard rubber . .	.024

292. Conduction in liquids and gases. Let a small piece of ice be held by means of a glass rod in the bottom of a test tube full of ice water. Let the upper part of the tube be heated with a Bunsen burner as in Fig. 206. The upper part of the water may be boiled for some time without melting the ice. Water is evidently then a very poor conductor of heat. The same thing may be shown more strikingly as follows. The bulb of an air thermometer is placed only a few millimeters beneath the surface of water contained in a large funnel arranged as in Fig. 207. If now a spoonful of ether is poured on the water and set on fire, the index of the air thermometer will show scarcely any change, in spite of the fact that the air thermometer is a very sensitive indicator of changes in temperature.

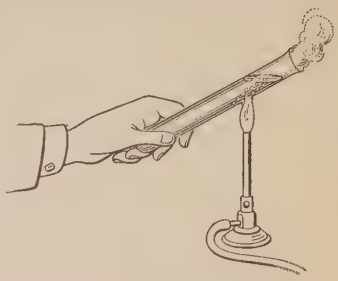


Fig. 206. Water a nonconductor

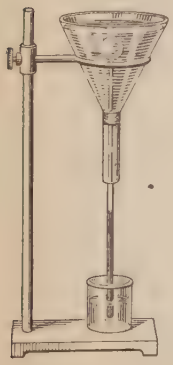


Fig. 207. Burning ether on the water does not affect the air thermometer

Careful measurements of the conductivity of water show that it is only about $\frac{1}{1200}$ of that of silver. The conductivity of gases is even smaller, not amounting on the average to more than $\frac{1}{25}$ that of water.

293. The nature of heat conduction. Since heat is regarded as the kinetic energy of vibration of the molecules of a substance, the conduction of heat must consist simply in the transfer of motion from molecule to molecule. Good conductors then are simply substances in which molecular energy is readily transferred from molecule to molecule.

294. Conductivity and sensation. It is a fact of common observation that on a cold day in winter a piece of metal feels much colder to the hand than a piece of wood, notwithstanding the fact that the temperature of the wood must be the same as that of the metal. On the other hand, if the same two bodies had been lying in the hot sun in midsummer, the wood might be handled without discomfort, but the metal would be uncomfortably hot. The explanation of these phenomena is found in the fact that the iron, being a much better conductor than the wood, removes heat from the hand much more rapidly in winter, and imparts heat to the hand much more rapidly in summer, than does the wood. In general, the better a conductor the hotter it will feel to a hand colder than itself, and the colder to a hand hotter than itself. Thus in a cold room oilcloth, a fairly good conductor, feels much colder to the touch than a carpet, a comparatively poor conductor. For the same reason linen clothing feels cooler to the touch in winter than woolen goods.

295. The rôle of air in nonconductors. Feathers, fur, felt, etc., make very warm coverings, because they are very poor conductors of heat and thus prevent the escape of heat from the body. Their poor conductivity is due in large measure to the fact that they are full of minute spaces containing air, and gases are the best nonconductors of heat. It is for this reason that freshly fallen snow is such an efficient protection to vegetation. Farmers always fear for their fruit trees and vines when there is a severe cold snap in winter, unless there is a coating of snow on the ground to prevent a deep freezing.

296. The Davy safety lamp. Let a piece of wire gauze be held above an open gas jet, and a match applied above the gauze. The flame will be found to burn above the gauze as in Fig. 208, (1); but it will not pass through to the lower side. If it is ignited below the gauze, the flame will not pass through to the upper side but will burn as shown in Fig. 208, (2).

The explanation is found in the fact that the gauze conducts the heat away from the flame so rapidly that the gas on the other side is not raised to the temperature of ignition. Safety lamps used by miners are completely incased in gauze, so that if the mine is full of inflammable gases, they are not ignited by the lamp outside of the gauze.

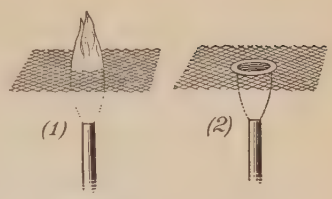


Fig. 208. A flame will not pass through wire gauze

QUESTIONS AND PROBLEMS

1. Why do firemen wear flannel shirts in summer to keep cool and in winter to keep warm?

2. If a piece of paper is wrapped tightly around a metal rod and held for an instant in a Bunsen flame, it will not be scorched. If held in a flame when wrapped around a wooden rod it will be scorched at once. Explain.

3. If one touches the pan containing a loaf of bread in a hot oven, he receives a much more severe burn than if he touches the bread itself, although the two are at the same temperature. Explain.

4. Why will a moistened finger or the tongue freeze instantly to a piece of iron on a cold winter's day, but not to a piece of wood?

5. Why are plants often covered with paper on a night when frost is expected?

6. Does clothing ever afford us heat in winter? How, then, does it keep us warm?

Convection

297. Convection in liquids. Although the conducting power of liquids is so small, as was shown in the experiment of § 292, they are yet able, under certain circumstances, to transmit heat much more effectively than solids. Thus if the ice in the experiment of Fig. 206 had been placed at the top and the flame at the bottom, the ice would have been melted very quickly. This shows that heat is transferred with enormously greater readiness from the bottom of the tube toward the top than

from the top toward the bottom. The mechanism of this heat transference will be evident from the following experiment.

Let a round-bottomed flask be half filled with water and a few crystals of magenta dropped into it. Then let the bottom of the flask be heated with a Bunsen burner. The magenta will reveal the fact that the heat sets up currents the direction of which is upward in the region immediately above the flame but downward at the sides of the vessel. It will not be long before the whole of the water is uniformly colored. This shows how thorough is the mixing accomplished by the heating.

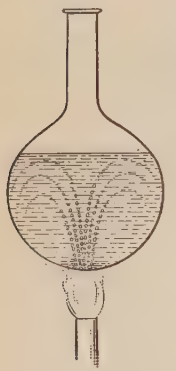


Fig. 209. Convection currents

The explanation of the phenomenon is as follows. The water nearest the flame became heated and expanded. It was thus rendered less dense than the surrounding water, and hence rose to the top, while the colder and therefore denser water from the sides came in and took its place.

It is obvious that this method of heat transfer is applicable only to fluids, and to them only when heat is applied to some point at which the expanded liquid has an opportunity to *rise*. The essential difference between it and conduction is that the heat is not transferred from molecule to molecule throughout the whole mass, but is rather transferred by the bodily movement of comparatively large masses of the heated liquid from one point to another. This method of heat transference is known as *convection*.

298. Winds and ocean currents. Winds are convection currents in the atmosphere caused by unequal heating of the earth by the sun. The air over a heated area expands and rises, while the air from the cooler surrounding regions rushes in to take its place.

The principles of convection easily explain the land and sea breezes so familiar to all dwellers near the coasts of large

bodies of water. During the daytime the land is heated more rapidly than the sea, because the specific heat of water is much greater than that of earth. Hence the hot air rises over the land and cold air from the sea rushes in to take its place. This constitutes the sea breeze which blows during the daytime, usually reaching its maximum strength in the late afternoon. At night the earth cools more rapidly than the sea and hence the direction of the wind is reversed. This constitutes the land breeze which blows during the night, reaching its maximum toward the early morning. These breezes are more pronounced in the tropics than they are in temperate climates, because the change of temperature between day and night is greatest in the tropics. Furthermore, the sea breeze is usually more pronounced than the land breeze, because the temperature of the land rises higher above that of the water in the daytime than it falls below it at night. The effect of these breezes is seldom felt more than twenty-five miles from shore.

Ocean currents are caused partly by the unequal heating of the sea and partly by the direction of the prevailing winds. In general both winds and currents are so modified by the configuration of the continents that it is only over broad expanses of the ocean that the direction of either can be predicted from simple considerations.

Radiation

299. A third method of heat transference. There are certain phenomena in connection with the transfer of heat for which conduction and convection are wholly unable to account. For example, if one sits in front of a hot grate fire, the heat which he feels cannot come from the fire by convection, because the currents of air are moving toward the fire rather than away from it. It cannot be due to conduction, because the conductivity of air is extremely small and the colder currents of air moving toward the fire would more than neutralize any

transfer outward due to conduction. There must therefore be some way in which heat travels across the intervening space other than by conduction or convection.

It is still more evident that there must be a third method of heat transfer when we consider the heat which comes to us from the sun. Conduction and convection take place only through the agency of matter; but we know that the space between the earth and the sun is not filled with ordinary matter, or else the earth would be retarded in its motion through space. *Radiation* is the name given to this third method by which heat travels from one place to another, and which is illustrated in the passing of heat from a grate fire to a body in front of it, or from the sun to the earth.

300. The nature of radiation. The nature of radiation will be discussed more fully in Chapter XXII. It will be sufficient here to call attention to the following differences between conduction, convection, and radiation.

First, while conduction and convection are comparatively slow processes, the transfer of heat by radiation takes place with the enormous speed with which light travels, namely 186,000 miles per second. That the two speeds are the same is evident from the fact that at the time of an eclipse of the sun the shutting off of heat from the earth is observed to take place at the same time as the shutting off of light.

Second, radiant heat travels in straight lines, while conducted or convected heat may follow the most circuitous routes. The proof of this statement is found in the familiar fact that radiation may be cut off by means of a screen placed directly between a source and the body to be protected.

Third, radiant heat may pass through a medium without heating it. This is shown by the fact that the upper regions of the atmosphere are very cold, even in the hottest days in summer, or that a hothouse may be much warmer than the glass through which the sun's rays enter it.

The Heating and Ventilating of Buildings

301. The principle of ventilation. The heating and ventilating of buildings are accomplished chiefly through the agency of convection.

To illustrate the principle of ventilation, let a candle be lighted and placed in a vessel containing a layer of water (Fig. 210). When a lamp chimney is placed over the candle so that the bottom of the chimney is under the water, the flame will slowly die down and will finally be extinguished. This is because the oxygen, which is essential to combustion, is gradually used up and no fresh supply is possible with the arrangement described. If the chimney is raised even a very little above the water, the dying flame will at once brighten. Why? If a metal or cardboard partition is inserted in the chimney, as in Fig. 210, the flame will burn continuously, even when the bottom of the chimney is under water. The reason will be clear if a piece of burning touch paper (blotting paper soaked in a solution of potassium nitrate and dried) is held over the chimney. The smoke will show the direction of the air currents. If the chimney is a large one, in order that the first part of the above experiment may succeed, it may be necessary to use two candles; for too small a heated area permits the formation of downward currents at the sides.

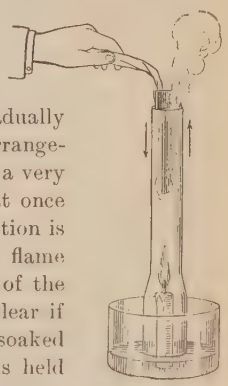


Fig. 210
Convection currents in air

302. Ventilation of houses. In order to secure satisfactory ventilation it is estimated that a room should be supplied with 2000 cu. ft. of fresh air per hour for each occupant (a gas burner is equivalent in oxygen consumption to four persons). A current of air moving with a speed great enough to be just perceptible has a velocity of about 3 ft. per second. Hence the area of opening required for each person when fresh air is entering at this speed is about 25 or 30 sq. in. The manner of supplying this requisite amount of fresh air in dwelling houses depends upon the method of heating employed.

If a house is heated by stoves or fireplaces, no special provision for ventilation is needed. The foul air is drawn up the chimney with the smoke, and the fresh air which replaces it finds entrance through cracks about the doors and windows and through the walls.

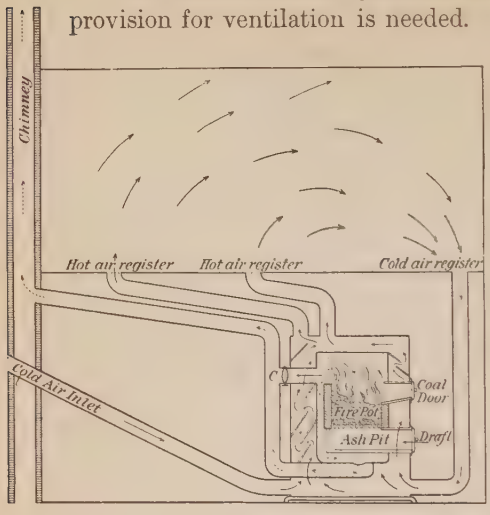


Fig. 211. Hot-air heating

303. Hot-air heating. In houses heated by hot-air furnaces an air duct ought always to be supplied for the entrance of fresh cold air, in the manner shown in Fig. 211 (see "cold-air inlet"). This cold air from out of doors is heated by passing in a circuitous way, as shown by the arrows, over the outer jacket of iron which covers the fire box. It is then delivered to the rooms. Here a part of it escapes through windows and doors and the rest returns through the cold-air register to be reheated, after being mixed with a fresh supply from out of doors.

The course of the air which feeds the fire is shown by the dotted arrows. When the fire is first started, in order to gain a strong draft the damper C is opened so that the smoke may pass directly up the chimney. After the fire is under way the damper C is closed so that the smoke and hot gases from the furnace must pass, as indicated by the arrows, over a roundabout path, in the course of which they give up the major part of their heat to the steel walls of the jacket, which in turn pass it on to the air which is on its way to the living rooms.

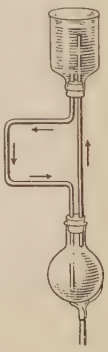


Fig. 212. Principle of hot-water heating

304. Hot-water heating. To illustrate the principle of hot-water heating let the arrangement shown in Fig. 212 be set up, the upper vessel being filled with colored water, and then let a flame be applied to the lower vessel. The colored water will show that the current moves in the direction of the arrows.

The actual arrangement of boiler and radiators in one system of hot-water heating is shown in Fig. 213. The water heated in the furnace rises directly through the pipe *A* to a radiator *R*, and returns again

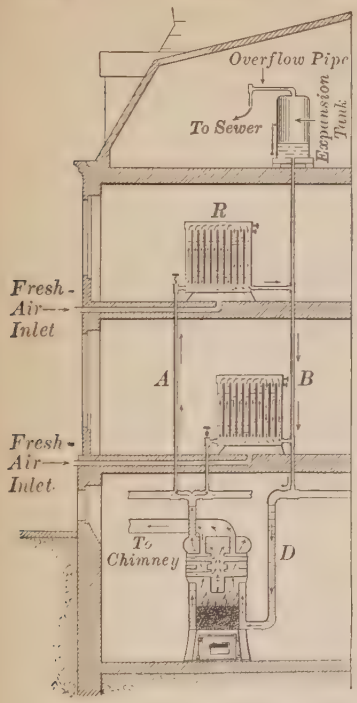


Fig. 213. Hot-water heater

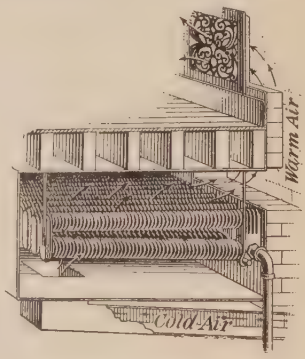


Fig. 214. Indirect system

to the bottom of the furnace through the pipes *B* and *D*. The circulation is maintained because the column of water in *A* is hotter and therefore lighter than the water in the return pipe *B*.

In the most common system of hot-water or steam heating, the so-called *direct-radiation* system, no provision whatever is made for ventilation. The occupants must depend entirely on open windows for their supply of fresh air. In the so-called *direct-indirect* system, shown in Fig. 213, fresh air is introduced through the radiator itself. The *indirect* system differs from this only in that steam or hot-water coils instead of being in the rooms are suspended from the ceiling of the basement in wooden boxes (Fig. 214). The arrows indicate air currents.

QUESTIONS AND PROBLEMS

1. Why is a hollow wall filled with sawdust a better nonconductor of heat than the same wall filled with air alone?

2. In a system of hot-water heating why does the return pipe always connect at the bottom of the boiler, while the outgoing pipe connects with the top?

3. When a room is heated by a fireplace, which of the three methods of heat transference plays the most important rôle?

4. Which methods of heat transfer are most important in systems of direct and of indirect radiation?

5. Why do you blow on your hands to warm them in winter and fan yourself for coolness in summer?

6. If hot water is poured into a thick glass vessel, the vessel will probably break; but if the glass is thin, it will not usually do so. Why?

7. If you open a door between a warm and a cold room, in what direction will a candle flame be blown which is placed at the top of the door? Explain.

8. Why is felt a better conductor of heat when it is very firmly packed than when loosely packed?

CHAPTER XII

MAGNETISM[1]

General Properties of Magnets

305. Magnets. It has been known for many centuries that some specimens of the ore known as magnetite (Fe_3O_4) have the property of attracting small bits of iron and steel. This ore probably received its name from the fact that it is especially abundant in the province of Magnesia, in Thessaly, although the Latin writer Pliny says that the word "magnet" is derived from the name of the Greek shepherd Magnes, who, on the top of Mount Ida, observed the attraction of a large stone for his iron crook. Pieces of this ore which exhibit this attractive property are known as *natural magnets.*

It was also known to the ancients that artificial magnets may be made by stroking pieces of steel with natural magnets, but it was not until about the twelfth century that the discovery was made that a suspended magnet will assume a north-and-south position. Because of this latter property natural magnets became known as lodestones (leading stones), and magnets, either artificial or natural, began to be used for determining directions. The first mention of the use of the compass in Europe is in 1190. It is thought to have been introduced from China.

Magnets are now made either by stroking bars of steel in one direction with a magnet, or by passing electric currents about the bars in a manner to be described later. The form

[1] This chapter should either be accompanied or preceded by laboratory experiments on magnetic fields and on the molecular nature of magnetism. See, for example, Experiments 25 and 26 of the authors' manual.

shown in Fig. 215 is called a *bar magnet,* that shown in Fig. 216 a horseshoe magnet.

Fig. 215. A bar magnet

Fig. 216. A horseshoe magnet

306. Poles of a magnet. If a magnet is dipped into iron filings the filings will be seen to cling in tufts near the ends but scarcely at all near the middle (Fig. 217). These places near the ends of a magnet at which its strength seems to be concentrated are called the *poles* of the magnet. The end of a freely swinging magnet which points to the north is designated as the north-seeking, or simply the *north pole* (N); and the other end as the south-seeking, or the *south pole* (S). The direction in which a compass needle points is called the magnetic meridian.

307. Laws of magnets. Let a magnet be suspended from a thread by means of a wire stirrup (Fig. 218) and its north pole marked. Then let another magnet be placed in the stirrup and its north pole marked. If now, while one magnet is at rest in a north-and-south plane, the other magnet is brought near it, the suspended magnet will be deflected. It will be found, moreover, that the *N* pole will be repelled by the *N* pole of the second magnet but attracted by the *S* pole. Also the *S* pole of the suspended magnet will be repelled by the *S* pole of the second one but attracted by its *N* pole.

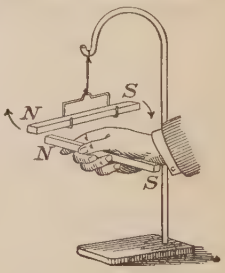


Fig. 218. Magnetic attractions and repulsions

These experiments indicate the general law of magnets: *Like poles repel each other, unlike poles attract.*

Fig. 217. Iron filings clinging to bar magnet

The force of attraction is found, like gravitation, to vary inversely as the square of the distance; e.g. separating two poles to three times their original distance reduces the force acting between them to $\frac{1}{9}$ its original value.

308. Measurement of magnetism. A pole is said to be *a pole of unit strength* when it will repel an exactly equal and similar pole a centimeter away with a force of one dyne. The number of units of magnetism in any pole is, then, measured by the number of dynes of force which it exerts upon a unit pole placed 1 cm. from it. Thus if this force is found to be 100 dynes, we say that the pole contains 100 units of magnetism, etc.

309. Magnetic materials. Iron and steel are the only substances which exhibit magnetic properties to any marked degree. Nickel and cobalt are also attracted appreciably by strong magnets. Bismuth, antimony, and a number of other substances are actually repelled instead of attracted, but the effect is very small. For practical purposes iron and steel may be considered as the only magnetic materials.

310. Induced magnetism. Let a small iron nail be suspended from one end of a bar magnet. A second nail may be suspended from the first, which itself acts like a magnet; a third from the second, etc., as shown in Fig. 219. But if the bar magnet is carefully pulled away from the first nail, the others will instantly fall away from each other, thus showing that the nails were strong magnets only so long as they were in contact with the bar magnet.

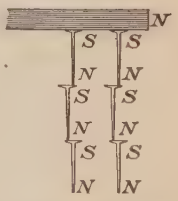


FIG. 219. Magnetism induced by contact

Any piece of soft iron may be made a temporary magnet in this way by touching one end of it to one end of a bar magnet. In fact, the soft iron will become a temporary magnet if one end of it is simply *brought near to* a bar magnet, even if not in contact with one of its poles. This may be shown by presenting some iron filings to one end of an iron nail when the latter is held close to one pole of a bar or horseshoe magnet, as in Fig. 220. Even inserting a plate of glass, or of copper, or of any other material except iron, between S and N, will not change

the number of filings which cling to the end S'. But as soon as the permanent magnet is removed most of the filings will fall. *Magnetism produced in this way by the mere presence of an adjacent magnet with or without contact is called induced magnetism.* If the induced magnetism of the nail of Fig. 220 is tested with a compass needle, it will be found that the *remote* induced pole S' is of the same sign as the inducing pole S, while the *near* pole N is of unlike sign. This is the general law of magnetic induction.

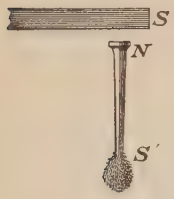


Fig. 220. Magnetism induced without contact

311. Permeability and retentivity. A piece of soft iron will very easily become a strong temporary magnet through the influence of a neighboring magnet. A substance which possesses this property is said to have a high degree of *permeability.* When, however, the magnetizing force is removed, the soft iron loses nearly all of its magnetism. Hard steel, on the other hand, does not become magnetized so readily, but once magnetized, it retains its magnetism even when removed from the magnetizing influence. Hard steel is said, therefore, to have high *retentivity.*

312. Magnetic lines of force. If we could separate the N and S poles of a small magnet so as to get an independent N pole, and were to place this N pole near the N pole of a bar magnet, it would move over to the S pole along some curved path similar to that shown in Fig. 221. The reason it would move in a curved path is that it would be simultaneously repelled by the N pole of the bar magnet, and attracted by its S pole, and the relative strengths of these two forces would continually change, as the relative distances of the moving pole from these two poles changed.

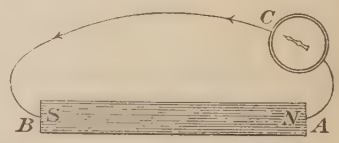


Fig. 221. A line of force set up by the magnet AB

To verify this conclusion let a strongly magnetized sewing needle be floated in a small cork in a shallow dish of water, and let a bar or horseshoe magnet be placed just above or just beneath the dish (see Fig. 222). The cork and needle will then move as would an independent pole, since the remote pole of the needle is so much farther from the magnet than the near pole that its influence on the motion is very small. The cork will actually be found to move in a curved path from N to S.

FIG. 222. Showing direction of a motion of an isolated pole near magnet

Any path which an independent N pole would take in going from N to S is called a *line of force.* The simplest way of finding the direction of this path at any point near a magnet is to hold a compass needle at the point considered. The compass needle sets itself along the line in which its poles would move if independent, i.e. along the line of force which passes through the given point (see C, Fig. 221).

313. Fields of force. The region about a magnet in which its magnetic forces can be detected is called its *field of force.* The easiest way of gaining an idea of the way in which the lines of force are arranged in the magnetic field about any magnet is to sift iron filings upon a piece of paper placed immediately over the magnet. Each little filing becomes a temporary magnet by induction, and therefore, like the compass needle, sets itself in the direction of the line of force at the point where it is. Fig. 223 shows how the filings arrange themselves about a bar magnet. Fig. 224 is the corresponding ideal diagram, showing the lines of force emerging from the N pole and passing about in curved paths to the S pole. It is customary to imagine these lines as returning

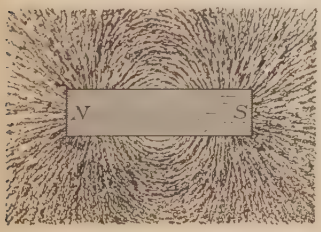


FIG. 223. Arrangement of iron filings about a bar magnet

through the magnet from S to N in the manner shown, so that each line is thought of as a closed curve. This convention was introduced by Faraday, and has been found of great assistance in correlating the facts of magnetism.

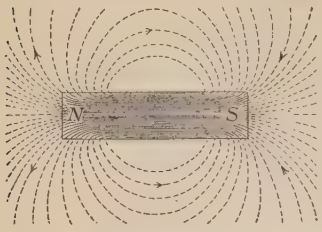


Fig. 224. Ideal diagram of field of a bar magnet

314. Strength of a magnetic field. The strength of a magnetic field at any point near a magnet is defined as the number of dynes of force which a unit magnet pole would experience at the point considered. Thus if at some particular point between the poles N, S, of a horseshoe magnet (Fig. 225) a unit N pole would be pushed from N toward S with a force of one dyne, then the magnetic field at that point would be a "field of unit strength," or a unit magnetic field. If the unit pole were pushed from N toward S with a force of 1000 dynes, then the field would be one of a 1000 units strength, etc. If we wish to represent graphically a field of unit strength, we draw one line per square centimeter through a surface such as $ABCD$, taken at right angles to the lines of force. A field of strength 2 would be represented by two lines per square centimeter, a field of strength n, by n lines per square centimeter, etc.; i.e. field strengths are represented by the number of lines of force drawn to the square centimeter.

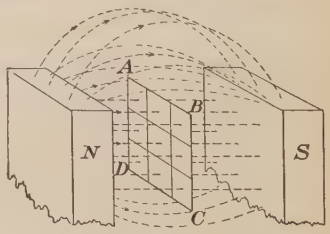


Fig. 225. The strength of a magnetic field is represented by the number of lines of force per square centimeter

315. Molecular nature of magnetism. If a small test tube full of iron filings be stroked from end to end with a magnet it will be found to have become itself a magnet; but it will lose its magnetism as soon as the filings are shaken up. If a magnetized

knitting needle is heated red-hot, it will be found to have lost its magnetism completely. Again, if such a needle is jarred, or hammered, or twisted, the strength of its poles, as measured by their ability to pick up tacks or iron filings, will be found to be greatly diminished.

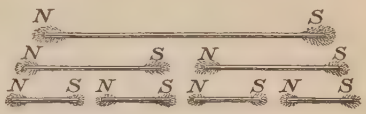


Fig. 226. Effect of breaking a magnet

These facts point to the conclusion that magnetism has something to do with the arrangement of the molecules, since causes which violently disturb the molecules of a magnet weaken its magnetism. Again, if a magnetized needle is broken, each part will be found to be a complete magnet; i.e. two new poles will appear at the point of breaking, a new N pole on the part which has the original S pole, and a new S pole on the part which has the original N pole. The subdivision may be continued indefinitely, but always with the same result, as indicated in Fig. 226. This points to the conclusion that the molecules of a magnetized bar are themselves little magnets arranged in rows with their opposite poles in contact.

If an unmagnetized piece of hard steel is pounded vigorously while it lies between the poles of a magnet, or if it is heated to redness and then allowed to cool in this position, it will be found to have become magnetized. This points to the conclusion that the molecules of the steel are magnets even when the bar as a whole is not magnetized, and that magnetization consists in causing them to arrange themselves in rows, end to end.

Fig. 227. Arrangement of molecules in an unmagnetized iron bar

316. Theory of magnetism. In an unmagnetized bar of iron or steel it is probable that the molecules themselves are tiny magnets which are arranged either haphazard, or in little closed

groups or chains, as in Fig. 227, so that, on the whole, opposite poles neutralize each other throughout the bar. But when the bar is brought near a magnet, the molecules are swung around by the outside magnetic force into some such arrangement as that shown in Fig. 228, in which the opposite poles completely neutralize each other only in the middle of the bar. According to this view, heating and jarring weaken the magnet because they tend to shake the molecules out of alignment. On the other hand, heating and jarring facilitate magnetization when the bar is between the poles of a magnet, because they assist the magnetizing force in breaking up the molecular groups and chains and getting the molecules into alignment. Soft iron has higher permeability than hard steel because the molecules of the former substance are much easier to swing into alignment than those of the latter substance. Steel has a very much greater retentivity than soft iron because its molecules are not so easily moved out of position once they have been aligned.

Fig. 228. Arrangement of molecules in a magnetized iron bar

317. Saturation. Strong evidence for the correctness of the above view is found in the fact that a piece of iron or steel cannot be magnetized beyond a certain limit, no matter how strong is the magnetizing force. This limit probably corresponds to the condition in which the axes of all the molecules are brought into parallelism, as in Fig. 229. The magnet is then said to be *saturated*, since it is as strong as it is possible to make it.

Fig. 229. Arrangement of molecules in a saturated magnet

Terrestrial Magnetism

318. The earth a magnet. It was in 1600 that Dr. William Gilbert, physician to Queen Elizabeth, in his great work entitled *De Magnete*, first explained the action of the compass needle by the assumption that the earth itself is a great magnet with an *S* pole near the geographical north pole and an *N* pole near the geographical south pole. The correctness of this assumption is now completely established. The *S* pole was found in 1831 by Sir James Ross in Boothia Felix, Canada, Lat. 70° 30′, Long. 95°. The *N* pole is in Lat. – 72° 35′, Long. – 152°.

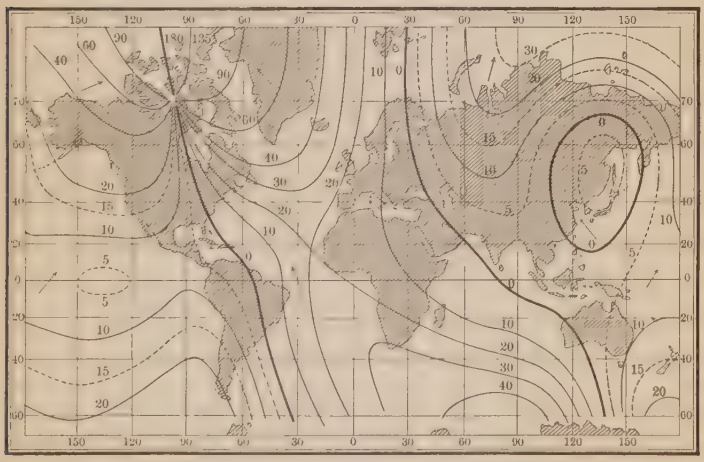


Fig. 230. The earth's isogonic lines

319. Declination. The earliest users of the compass were aware that it did not point exactly north; but it was Columbus who, on his first voyage to America, made the discovery, much to the alarm of his sailors, that the direction of the compass needle changes as one moves about over the earth's surface. The chief reason for this variation is found in the fact that the

magnetic poles do not coincide with the geographical poles; but there are also other causes, such as the existence of large deposits of iron ore, which produce local effects upon the needle. The number of degrees by which the needle varies from a true north-and-south line is called its *declination.* Each of the lines in Fig. 230 is so drawn that at each point on it the declination is the same. Such lines are called *isogonic lines.* The heavy lines pass through all the points where the needle points exactly to the north. These lines correspond, therefore, to places where the declination is zero. Lines of zero declination are called *agonic lines.*

320. Dip of the compass needle. Let an unmagnetized knitting needle a (Fig. 231) be thrust through a cork, and let a second needle b be passed through the cork at right angles to a, and as close to it as possible. Let a pin c be adjusted until the system is in neutral equilibrium about b as an axis, when a is pointing east and west. Then let a be carefully magnetized by stroking one end of it from the middle out with the N pole of a strong magnet, and the other end from the middle out with the S pole of the same magnet. When now the needle is replaced on its supports and turned into a north-and-south position, its N pole will be found to dip so as to cause the needle to make an angle of 60° or 70° with the horizontal.

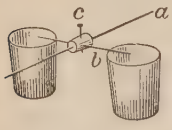


FIG. 231. Arrangement for showing dip

The experiment shows that in this latitude the earth's magnetic lines make a large angle with the horizontal. This angle between the earth's surface and the direction of the magnetic lines is called the *dip,* or *inclination,* of the needle. At Washington it is 71° 5′ and at Chicago 72° 50′. At the magnetic pole it is of course 90°, and at the so-called *magnetic equator,* which is an irregular curved line near the geographical equator, the dip is 0°.

321. The earth's inductive action. That the earth acts like a great magnet may be very strikingly shown in the following way.

WILLIAM GILBERT (1540–1603)

English physician and physicist; first Englishman to appreciate fully the value of experimental observations; first to discover through careful experimentation that the compass points to the north, not because of some influence of the stars, but because the earth is itself a great magnet; first to use the word "electricity"; first to discover that electrification can be produced by rubbing a great many different kinds of substances; author of the epoch-making book entitled *De Magnete*, etc., published in London in 1600.

Let a steel rod, e.g. a tripod rod, be held parallel to the earth's magnetic lines (the north end slanting down at an angle of about 70° or 75°) and struck a few sharp blows with a hammer. The rod will be found to have become a magnet with its upper end an *S* pole, like the north pole of the earth, and its lower end an *N* pole. If the rod is reversed and tapped again with the hammer, its magnetism will be reversed. If held in an east-and-west position and tapped, it will become demagnetized, as will be shown by the fact that either end of it will attract either end of a compass needle.

QUESTIONS AND PROBLEMS

1. If a bar magnet is floated on a piece of cork, will it tend to float toward the north? Why?

2. Will a bar magnet pull a floating compass needle toward it? Compare the answer to this question with that to 1.

3. Why should the needle used in the experiment of § 320 be placed east and west, when adjusting for neutral equilibrium, before it is magnetized?

4. The dipping needle is suspended from one arm of a steel-free balance and carefully weighed. It is then magnetized. Will its apparent weight increase?

5. With what force will an *N* magnet pole of strength 6 attract an *S* pole of strength 1 which is 5 cm. away? What will be the force of attraction if the *S* pole is of strength 9?

6. Explain on the basis of induced magnetization the process by which a magnet attracts a piece of soft iron.

7. When a piece of soft iron is made a temporary magnet by bringing it near the *N* pole of a bar magnet, will the end of the iron nearest the magnet be an *N* or an *S* pole?

8. Devise an experiment which will show that a piece of iron attracts a magnet just as truly as the magnet attracts the iron.

9. How would an ordinary compass needle act if placed over one of the earth's magnetic poles? How would a dipping needle act at these points?

10. Do the facts of induction suggest to you any reason why a horseshoe magnet retains its magnetism better when a bar of soft iron (a keeper, or armature) is placed across its poles than when it is not so treated? (See Fig. 228.)

CHAPTER XIII

STATIC ELECTRICITY

General Facts of Electrification

322. Development of electrification by friction. Let a hard rubber (ebonite) rod be rubbed with flannel or cat's fur and then brought near some dry pith balls or bits of paper (Fig. 232). The small bodies will jump toward the rod.

This sort of attraction was observed by the ancient Greeks as early as 600 B.C., when it was found that amber which had been rubbed with silk attracted light objects. It was not, however, until A.D. 1600 that Gilbert, "the father of the modern science of electricity and magnetism," discovered that the effect could be produced by rubbing together a great variety of other substances, such, for example, as glass and silk, sealing wax and wool, ebonite and cat's fur.

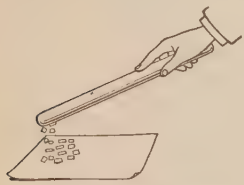


Fig. 232. Electrical attractions

Gilbert named the effect which is produced upon these various substances by friction, *electrification*, after the Greek name for amber, *electron*. Thus a body which, like rubbed amber, has been endowed with the property of attracting light bodies is said to have been *electrified*, or to have been given a charge of *electricity*. In this statement nothing whatever is said about the nature of electricity. We simply define an electrically charged body as one which has been put into the condition in which it acts like the rubbed amber.

323. Two opposite kinds of electrification. Let a glass rod which has been electrified by rubbing it with silk be suspended by a silk

thread, as shown in Fig. 233. Let an ebonite rod which has been rubbed with cat's fur be suspended in a second stirrup. If now a second glass rod which has been rubbed with silk is brought near the suspended glass rod, it will be found to repel it strongly; but when it is brought near the suspended ebonite it will attract it no less strongly. On the other hand, a second electrified ebonite rod will attract the glass and repel the ebonite.

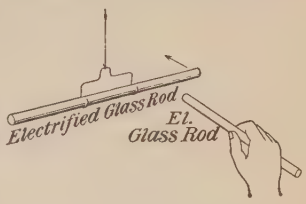


Fig. 233. Electrical repulsions

Evidently, then, the electrifications which have been imparted to the glass and to the ebonite are opposite, in the sense that an electrified body which attracts one repels the other. We say, therefore, that there are two kinds of electrification, and we arbitrarily call one *positive* and the other *negative*. Thus *a positively electrified body* is defined as one which acts with respect to other electrified bodies like *a glass rod which has been rubbed with silk*, and *a negatively electrified body* is one which acts like *an ebonite rod which has been rubbed with cat's fur*.

324. Laws of electrical attraction and repulsion. The facts presented in the preceding experiment may be stated in the following law. *Electrical charges of like kind repel each other; those of unlike kind attract each other.* The forces of attraction or repulsion are found, like those of gravitation and of magnetism, *to decrease as the square of the distance increases.*

325. Measurement of electrical quantities. The fact of attraction and repulsion is taken as the basis for the definition and measurement of so-called *quantities* of electricity. Thus a small charged body is said to contain 1 unit of electricity when it will repel an exactly equal and similar charge placed 1 cm. away with a force of 1 dyne. The number of units of electricity on any charged body is then measured by the force which it exerts upon a unit charge placed at a given distance from it; for example, a charge which at a distance of 10 cm. repels a unit charge with a force of 1 dyne contains 100 units of electricity,

for this means that at a distance of 1 cm. it would repel the unit charge with a force of 100 dynes (see § 324).

326. Conductors and nonconductors. Let an electroscope E (Fig. 234), consisting of a pair of gold leaves a and b, suspended from an insulated metal rod r, and protected from air currents by a case J, be connected with the metal ball B by means of a wire. Let the ebonite rod be now electrified and rubbed over B. The immediate divergence of the gold leaves will show that a portion of the electric charge placed upon B has been carried by the wire to the gold leaves, where it causes them to diverge in accordance with the law that bodies charged with the same kind of electricity repel each other.

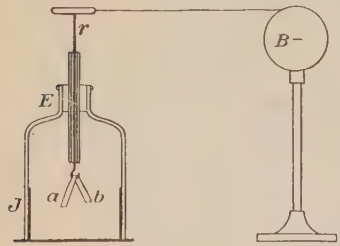


Fig. 234. Illustrating conduction

Let the experiment be repeated when E and B are connected with a thread of silk or a long rod of wood instead of the metal wire. No divergence of the leaves will be observed. If a moistened thread connects E and B the leaves will be seen to diverge slowly when the ball B is charged, showing that a charge is carried slowly by the moist thread.

These experiments make it clear that while electric charges pass with perfect readiness from one point to another in a wire, they are quite unable to pass along dry silk or wood, and pass with difficulty along moist silk. We are therefore accustomed to divide substances into two classes, *conductors* and *nonconductors* or *insulators*, according to their ability to transmit electrical charges from point to point. Thus metals and solutions of salts and acids in water are all conductors of electricity, while glass, porcelain, rubber, mica, shellac, wood, silk, vaseline, turpentine, paraffin, and oils generally are insulators. No hard and fast line, however, can be drawn between conductors and nonconductors, since all so-called insulators conduct to some slight extent, while the so-called conductors differ greatly among themselves in the facility with which they transmit charges.

The facts of conduction bring out sharply one of the most essential distinctions between electricity and magnetism. Magnetic poles exist only in iron and steel bodies and they remain fixed in position in these bodies. Electric charges may exist on any bodies and may pass from one body to another by conduction.

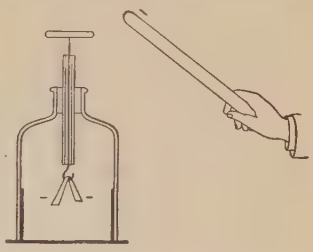


Fig. 235. Illustrating induction

327. Electrostatic induction. Let the ebonite rod be electrified by friction and slowly brought toward the knob of the gold-leaf electroscope (Fig. 235). The leaves will be seen to diverge, even though the rod does not approach to within a foot of the electroscope.

This makes it clear that the mere *influence* which an electric charge exerts upon a conductor placed in its neighborhood is able to produce electrification in that conductor. This method of producing electrification is called *electrostatic induction.*

As soon as the charged rod is removed the leaves will be seen to collapse completely. This shows that this form of electrification is only a temporary phenomenon which is due simply to the presence of the charged body in the neighborhood.

328. Nature of electrification produced by induction. Let a metal ball A (Fig. 236) be strongly charged by rubbing it with a charged rod, and let it then be brought near an insulated[1] metal body B which is provided with pith balls or strips of paper a, b, c, as shown. The divergence of a and c will show that the ends of B have received electrical charges because of the presence of A, while the failure of b to diverge will show that the middle of B is uncharged. Further, the rod which charged A will be found to repel c but to attract a.

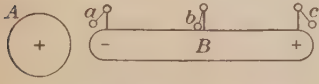


Fig. 236. Nature of induced charges

[1] Sulphur is practically a perfect insulator in all weathers, wet or dry. Metal conductors of almost any shape resting upon pieces of sulphur will serve the purposes of this experiment in summer or winter.

We conclude, therefore, that when a conductor is brought near a charged body, the end away from the inducing charge is electrified with the same kind of electricity as that on the inducing body, while the end toward the inducing body receives electricity of opposite sign.

329. Two-fluid theory of electricity. We can describe the facts of induction conveniently by assuming that in every conductor there exists an equal number of positively and negatively charged corpuscles which are very much smaller than atoms, and which are able to move about freely between the molecules of the conductor. When no electrified body is near the conductor B it appears to have no charge at all, because all the little positive charges within it counteract the effects upon outside bodies of all the little negative charges. But as soon as an electrical charge is brought near B, it drives as far away as possible the little corpuscles which carry charges of sign like its own, while it attracts the little corpuscles of unlike sign. B therefore becomes electrified like A at its remote end and unlike A at its near end. As soon as the inducing charge is removed, B immediately becomes neutral again because the little positive and negative corpuscles come together under the influence of their mutual attractions. This picture of the mechanism of electrification by induction is a modern modification of the so-called *two-fluid theory* of electricity, which conceived of all conductors as containing equal amounts of two weightless electrical fluids called positive electricity and negative electricity. Although it is extremely doubtful whether this theory represents the actual conditions within a conductor, yet we are able to say with perfect positiveness that *the electrical behavior of a conductor is exactly what it would be if it did contain equal amounts of positive and negative electrical fluids,* or equal numbers of minute positive and negative corpuscles which are free to move about among the molecules of the conductor under the influence of outside electrical forces. Hence we shall habitually

speak of the positive electricity within a conductor as being repelled to the remote end and the negative electricity as attracted to the near end by an outside positive charge, and vice versa. This is merely a universally recognized convention. It does not imply the acceptance of the two-fluid theory.

330. The electron theory. A slightly different theory has recently been put forward by physicists of high standing, notably by J. J. Thomson of Cambridge, England. According to this theory a certain amount of positive electricity is supposed to constitute the nucleus of the atom of every substance. About the center of this positive charge are grouped a large number of very minute negatively charged corpuscles or electrons, the mass of each of which is approximately $\frac{1}{2000}$ of that of the hydrogen atoms. The sum of the negative charges of these electrons is supposed to be just equal to the positive charge of the atom, so that in its normal condition the whole atom is neutral or uncharged. But in the jostlings of the molecules of a conductor electrons are continually getting loose from the atoms, moving about freely between the molecules, and then reëntering other atoms which have lost electrons. Therefore at any given instant there are always in every conductor a large number of free negative electrons and an exactly equal number of atoms which have lost electrons and which are therefore positively charged. Such a conductor would, as a whole, show no charge of either positive or negative electricity. But as soon as a body charged, for example negatively, is brought near such a conductor, the negatively charged electrons stream away to the remote end, leaving behind them the positively charged atoms which are not free to move from their positions. On the other hand, if a positively charged body is brought near the conductor, the negative electrons are attracted and the remote end is left with the immovable plus atoms.

The only advantage of this theory over that suggested in § 329, in which the existence of both positive and negative

corpuscles was assumed, is that there is much direct experimental evidence for the existence of such negatively charged corpuscles or electrons of about $\frac{1}{2000}$ the mass of the hydrogen atom (see Chapter XXII), but no direct evidence as yet for the existence of positively charged electrons.

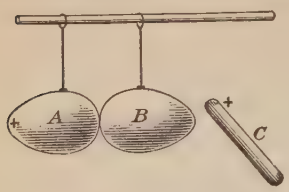


Fig. 237. Obtaining a plus and a minus charge by induction

331. Charging by induction. Let two metal balls or two eggshells A and B, which have been gilded or covered with tin foil, be suspended by silk threads and touched together, as in Fig. 237. Let a positively charged body C be brought near them. As described above, A and B will at once exhibit evidences of electrification, i.e. A will repel a positively charged pith ball while B will attract it. If C is removed while A and B are still in contact, the separated charges reunite and A and B cease to exhibit electrification. But if A and B are separated from each other while C is in place, A will be found to be permanently positively charged and B negatively charged. This may be proved either by the attractions and repulsions which they show for charged rods brought near them, or by the effects which they produce upon a charged electroscope brought into their vicinity, the leaves of the latter falling together when it is brought near one and spreading farther apart when brought near the other.

We see, therefore, that if we cut in two, or separate into two parts, a conductor while it is under the influence of an electric charge, we obtain two permanently charged bodies, the remoter part having a charge of the same sign as that of the inducing charge, and the near part having a charge of unlike sign.

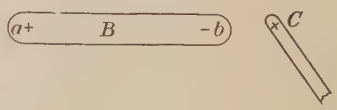


Fig. 238. A body charged by induction has a charge of sign opposite to that of the inducing charge

Let the conductor B (Fig. 238) be touched by the finger while a charged rod C is near it. Then let the finger be removed and after it the rod C. If now a negatively charged pith ball is brought near B, it will be repelled, showing that B has become negatively charged. In this experiment the body of the

experimenter corresponds to the egg A of the preceding experiment, and removing the finger from B corresponds to separating the two eggshells. Let the last experiment be repeated with only this modification, that B is touched at b rather than at a. When B is again tested with the pith ball it will still be found to have a negative charge, exactly as when the finger was touched at a.

We conclude, therefore, that no matter where the body B is touched, *the sign of the charge left upon it is always opposite to that of the inducing charge.* This is because the negative electricity can under no circumstances escape from B so long as C is present, for it is "bound" by the attraction of the positive charge on C. We may think of the final negative charge on B as due either to the fact that the positive escapes through the finger, or that enough more negative is drawn up to B to counteract this positive which was at a.

332. Charging the electroscope by induction. Let an ebonite rod which has been rubbed with cat's skin be brought near the knob of the electroscope (Fig. 235). The leaves at once diverge. Let the knob be touched with the finger while the rod is held in place. The leaves will fall together. Let the finger be removed and then the rod. The leaves will fly apart again.

The electroscope has been charged by induction, and since the charge on the ebonite rod was negative, the charge on the electroscope must be positive. If this conclusion is tested by bringing the ebonite rod near the electroscope, the leaves will fall together as the rod approaches the knob. How does this prove that the charge on the electroscope is positive?

Fig. 239. Plus and minus electricities always developed in equal amounts

333. Plus and minus electricities always appear simultaneously and in equal amounts. Let an ebonite rod be completely discharged by passing it quickly through a Bunsen flame. Let a flannel cap having a silk thread attached be slipped over the rod, as in Fig. 239, and twisted rapidly around a number of times. When rod and cap together are

held near a charged electroscope no effect will be observed. But if the cap is pulled off, it will be found to be positively charged, while the rod will be found to have a negative charge.

Since the two together produce no effect, the experiment shows that the plus and minus charges were equal in amount. This experiment confirms the view already brought forward in connection with induction, that electrification always consists in a separation of plus and minus charges which already exist in equal amounts within the bodies in which the electrification is developed.

QUESTIONS AND PROBLEMS

1. Charge a gold-leaf electroscope by induction from a glass rod. Warm a piece of paper and stroke it on the clothing. Hold it over the charged electroscope. If the divergence of the gold leaves is increased, is the charge on the paper + or − ? If the divergence of the gold leaves is decreased, what is the sign of the charge on the paper ?

2. Given a gold-leaf electroscope, a glass rod, and a piece of silk, how, in general, would you proceed to test the sign of the electrification of an unknown charge ?

3. If pith balls, or any light figures, are placed between two plates (Fig. 240), one of which is connected to earth and the other to one knob of an electrical machine in operation, the figures will bound back and forth between the two plates as long as the machine is operated. Explain.

Fig. 240

4. If you are given a positively charged insulated sphere, how could you charge two other spheres, one positively and the other negatively, without diminishing the charge on the first sphere ?

5. If you bring a positively charged glass rod near the knob of an electroscope and then touch the knob, why do you not remove the negative electricity which is on the knob ?

6 In charging an electroscope by induction, why must the finger be removed before the removal of the charged body ?

7. If you hold a brass rod in the hand and rub it with silk, the rod will show no sign of electrification ; but if you hold the brass rod with a piece of sheet rubber and then rub it with silk, you will find it electrified. Explain.

8. Why is a pith ball first attracted to an electrified rod and then repelled from it ?

9. Why is repulsion between an unknown body and an electrified pith ball a surer sign that the unknown body is electrified than is attraction?

10. State as many differences as you can between the phenomena of magnetism and those of electricity.

Distribution of Electric Charge upon Conductors

334. Electric charges reside only upon the outside surface of conductors. Let a deep tin cup be placed upon an insulating stand and charged as strongly as possible either from an ebonite rod or from an electrical machine (Fig. 241). If now a smooth metal ball suspended by a silk thread is touched to the *outside* of the charged cup, and then brought near the knob of a charged electroscope, it will show a strong charge. But if it is touched to the *inside* of the cup, it will show no charge at all. Again, if the ball is first charged and then touched to the inside of the cup, it will be found to lose its entire charge, even though a strong charge is on the outside of the cup.

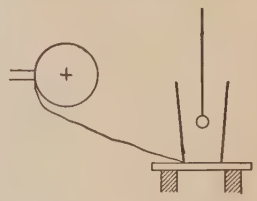


Fig. 241. Proof that charge resides on surface

These experiments show that *an electric charge resides entirely on the outside surface of a conductor.* This is a result which might have been inferred from the fact that all the little electrical charges of which the total charge is made up repel each other and therefore move through the conductor until they are, on the average, as far apart as possible.

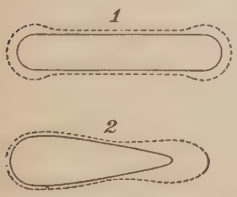


Fig. 242. Distribution of charge over oblong bodies

335. Density of charge greatest where curvature of surface is greatest. Since all of the parts of an electric charge tend, because of their mutual repulsions, to get as far apart as possible, we should infer that if a charge of either sign is placed upon an oblong conductor like that of Fig. 242, (1), it will distribute itself so that the electrification at the ends will be stronger than that at the middle.

To test this inference let a proof plane (a small flat metal disk provided with an insulating handle, Fig. 243) be touched to one end of such a charged body, the charge conveyed to a gold-leaf electroscope, and the amount of separation of the leaves noted. Then let the experiment be repeated when the proof plane touches the middle of the body. The separation of the leaves in the latter case will be found to be very much less than in the former. If we should test the distribution on a pear-shaped body [Fig. 242, (2)], in the same way, we should find the density of electrification considerably greater on the small end than on the large one. By density of electrification is meant the quantity of electricity on unit area of the surface.

Fig. 243. Proof plane

336. Discharging effect of points. The above experiments indicate that if one end of a pear-shaped body is made more and more pointed, then when the body is charged the electric density on this end will become greater and greater. The following experiment will show what happens when the conductor is provided with a sharp point.

Let a very sharp needle be attached to any smooth insulated metal body provided with paper or pith-ball indicators, as in Fig. 236, p. 241. If the body is now charged either with a rubbed rod or with an electric machine, as soon as the supply of electricity is stopped the paper indicators will immediately fall, showing that the body is losing its charge. To show that this is certainly due to the effect of the point, remove the needle and repeat. The indicators will fall very slowly, if at all.

The experiment shows that the electrical density upon the point is so great that the charge escapes from it into the air. This is probably because the intense charge on the point breaks apart the molecules of air into little positive and negative parts, one set of which is attracted to the point and the other repelled away from it. The former set neutralize the charge on the conductor.

The effect of points may be shown equally well by charging the gold-leaf electroscope and holding a needle in the hand within a few inches

of the knob. The leaves will fall together rapidly. In this case the needle point becomes electrified by induction and discharges to the knob electricity of the opposite kind to that on the knob, thus neutralizing its charge. An entertaining variation of the last experiment is to attach a tassel of tissue paper to an insulated conductor and electrify it strongly. The paper streamers under their mutual repulsions will stand out in all directions, but as soon as a needle point is held in the hand near them they will at once fall together (Fig. 244).

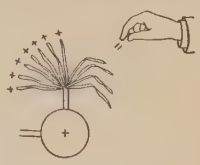


Fig. 244. Discharging effect of points

337. The electric whirl. Let an electric whirl (Fig. 245) be balanced upon a pin point and attached to one knob of an electric machine. As soon as the machine is started the whirl will rotate rapidly in the direction of the arrows.

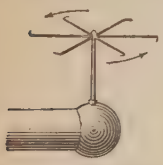


Fig. 245. The electric whirl

The explanation is as follows. On account of the great magnitude of the electric force near a point the molecules of the gas just in front of it are broken into + and − parts. The part of sign unlike that of the charge on the point is drawn to the point, the other part is repelled. But since this repulsion is mutual, the point is pushed back with the same force with which the particles are pushed forward; hence the rotation. The repelled particles in their turn drag the air with them in their forward motions, and thus produce the "electric wind," which may be detected easily by the hand or by a candle flame (Fig. 246).

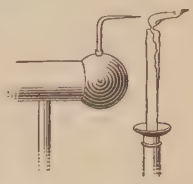


Fig. 246. The electric wind

338. Lightning and lightning rods. It was in 1752 that Franklin, during a thunderstorm, sent up his historic kite. This kite was provided with a pointed wire at the top. As soon as the hempen kite string had become wet he succeeded in drawing ordinary electric sparks from a key attached to the lower end. This experiment demonstrated for the first time that thunderclouds

carry ordinary electrical charges which may be drawn from them by points just as the charge was drawn from the tassel in the experiment of § 336. It also showed that lightning is nothing but a huge electric spark. Franklin applied this discovery in the invention of the lightning rod. The way in which the rod discharges the cloud and protects the building is as follows. As the charged cloud approaches the building it induces an opposite charge in the rod. This induced charge escapes rapidly and quietly from the sharp point and thus neutralizes the charge of the cloud.

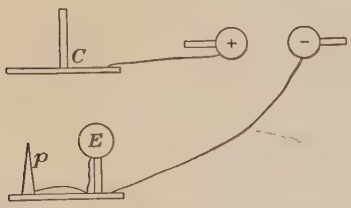
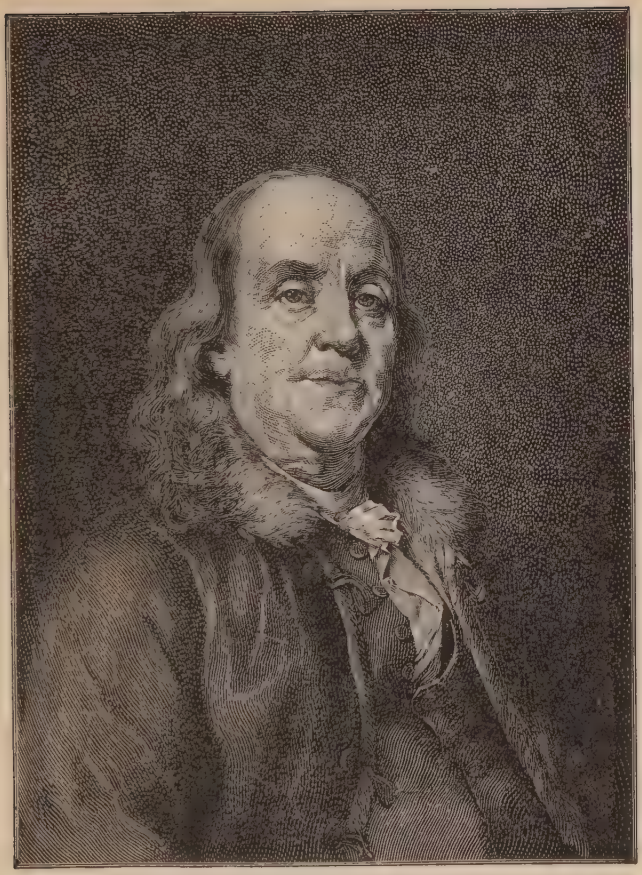


Fig. 247. Illustrating the action of a lightning rod

To illustrate, let a metal plate C (Fig. 247) be supported above a metal ball E, and let C and E be attached to the two knobs of an electrical machine. When the machine is started sparks will pass from C to E, but if a point p is connected to E, the sparking will cease; i.e. the point will protect E from the discharges even though the distance Cp be considerably greater than CE.

The lower end of a lightning rod should be buried deep enough so that it will always be surrounded by moist earth, since dry earth is a poor conductor. It will be seen, therefore, that lightning rods protect buildings not because they conduct the lightning to earth, but because they prevent the formation of powerful charges in the neighborhood of the buildings on which they are placed. There are certain kinds of discharges from which lightning rods do not protect a building. In general, however, they do diminish greatly the liability to lightning stroke.

339. Electric screens. That the charge on the outside of a conductor always distributes itself in such a way that there is no electric force within the conductor was first proved experimentally by Faraday. He covered a large box with tin foil and



BENJAMIN FRANKLIN (1706–1790)

Celebrated American statesman, philosopher, and scientist; born at Boston, the sixteenth child of poor parents; printer and publisher by occupation; pursued scientific studies in electricity as a diversion; first proved that the two coats of a Leyden jar are oppositely charged; demonstrated the identity of lightning and frictional electricity by flying a kite in a thunderstorm and drawing sparks from the insulated lower end of the kite string; invented the lightning rod; originated the one-fluid theory of electricity which regarded a positive charge as indicating an excess, a negative charge a deficiency, in a certain normal amount of an all-pervading electrical fluid.

went inside with the most delicate electroscopes obtainable. He found that the outside of the box could be charged so strongly that long sparks were flying from it without any electrical effects being observable anywhere inside the box.

To repeat the experiment in modified form, let an electroscope be placed beneath a bird cage or wire netting, as in Fig. 248. Let charged rods or other powerfully charged bodies be brought near the electroscope outside the cage. The leaves will be found to remain undisturbed.

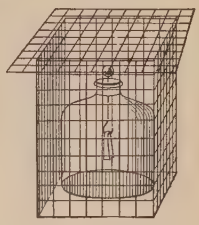


FIG. 248. Electroscope protected by a wire cage

Hence, if we wish to protect an electrical instrument from outside electrical disturbances, we have only to surround it with a metal covering.[1]

POTENTIAL AND CAPACITY

340. Potential difference. There is a very instructive analogy between the use of the word "potential" in electricity and "pressure" in hydrostatics. For example, if water will flow from tank A to tank B through the connecting pipe R (Fig. 249), we infer that the hydrostatic pressure at a must be greater than that at b, and we attribute the flow directly to this difference in pressure. In exactly the same way, if, when two bodies A and B (Fig. 250) are connected by a conducting wire r, a charge of + electricity is found to pass from A to B, we say that the electrical potential is higher at A than at B, and we assign this *difference of potential* as the cause of the flow. Thus, just as water tends to flow from points of higher hydrostatic pressure to points of lower hydrostatic pressure, so electricity tends to

FIG. 249. Illustrating hydrostatic pressure

[1] A laboratory exercise on static electrical effects should follow the discussion of this section. See, for example, Experiment 27 of the authors' manual.

flow from points of higher electrical pressure or potential to points of lower electrical pressure or potential.

Again, if water is not continuously supplied to one of the tanks A or B of Fig. 249, we know that the pressures at a and b must soon become the same. Similarly, if no electricity is supplied to the bodies A and B of Fig. 250, their potentials very quickly become the same. In other words, *all points on a system of connected conductors in which the electricity is in a stationary or static condition are at the same potential.* This result follows at once from the fact of mobility of electric charges through conductors.

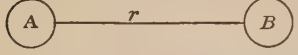


FIG. 250. Illustrating electrical pressure

But if water is continuously poured into A and removed from B (Fig. 249), the pressure at a will remain permanently above the pressure at b, and a continuous flow of water will take place through R. So if A (Fig. 250) is connected with an electrical machine and B to earth, a permanent potential difference will exist between A and B, and a continuous current of electricity will flow through r. Difference in potential is commonly denoted simply by the letters P.D. (Potential Difference).

341. The earth the zero of potential. When we speak simply of the "potential" of a body, we mean the *difference of potential* which exists between the body and the earth, for the electrical condition of the earth is always taken as the zero to which the electrical conditions of all other bodies are referred. Thus a body which is positively charged is regarded as one which has a potential higher than that of the earth, while a body which is negatively charged is looked upon as one which has a potential lower than that of the earth. Fig. 251 represents the hydrostatic analogy of positively and negatively charged bodies. Since it has been decided to regard the flow of electricity as taking place from the point of higher to that of lower potential, it will be seen that when a discharge takes place

between a negatively charged body and the earth we must regard the positive electricity as passing from the earth to the body rather than the negative as passing from the body to the earth. This is, indeed, a mere convention; but it is one which it is very important to remember in connection with the study of the next chapter. From the point of view of the electron theory (§ 330) it would be natural to exactly invert this convention, since this theory regards the negative electricity alone as moving through conductors; but since the opposite convention has become established it will not be wise to attempt to change it until the electron theory has become more thoroughly established than is at present the case.

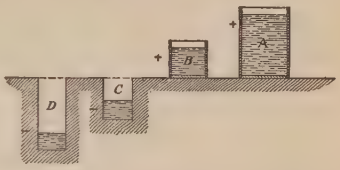


FIG. 251. A positively charged body is one which has a potential higher than that of the earth; a negatively charged body, one which has a potential lower than that of the earth

342. The unit of P.D. The volt. The measure of the P.D. between two bodies A and B (Fig. 250) is the amount of work in ergs which is required to carry a unit positive charge from the body of lower potential B to the body of higher potential A against the electrical forces exerted upon the moving unit charge by the charges already upon A and B. Thus, if A is positively charged and B negatively charged, it will require work to move a unit positive charge from B to A, because this charge will be attracted by B and repelled by A. The amount of this work in ergs is the measure, in so-called absolute units, of the potential difference between A and B. For practical purposes, however, it has been found convenient to choose a unit of P.D. which is exactly $\frac{1}{300}$ of this absolute unit. This practical unit is named the *volt* in honor of the famous Italian physicist, Alessandro Volta (1745–1827), who, about 1800, at the University of Pavia, first made quantitative measurements upon the potentials of charged bodies. A volt is then defined as the P.D. between

two bodies when it requires $\frac{1}{300}$ ergs of work to carry a unit charge from one body to the other against the electrical forces existing between the two bodies.

343. Some methods of measuring potentials. The simplest and most direct way of comparing the potential differences which exist between various charged bodies and the earth is to connect the charged bodies successively to the knob of an electroscope, the conducting case[1] of which is in electrical connection with the earth. The amount of separation of the gold leaves is a measure of the P.D. between the earth and the charged body.

Another very convenient way of measuring a large P.D. is to measure the length of the spark which will pass between the two bodies whose P.D. is sought. The P.D. is roughly proportional to spark length, each centimeter of spark length representing, with large electrodes, a P.D. of about 30,000 volts.

344. Condensers. Let a metal plate A be mounted on an insulating plate and connected with an electroscope, as in Fig. 252. Let a second plate B be similarly mounted and connected to the earth by a conducting wire. Let A be charged and the deflection of the gold leaves noted. If now we push B toward A, we shall observe that as it comes near the leaves begin to fall together, showing that the potential of A is diminished by the presence of B, although the quantity of electricity on A has remained unchanged. If we convey additional + charges to A with the aid of a proof plane we shall find that many times the original amount of electricity may now be put on A before the leaves return to their original divergence, i.e. before the body regains its original potential.

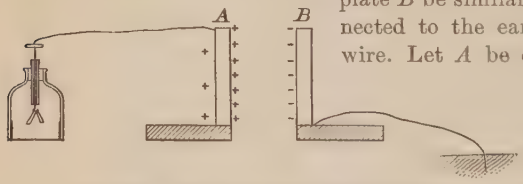


Fig. 252. The principle of the condenser

[1] If the case is of glass it should always be made conducting by pasting tin foil strips on the inside of the jar opposite the leaves and extending these strips over the edge of the jar and down on the outside to the conducting support on which the electroscope rests. The object of this is to maintain the walls always at the potential of the earth.

We say, therefore, that the *capacity* of A for holding electricity has been very greatly increased by bringing near it another conductor which is connected to earth. It is evident from this statement that *we measure the capacity of a body by the amount of electricity which must be put upon it to raise it to a given potential.* The explanation of the increase in capacity in this case is obvious. As soon as B was brought near to A it became charged, by induction, with electricity of opposite sign to A, the electricity of like sign to A being driven off to earth through the connecting wire. The attraction between these opposite charges on A and B drew the electricity on A to the face nearest to B and removed it from the more remote parts of A, so that it became possible to put a very much larger charge on A before the tendency of the electricity on A to pass over to the electroscope became as great as it was at first; i.e. before the potential of A rose to its initial value. In such a condition the electricity on A is said to be "bound" by the opposite electricity on B. An arrangement of this sort consisting of two conductors separated by a nonconductor is called a *condenser.* If the conducting plates are very close together and one of them grounded, the capacity of the system may be thousands of times as great as that of one of the plates alone. The most common form of condenser is a glass jar coated part way to the top inside and outside with tin foil (Fig. 253). The inside coating is connected by a chain to the knob, while the outside coating is connected to earth. Condensers of this sort first came into use in Leyden, Holland, in 1745. Hence they are now called *Leyden jars.*

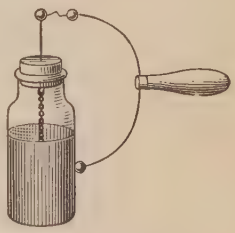


FIG. 253. The Leyden jar

345. Method of using a Leyden jar. To charge a Leyden jar the outer coating is held in the hand while the knob is brought into contact with one terminal of an electrical machine. As fast as electricity

passes to the knob it spreads to the inner coat of the jar where it attracts electricity of the opposite kind from the earth to the outer coat, repelling from it electricity of the same kind. If the inner and outer coatings are now connected by a discharging rod, as in Fig. 253, a powerful spark will be produced. Let a charged jar be placed on a glass plate so as to insulate the outer coat. Let the knob be touched with the finger. No appreciable discharge will be noticed. Let the outer coat be in turn touched with the finger. Again no appreciable discharge will appear. But if the inner and outer coatings are connected with the discharger, a powerful spark will pass.

The experiment shows that it is impossible to discharge one side of the jar alone. Touching the knob of the insulated jar will, indeed, draw off a very slight spark, but the greater amount of the charge is bound by the opposite charge on the outer coat. The full discharge can therefore occur only when the inner and outer coats are connected.

Electrical Generators

346. The electrophorus. Fig. 254 represents an *electrophorus*, an instrument invented by Volta in 1777, and one which illustrates well the principle underlying the action of all electrostatic machines. All such machines generate electricity by *induction*, not by friction. Let the ebonite plate *B* be rubbed with cat's fur or flannel. It will thus receive a negative charge. Let the metal plate *A* be placed upon *B* and touched with the finger. Then let the finger be removed and *A* lifted by the insulating handle away from *B*. If the weather is dry it will probably be found that a spark a quarter of an inch long may now be drawn from *A*. If *A* is replaced upon *B*, touched with the finger, and again withdrawn, another spark of equal length may be obtained. The process may be repeated an indefinite

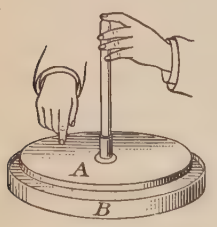


Fig. 254. The electrophorus

number of times without producing any apparent diminution in the size of the spark which may be taken from A.

If the charged plate A is brought near a charged electroscope, it will be found that it carries a positive charge. This proves that it has been charged by induction, and not by contact with B, for it will be remembered that the latter is charged negatively. The reason for this is that even when A rests upon B it is in reality separated from it, in all but a very few points, by an insulating layer of air; and since B is a nonconductor, its charge cannot pass off appreciably through these few points of contact. Hence only a negligible fraction of the surface of A is discharged each time that B is placed upon it.

347. Source of the energy of the charge obtained from an electrophorus. Although an indefinite amount of electricity may be developed by an electrophorus and stored up, for example, in a Leyden jar, without diminishing at all the charge on B, yet this electrical energy does not come into existence without the expenditure of a corresponding amount of mechanical work; for each time that the plate A is removed from B it must be lifted against the attractions of the opposite charges on A and B (see Fig. 255).

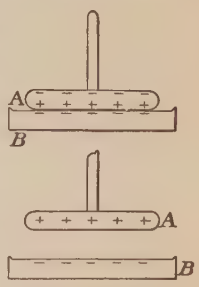


Fig. 255. Illustrating action of electrophorus

According to the principle of the conservation of energy, the energy in one charge on the plate A is then exactly equal to the work done against these electrical attractions in lifting the plate. This electrical energy is transformed into heat energy in the spark which is produced by the discharge.

348. The Toepler-Holtz electrical machine. The ordinary static machine is nothing but a continuously acting electrophorus. Fig. 256 represents the so-called Toepler-Holtz type of such a machine. Upon the back of the stationary plate E are pasted paper sectors, beneath which are strips of tin foil AB and CD, called *inductors*. In front of E

is a revolving glass plate carrying disks l, m, n, o, p, and q, called *carriers*. To the inductors AB and CD are fastened metal arms t and u which bring C and D into electrical contact with the disks l, m, n, o, p, and q, when these disks pass beneath the tinsel brushes carried by t and u. A stationary metallic rod rs carries at its ends stationary brushes as well as sharp-pointed, metallic combs. The two knobs R and S have their capacity increased by the Leyden jars L and L'.

349. Action of the Toepler-Holtz machine. The action of the machine described above is best understood from the diagram of Fig. 257. Suppose that a small + charge is originally placed on the inductor CD. Induction takes place in the metallic system consisting of the disks l and o and the rod rs, l becoming negatively charged and o positively charged. As the plate carrying l, m, n, o, p, q rotates in the direction

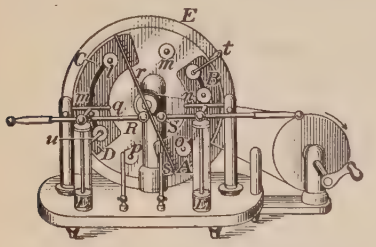


Fig. 256. Toepler-Holtz induction machine

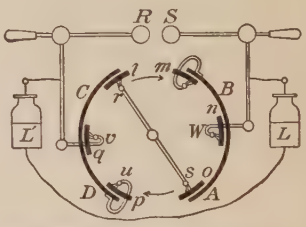


Fig. 257. Principle of Toepler-Holtz machine

of the arrow the negative charge on l is carried over to the position m, where a part of it passes over to the inductor AB, thus charging it negatively. When l reaches the position n the remainder of its charge, being repelled by the negative which is now on AB, passes over into the Leyden jar L. When l reaches the position o it again becomes charged by induction, this time positively, and more strongly than at first, since now the negative on AB, as well as the positive on CD, is acting inductively upon the rod rs. When l reaches the position u, a part of its now strong positive charge passes to CD, thus increasing the positive charge upon this inductor. In the position v the remainder of the positive charge on l passes over to L'. This completes the cycle for l. Thus as the rotation continues AB and CD acquire stronger and stronger charges, the inductive action upon rs becomes more and more intense, and positive and negative charges are continuously imparted to L' and L until a discharge takes place between the knobs R and S.

There is usually sufficient charge on one of the inductors to start the machine, but in damp weather it will often be found necessary to apply a charge to one of the inductors by means of the ebonite or glass rod before the machine will work.

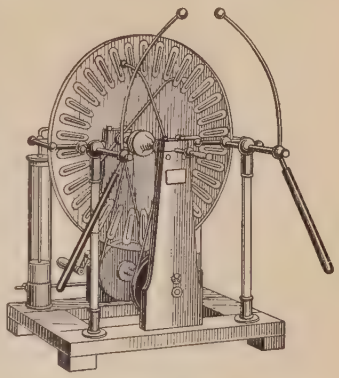


Fig. 258. The Wimshurst induction machine

350. The Wimshurst electrical machine. The essential difference between the Toepler-Holtz (Fig. 256) and the Wimshurst electrical machine (Fig. 258) is that the latter has two plates revolving in opposite directions and that these plates carry a large number of tin-foil strips which act alternately as inductors and as carriers, thus dispensing with the necessity of separate inductors. The action of the machine may be understood readily from Fig. 259. Suppose that a small negative charge is placed on a. This, acting inductively on the rod rs, charges a' positively. When a' in the course of the rotation reaches the position b' it acts inductively upon the rod $s'r'$ and thus charges the disk b negatively. It will be seen that henceforth all the disks in the inner circle receive + charges as they pass the brush r, and that all the disks in the outer circle, i.e. on the back plate, receive − charges as they pass the brush s'. Similarly, on the lower half of the plates all the disks on the inner circle receive − charges as they pass the brush s, and all the disks on the outer circle receive + charges as they pass the brush r'.

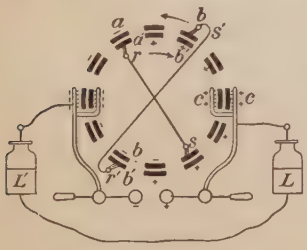


Fig. 259. Principle of Wimshurst machine

When the positive charges on the inner disks come opposite the combs c they pass off to the + knob of the machine or to the Leyden jar connected with it. The same process is occurring on the other side, where − charges are being taken off. When a spark passes, the Leyden jars and the connecting system of conductors are restored to their initial conditions and the process begins again.

QUESTIONS AND PROBLEMS

1. If the Leyden jar of Fig. 260 is charged, the clapper suspended by a silk thread between the two bells will bound back and forth for a long time, thus ringing the bells. Explain.

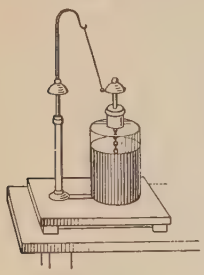


Fig. 260

2. Will a solid sphere hold a larger charge of electricity than a hollow one of the same diameter?

3. How would you proceed to protect an electroscope from the influences of an electrical machine which was in operation near it?

4. The potential of a body A is $+12$ and of a body B is -8. How much work will be required to carry three positive units of electricity from B to A?

5. When a negatively electrified cloud passes over a house provided with a lightning rod, the rod discharges positive electricity into the cloud. Explain.

6. Why is the capacity of a conductor greater when another conductor connected to the earth is near it than when it stands alone?

7. A Leyden jar is placed on a glass plate and 10 units of electricity placed on the inner coating. The knob is then connected to a gold-leaf electroscope. Will the leaves of the electroscope stand farther apart now or after the outside coating has been connected to the earth?

8. Why cannot a Leyden jar be appreciably charged if the outer coat is insulated?

9. Why is it not necessary to connect the outer coatings of the Leyden jars on an electrical machine to earth to charge them fully, provided they are connected to one another?

CHAPTER XIV

ELECTRICITY IN MOTION[1]

Detection and Measurement of Electric Currents

351. Electricity in motion produces a magnetic effect. Let a powerfully charged Leyden jar be discharged through a coil which surrounds an unmagnetized knitting needle in the manner shown in Fig. 261. After the discharge the needle will be found to be distinctly magnetized. If the sign of the charge on the jar is reversed, the poles will be reversed.

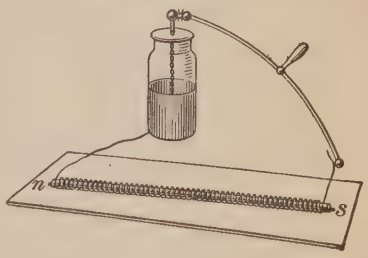


Fig. 261. Magnetizing effect of spark on knitting needle

The experiment shows that there is some connection between electricity and magnetism. Just what this connection is we do not yet know with certainty, but we do know that magnetic effects are always observable near the path of a moving electrical charge, while no such effects can ever be observed near a charge at rest.

To prove that a charge at rest does not produce a magnetic effect, let a charged body be brought near a compass needle. It will attract either end of the needle with equal readiness. While the needle is deflected insert between it and the charge a sheet of zinc, aluminium, brass, or copper. This will act as an electric screen (see § 339, p. 250) and will therefore cut off all effect of the charge. The compass needle will at once swing back to its north and south position.

[1] This chapter should be accompanied or, better, *preceded* by laboratory experiments on the simple cell and on the magnetic effects of a current. See, for example, Experiments 28, 29, and 30, of the authors' manual.

Let the compass needle be deflected by a bar magnet and let the screen be inserted again. The sheet of metal does not cut off the magnetic forces in the slightest degree.

The fact that an electric charge exerts no magnetic force is shown, then, both by the fact that it attracts either end of the compass needle with equal readiness, and also by the fact that the screen cuts off its action completely, while the same screen does not have any effect in cutting off the magnetic force.

An electrical charge in motion is called an electric current, and its presence is most commonly detected by the magnetic effect which it produces.

352. The galvanic cell. When a Leyden jar is discharged, but a very small quantity of electricity passes through the connecting wires, since the current lasts for but a small fraction of a second. If we could keep a current flowing continuously through the wire, we should expect the magnetic effect to be much more pronounced. It was in 1786 that Galvani, an Italian anatomist at the University of Bologna, accidentally discovered that there is a chemical method for producing such a continuous current. His discovery was not understood, however, until Volta, while endeavoring to throw light upon it, in 1800 invented an arrangement which is now known sometimes as the *voltaic* and sometimes as the *galvanic* cell. This consists, in its simplest form, of a strip of copper and a strip of zinc immersed in dilute sulphuric acid (Fig. 262).

Fig. 262. Simple voltaic cell

Let the terminals of such a cell be connected for a few seconds to the ends of the coil of Fig. 261 when an unmagnetized needle lies within the glass tube. The needle will be found to have become magnetized much more strongly than before. Again, let the wire which connects the terminals of the cell be held above a magnetic needle, as in Fig. 263; the needle will be strongly deflected.

Evidently, then, the wire which connects the terminals of a galvanic cell carries a current of electricity. Historically the

COUNT ALESSANDRO VOLTA (1745–1827)

Great Italian physicist; professor at Como and at Pavia; inventor of the electroscope, the electrophorus, the condenser, and the voltaic pile (a form of galvanic cell); first measured the potential differences arising from the contact of dissimilar substances; ennobled by Napoleon for his scientific services; the volt, the practical unit of potential difference, is named in his honor.

second of these experiments, performed by the Danish physicist Oersted in 1819, preceded the discovery of the magnetizing effects of currents upon needles. It created a great deal of excitement at the time, because it was the first clew which had been found to a relationship between electricity and magnetism.

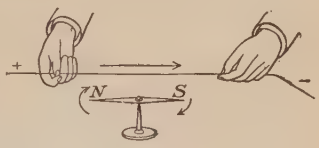


Fig. 263. Oersted's experiment

353. Plates of a galvanic cell are electrically charged. Since an electric current flows through a wire as soon as it is touched to the zinc and copper strips of a galvanic cell, we at once infer that the terminals of such a cell are electrically charged before they are connected. That this is indeed the case may be shown as follows.

Let a metal plate A (Fig. 264), covered with shellac on its lower side and provided with an insulating handle, be placed upon a similar plate B which is in contact with the knob of an electroscope. Let the copper plate of a galvanic cell be connected with A and the zinc plate with B, as in Fig. 264. Then let the connecting wires be removed and the plate A lifted away from B. The opposite electrical charges which were bound by their mutual attractions to the adjacent faces of A and B so long as these faces were separated only by the thin coat of shellac, are freed as soon as A is lifted; and hence part of the charge on B passes to the leaves of the electroscope. These leaves will indeed be seen to diverge. If an ebonite rod which has been rubbed with flannel or cat's fur is brought near the electroscope, the leaves will diverge still farther, thus showing that the zinc plate of the galvanic cell is negatively charged.[1] If the experiment is repeated with the copper plate in contact with B, and the zinc in contact with A, the leaves will be found to be positively charged.

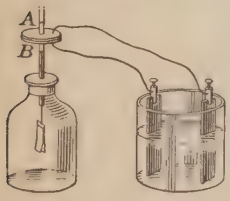


Fig. 264. Showing charges on plates of a voltaic cell

[1] If the deflection of the gold leaves is too small for purposes of demonstration, let a battery of from five to ten cells be used instead of the single cell. However, if the plates A and B are three or four inches in diameter, and if their surfaces are very flat, a single cell is sufficient.

The terminals of a galvanic cell therefore carry positive and negative charges just as do the terminals of an electrical machine in operation. The + charge is always found upon the copper and the − charge upon the zinc. The source of these charges is the chemical action which takes place within the cell. When these terminals are connected by a conductor a current flows through the latter from positive to negative (see § 341), i.e. from copper to zinc, just as in the case of the electrical machine.

354. Comparison of a galvanic cell and static machine. If one of the terminals of a galvanic cell is touched directly to the knob of a gold-leaf electroscope, without the use of the condenser plates *A* and *B* of Fig. 264, no divergence of the leaves will be detected. But if one knob of the static machine in operation were so touched, the leaves would probably be torn apart by the violence of the divergence. Since we have seen in § 343 that the divergence of the gold leaves is a measure of the potential of the body to which they are connected, we learn from this experiment that the chemical actions in the galvanic cell are able to produce between its terminals but a very small potential difference in comparison with that produced by the static machine between its terminals. As a matter of fact the potential difference between the terminals of the cell is about one volt, while that between the knobs of the electrical machine may be as much as 1,000,000 volts. (This corresponds to about a 13-inch spark.)

But if the knobs of the static machine are connected to the ends of the wire of Fig. 263, and the machine operated, the current sent through the wire will not be large enough to produce any appreciable effect upon the needle. Since under these same circumstances the galvanic cell produced a very large effect upon the needle, we learn that although the cell develops a very small P.D. between its terminals, it nevertheless sends through the connecting wire very much more electricity per second than the static machine is able to send. This is because the

chemical action of the cell is able to recharge the plates to their small P.D. practically as fast as they are discharged through the wire, whereas the static machine requires a relatively long time to recharge its terminals to their high P.D. after they have been once discharged.

Fig. 265. Behavior of a compass needle near a current

355. Shape of magnetic field about a current. Right-hand rule. Let the terminals of a galvanic cell be connected with a vertical wire and the compass needle held in the positions 1, 2, 3, 4, 5, etc., of Fig. 265. It will be found that the needle always sets itself tangent to the circumference of a circle whose center is the wire and whose plane is perpendicular to the wire.

Since we have already seen that a compass needle sets itself parallel to the magnetic lines of force, we learn from this experiment that an electric current is surrounded by a magnetic field whose lines of force are concentric circles about the current.

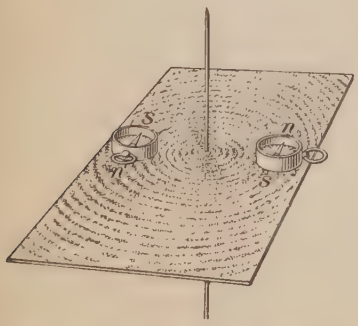


Fig. 266. Magnetic field about a current

If a heavy current (10 or 15 amperes) is available, the experiment may be made more striking by passing the wire through a piece of cardboard and sprinkling iron filings over the board. When the board is gently tapped the filings will be seen to arrange themselves in concentric circles about the wire (Fig. 266).

If the direction of the current is reversed by means of a pole changer, or by simply inverting the wire, the direction in which the compass needle points will be reversed also, thus showing that there is a definite relation between the direction in which the current flows through the wire and the direction in which the magnetic lines of force encircle it.

The following rule will give this relation in all cases.

If the right hand grasps the wire, as in Fig. 267, so that the thumb points in the direction in which the positive electricity is flowing, then the magnetic lines of force encircle the wire in the same direction as do the fingers of the hand. This rule was first stated in 1822, in slightly different form, by the great French physicist, André Marie Ampère (1775–1836), who began a careful study of the relation between electricity and magnetism as soon as he heard of Oersted's experiment. The rule is now known as the "right-hand rule," or sometimes as "Ampère's rule."

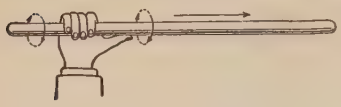


Fig. 267. The right-hand rule

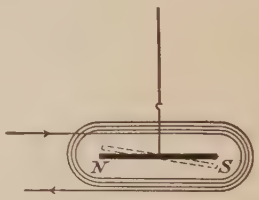


Fig. 268. Simple suspended-needle galvanometer

356. Galvanometers. Let the terminals of a simple cell be connected to the ends of a coil which passes around a magnetic needle as in Fig. 268. The needle will be deflected more strongly than in the preceding experiment in which a single wire was used, for in accordance with the right-hand rule both the upper and lower portions of the coil tend to make the needle turn in the same direction. It will come to rest in a position at right angles to the plane of the coil.

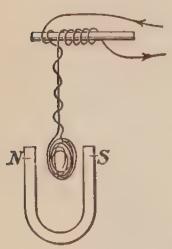


Fig. 269. Simple suspended-coil galvanometer

Again, let a coil of say 200 turns of No. 30 copper wire be suspended between the poles of a magnet NS, as in Fig. 269. The suspending wires should be of No. 40 copper wire. As soon as the current from the galvanic cell is sent through the coil it will turn so as to set itself at right angles to the line connecting N and S, i.e. at right angles to the lines of force of the magnet. The only essential difference between the two experiments is that in the first the coil is fixed and the magnet free to turn, while in the second the magnet is fixed and the coil is free to turn. In both cases the passage of the current tends to cause the suspended system to rotate through 90°.

André Marie Ampère (1775–1836)

French physicist and mathematician; son of one of the victims of the guillotine in 1793; professor at the Polytechnic School in Paris and later at the College of France; began his experiments on electro-magnetism in 1820, one year after Œrsted's discovery; published his great memoir on the magnetic effects of currents in 1823; the ampere, the practical unit of electric current, is named in his honor.

These two experiments illustrate the principle underlying nearly all current-measuring instruments or *galvanometers.* Instruments of the suspended-coil type are the more common and the more convenient. They are usually called D'Arsonval galvanometers. Fig. 270 represents one form of such a galvanometer.

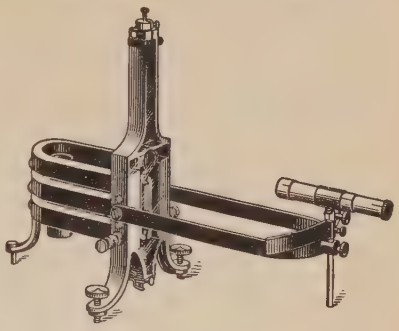


Fig. 270. The D'Arsonval galvanometer

357. The unit of current — the ampere. Ampère was the first investigator who made quantitative measurements on continuous currents by means of their magnetic effects. Hence the practical unit of current is named in honor of him, the *ampere.* It may be defined as the current which, when flowing through a circular coil of 100 turns and 10 cm. radius, will produce at its center a magnetic field of strength equal to 2π dynes. Fig. 271 shows the shape of the magnetic field about such a coil. The ampere is approximately the same as the current which, flowing through a circular coil of 3 turns and 10 cm. radius, set in a north-and-south plane, will produce a deflection of 45° in a small compass needle placed at its center as in Fig. 271.

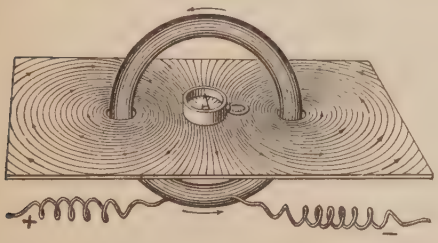


Fig. 271. Magnetic field about a circular coil carrying a current

358. The ammeter. It will be clear from the definition of the last paragraph that in order to measure an electrical current in amperes it is only necessary to pass it through a circular coil, like that of Fig. 271,

of a known number of turns and known radius, and to observe the deflection of the compass needle at the center. If the scale beneath the compass needle has been graduated, once for all, so as to read amperes directly, then the instrument is called an *ammeter*. Most commercial ammeters, however, are ammeters of the movable-coil rather than of the movable-needle type. Fig. 272 shows the form of the Weston ammeter. It consists simply of a coil C (Fig. 273), which is pivoted on jewel bearings and held in place between the poles MM' of a strong magnet by means of a spiral spring S. When a current is sent through the coil it tends to turn so as to set itself at right angles to the line connecting the poles MM'. Hence the pointer p moves over the scale. Such an instrument may be calibrated by sending the same current simultaneously through it and through another standard instrument like that of Fig. 271.

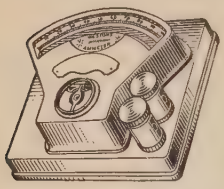


Fig. 272. The Weston ammeter

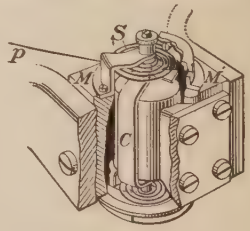


Fig. 273. Coil and magnets of Weston ammeter

QUESTIONS AND PROBLEMS

1. Under what conditions will an electric charge produce a magnetic effect?

2. In what direction will the north pole of a magnetic needle be deflected if it is held above a current flowing from north to south?

3. A man stands beneath a north-and-south trolley line and finds that a magnetic needle in his hand has its north pole deflected toward the east. What is the direction of the current flowing in the wire?

4. A loop of wire lying on the table carries a current which flows around it in clockwise direction. Would a north magnetic pole at the center of the loop tend to move up or down?

5. When a compass needle is placed, as in Fig. 271, at the middle of a coil of wire which lies in a north-and-south plane, the deflection produced in the needle by a current sent through the coil is approximately proportional to the strength of the current, provided the deflection is small — not more, for example, than 20° or 25°; but when the deflection becomes large — say 60° or 70° — it increases very much more slowly than does the current which produces it. Can you see any reason why this should be so?

Measurement of Potential Difference[1]

359. Measurement of P. D. with galvanometers. Voltmeters. The only methods which we have thus far considered for the determination of P.D. have consisted in (1) the measurement of spark lengths, and (2) the observation of the divergence of the gold leaves in an electroscope (§ 343). These methods are not suitable, however, for the measurement of the very small P.D. developed by a galvanic cell. Hence it is customary to use for this purpose so-called high-resistance galvanometers, which, if calibrated so as to read volts directly, are known as *voltmeters.* These instruments are precisely like the galvanometers just described, except that the coil consists of a very large number of turns (usually several thousand) of very fine wire, and hence is capable of carrying but a small current of electricity.

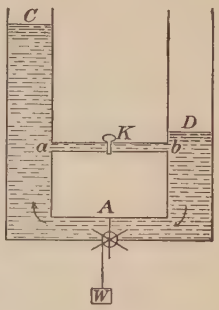


Fig. 274. Hydrostatic analogy of a galvanic cell

The reason why galvanometers which are to be used as voltmeters must be provided with coils which will carry very little current will be apparent from the following water analogy. If the stopcock K (Fig. 274) in the pipe connecting the water tanks C and D is closed, and if the water wheel A is set into motion by applying a weight W, the wheel will turn until it creates such a difference in the water levels between C and D that the back pressure against the left face of the wheel stops it and brings the weight W to rest. In precisely the same way the chemical actions within a galvanic cell whose terminals are not joined (Fig. 275) develops positive and negative charges upon these terminals, i.e. creates a P.D. between them, until the back electrical pressure through the cell due to this P.D. is sufficient to put a stop to further chemical action.

Now suppose the water reservoirs (Fig. 274) are put into communication by opening the stopcock K. The difference in level will at once

[1] This subject should be either preceded or accompanied by a laboratory experiment on electromotive forces. See, for example, Experiment 31 of the authors' manual.

begin to fall and the wheel will begin to build it up again. But if the carrying capacity of the pipe *ab* is small in comparison with the capacity of the wheel to remove water from *D* and supply it to *C*, then the difference of level which permanently exists between *C* and *D* when *K* is open will not be appreciably smaller than when it is closed. In this case the current which flows through *ab* may be taken as a measure of the difference in pressure which the pump is able to maintain between *C* and *D* when *K* is closed.

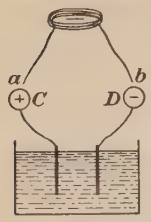


Fig. 275. Measurement of P.D. between terminals of galvanic cell

In precisely the same way, if the terminals *C* and *D* of the cell (Fig. 275) are connected by touching to them the terminals *a* and *b* of any conductor, they at once begin to discharge through this conductor, and their P.D. therefore begins to fall. But if the chemical action in the cell is able to recharge *C* and *D* very rapidly in comparison with the ability of the wire to discharge them, then the P.D between *C* and *D* will not be appreciably lowered by the presence of the connecting conductor. In this case the current which flows through the conducting coil, and therefore the deflection of the needle at its center, may be taken as a measure of the electrical pressure developed by the cell, i.e. of the P.D. between its unconnected terminals.

There is a very simple way of determining experimentally whether or not touching the ends of any particular galvanometer to the terminals of a cell does appreciably lower their P.D.

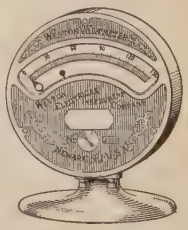


Fig. 276. Lecture-table voltmeter

Let the galvanometer in question[1] be connected directly to the terminals of the cell and the deflection noted; then let the ends of a second coil of wire which has exactly the same carrying capacity as the galvanometer coil be also touched to the

[1] A vertical lecture-table voltmeter (Fig. 276) and a similar ammeter are desirable for this and some of the following experiments, but homemade high- and low-resistance galvanometers, like those described in the authors' manual, are thoroughly satisfactory, save for the fact that one student must take the readings for the class.

terminals, the galvanometer coil being still in circuit. If the second coil has sufficient carrying capacity to appreciably discharge the terminals, the deflection of the needle of the galvanometer will be instantly diminished when the ends of the second coil are brought into contact with them. If no such diminution is observed, we may know that the second coil does not discharge the terminals of the cell fast enough to appreciably lower their P.D., and hence that the introduction of the first coil, which was of equal carrying capacity, also did not appreciably lower the P.D. between the terminals. To show that a coil of greater carrying capacity will at once lower the P.D. between C and D as soon as it is touched across them, let any coil of thicker wire be so touched. The deflection of the needle will be diminished instantly.

360. Electromotive force. The P.D. which a cell is able to maintain between its terminals when the circuit is completely broken, i.e. the total electrical pressure which it is capable of exerting, is called its *electromotive force,* — commonly abbreviated to E.M.F. *The E.M.F. of any electrical generator may then be defined as its power of producing electrical pressure,* or P.D. The E.M.F. of a cell consisting of zinc and copper strips in sulphuric acid is approximately 1 volt. The seat of this E.M.F. is at the surfaces of contact of the metals with the acid, where the chemical actions take place. The E.M.F. of the cell has its water analogy in the wheel A of Fig. 274, for it is here that the force is applied which creates the differences in hydrostatic pressure in the various parts of the water circuit.

361. E.M.F. independent of size and shape of plates. Let a voltmeter, or any high-resistance galvanometer, be connected to the terminals of a simple cell and the deflection noted. This deflection measures the E.M.F. of the cell. Then let the distance between the plates and the amount of immersion of the plates be changed through wide limits. The deflection will undergo no change whatever, thus showing that the E.M.F. of a cell is wholly independent of size or distance apart of the plates. Then let a carbon strip be substituted for the copper. The deflection will at once be increased. Again, let hydrochloric acid be substituted for sulphuric. The deflection will be diminished.

We learn, therefore, that *the E.M.F. of a cell depends simply upon the materials of which the cell is made, not at all upon the size or shape of the plates.*

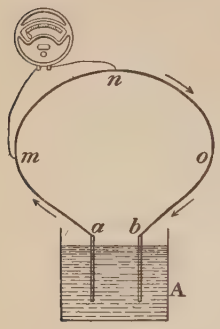


FIG. 277. Showing method of connecting voltmeter to find P.D. between any two points m and n on an electrical circuit

362. Fall of potential along a conductor carrying a current. Not only does a P.D. exist between the terminals of a cell on open circuit, but also between any two points on a conductor through which a current is passing. For example, in the electrical circuit shown in Fig. 277 the potential at the point a is higher than that at m, that at m higher than that at n, etc., just as in the water circuit, shown in Fig. 278, the hydrostatic pressure at a is greater than that at m, that at m greater than that at n, etc. The fall in the water pressure between m and n (Fig. 278) is measured by the water head $n's$. If we wish to measure the fall in electrical potential between m and n (Fig. 277), we touch the terminals of a voltmeter to these points in the manner shown in the figure. Its reading gives us at once the P.D. between m and n in volts, provided always that its own current-carrying capacity is so small that it does not appreciably lower the P.D. between the points m and n by being touched across them, i.e. provided the current which flows through it is negligible in comparison with that which flows through the conductor which already joins the points m and n.

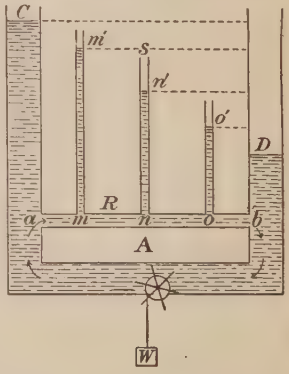


FIG. 278. Hydrostatic analogy of fall of potential in an electrical circuit

Measurement of Resistance[1]

363. Definition of resistance. If two vessels containing water at different levels are connected by a pipe, the rate at which the water will flow from one to the other because of the difference in the two levels, will evidently depend upon the length, diameter, and nature of the conducting pipe. In precisely the same way, when the terminals of a galvanic cell are connected by a conductor, the strength of the electric current which flows from one terminal to the other because of the E.M.F. of the cell, is found to depend upon the length, diameter, and material of the connecting wire. If now with a given P.D. between its terminals, one wire is found to carry twice as much current as another, the first wire is said to have twice the conductivity, or one half the resistance, of the second; i.e. *the resistances of various conductors are taken as inversely proportional to the currents which these conductors transmit when a given potential difference exists between their ends.*

364. Specific resistance. Let the circuit of a galvanic cell be connected through a lecture-table ammeter,[2] or any low-resistance galvanometer, and, for example, 20 feet of No. 30 copper wire, and let the deflection of the needle be noted. Then let the copper wire be replaced by an equal length of No. 30 German silver wire. The deflection will be found to be a very small fraction of what it was at first.

German silver wire, therefore, evidently has a much higher resistance than a copper wire of the same length and diameter. It is said, therefore, to have a higher *specific resistance* than copper. The following numbers represent the specific resistances of a number of metals in terms of silver as a standard; i.e. the numbers give the ratio of the resistance of a wire of any

[1] This subject should be accompanied and followed by laboratory experiments on Ohm's law, on the comparison of wire resistances, and on the measurement of internal resistances. See, for example, Experiments 32, 33, and 34, of the authors' manual.

[2] See note on p. 270.

metal to that of a silver wire of the same length and diameter. Silver is the best conductor of any known substance.

Silver	1.00	Soft iron . . .	7.40	German silver .	20.4
Copper. . . .	1.13	Nickel	7.87	Hard steel . .	21.0
Aluminium . .	2.00	Platinum . . .	9.00	Mercury . . .	62.7

365. Law of lengths and diameters. If the length of either of the wires in the last experiment had been increased, the current would have been found to have been decreased; but if a wire of the same length but of larger diameter had been substituted, the current would have been found to be greater. Careful experiments have shown that the resistances of conductors of the same material are *directly proportional to their lengths and inversely proportional to their cross sections;* i.e. doubling the length of a wire doubles its resistance, and doubling its diameter makes its resistance one fourth as great, since it makes the cross section four times as great.

366. Resistance and temperature. Let the circuit of a galvanic cell be closed through a very low resistance galvanometer and about 10 feet of No. 30 iron wire wrapped about a strip of asbestus. Let the deflection of the galvanometer be observed as the wire is heated in a Bunsen flame. As the temperature rises higher and higher the current will be found to fall continually.

The experiment shows that *the resistance of iron increases with rising temperature.* This is a general law which holds for all metals. In the case of liquid conductors, on the other hand, the resistance usually decreases with increasing temperature. Carbon and a few other solids show a similar behavior, the filament in an incandescent electric lamp having only about half the resistance when hot which it has when cold.

367. The unit of resistance — the ohm. A conductor which carries a current of one ampere when a P.D. of one volt is maintained between its terminals is said to have an electric resistance of one *ohm,* the unit having been named in honor of the great

German physicist, Georg Ohm (1789–1854), who first established the laws of electrical resistance. A column of mercury 106.3 cm. long and 1 sq. mm. in cross section has at 0° C. a resistance of exactly one ohm. A length of 9.35 ft. of No. 30 copper wire or 6.2 in. of No. 30 German silver wire has a resistance of one ohm. Copper wire of the size shown in Fig. 279 (No. 7) has a resistance of but 2.62 ohms per mile.

FIG. 279. No. 7 wire

368. Ohm's law. In 1827 Ohm announced the discovery that *the currents furnished by different galvanic cells or combinations of cells are always directly proportional to the E.M.F.'s existing in the circuits in which the currents flow, and inversely proportional to the total resistances of these circuits;* i.e. if C represents the current in amperes, E the E.M.F. in volts, and R the resistance of the circuit in ohms, then Ohm's law as applied to the complete circuit is

$$C = \frac{E}{R}; \text{ i.e. current} = \frac{\text{electromotive force}}{\text{resistance}}. \qquad (1)$$

As applied to any portion of an electrical circuit Ohm's law is

$$C = \frac{PD}{r}; \text{ i.e. current} = \frac{\text{potential difference}}{\text{resistance}}, \qquad (2)$$

where PD represents the difference of potential in volts between any two points in the circuit and r the resistance in ohms of the conductor connecting these two points. This is one of the most important laws in physics. The experimental demonstration of its correctness will be left to the laboratory. (See, for example, Experiment 32 of the authors' manual.)

Both of the above statements of Ohm's law are included in the equation

$$\text{amperes} = \frac{\text{volts}}{\text{ohms}}. \qquad (3)$$

369. Internal resistance. Let the zinc and copper plates of a simple voltaic cell be connected to an ammeter or to a single-turn coil galvanometer, as shown in Fig. 280. Then let the distance between the plates be increased. The deflection of the needle will be found to decrease. Let the amount of immersion be decreased. This too will be found to decrease the current.

FIG. 280

Now since the E.M.F. of a cell was shown in § 361 to be wholly independent of the area of the plates immersed, or of the distance between them, it will be seen from Ohm's law that the change in the current must be due to some change in the total resistance of the circuit. Since the wire which constitutes the outside portion of the circuit remains the same, we conclude that *the liquid within the cell as well as the external wire offers resistance to the passage of the current,* and that this resistance of the liquid between the plates increases as the distance between the plates increases, and decreases as the area of the immersed portion of the plates increases.

In the algebraic statement of Ohm's law given above, i.e. $C = E/R$, R represents the total resistance of the electrical circuit, i.e. the resistance of the liquid between the plates as well as that of the outside wire. The resistance of the liquid is usually called the *internal resistance* of the cell, and the resistance of the wire the *external resistance.* If, then, we represent the external resistance by R_e and the internal resistance by R_i, Ohm's law as applied to a complete circuit takes the form

$$C = \frac{E}{R_e + R_i}. \tag{4}$$

Thus, if a simple cell has an internal resistance of 2 ohms and an E.M.F. of 1 volt, the current which will flow through the circuit when its terminals are connected by 9.3 ft. of No. 30 copper wire (1 ohm) is $\frac{1}{1+2} = .33$ amperes.

370. Measurement of internal resistance. A simple and direct method of finding a length of wire which has a resistance equivalent to the internal resistance of a cell is to connect the cell first to an ammeter or any galvanometer of negligible resistance[1] and then to introduce enough German silver wire into the circuit to reduce the galvanometer reading to half its original value. The internal resistance of the cell is then the same as that of the German silver wire; for, since the E.M.F. has remained unchanged, by Ohm's law the total resistance of the circuit must have been doubled when the current was halved. A still easier method in case both an ammeter and a voltmeter are available is to divide the E.M.F. of the cell as given by the voltmeter by the current which the cell is able to send through the ammeter when connected directly to its terminals; for in this case R_e of equation (4) is 0; therefore $R_i = \frac{E}{C}$. This gives the internal resistance directly in ohms.

371. Measurement of any resistance by ammeter-voltmeter method. The simplest way of measuring the resistance of a wire, or in general of any conductor, is to connect it into the circuit of a galvanic cell in the manner shown in Fig. 281. The ammeter A is inserted to measure the current, and the voltmeter V to measure the P.D. between the ends a and b of the wire r, the resistance of which is sought. The resistance of r in ohms is obtained at once from the ammeter and voltmeter readings with the aid of the law, $C = \frac{\text{P.D.}}{r}$, from which it follows that $r = \frac{\text{P.D.}}{C}$. Thus, if the voltmeter indicates .4 volts and the ammeter .5 amperes, the resistance of r is $\frac{.4}{.5} = .8$ ohms.

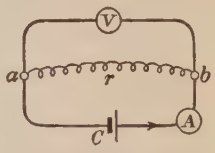


FIG. 281. Ammeter-voltmeter method of measuring resistance

372. Measurement of resistance by a high-resistance galvanometer. If an ammeter and a voltmeter are not available, an unknown resistance may be found in the following way. The unknown resistance r is connected into the circuit of a galvanic cell C through a suitable length bd

[1] A lecture-table ammeter is best, but see note on page 270.

of No. 30 German silver wire, or any other wire of known resistance per foot, in the manner shown in Fig. 282. The terminals of the high-resistance galvanometer are placed across the ends ab of the unknown resistance r, and the deflection noted. Then one end of the galvanometer is touched at b and the other end is moved along the German silver wire to some point c at which the galvanometer shows the same deflection as when touched across ab. The difference of potential between the points a and b must then be the same as that between the points b and c. The current flowing from a to b is also the same as that flowing from b to c, since the unknown resistance and the German silver wire are parts of the same circuit. Hence, by Ohm's law, $r = \text{P.D.}/C$, the unknown resistance r is equal to the resistance of the German silver wire between b and c. The resistance of r in ohms may be found by dividing the length of bc by such a length of this wire as has a resistance of 1 ohm.[1]

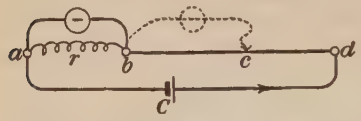


Fig. 282. High-resistance galvanometer method of measuring resistance

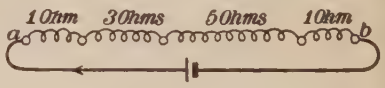


Fig. 283. Series connections

373. Joint resistance of conductors connected in series and in parallel. When resistances are connected as in Fig. 283, so that the same current flows through each of them in succession, they are said to be connected in series. The total resistance of a number of conductors so connected is the sum of the several resistances. Thus, in the case shown in the figure, the total resistance between a and b is 10 ohms.

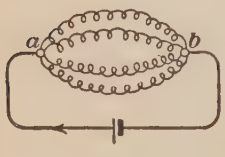


Fig. 284. Parallel connections

When n exactly similar conductors are joined in parallel, i.e. in the manner shown in Fig. 284, the total resistance between a and b is $\frac{1}{n}$ of the resistance of one of them; for, obviously, with a given P.D. between the points a

[1] The Wheatstone's bridge method of measuring resistances is recommended for laboratory study. See, for example, Experiment 33 of the authors' manual.

and b, four conductors will carry four times as much current as one, and n conductors will carry n times as much current as one. Therefore the resistance, which is inversely proportional to the carrying capacity (see § 363), is $\frac{1}{n}$ as much as that of one.

374. Shunts. A wire connected in parallel with another wire is said to be a *shunt* to that wire. Thus the conductor X (Fig. 285) is said to be shunted across the resistance R. Under such conditions the currents carried by R and X will be inversely proportional to their resistances, so that, if X is 1 ohm and R 10 ohms, R will carry $\frac{1}{10}$ as much current as X, or $\frac{1}{11}$ of the whole current flowing from a to b.

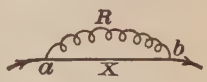


FIG. 285. A shunt

QUESTIONS AND PROBLEMS

1. If the potential difference between the terminals of a cell on open circuit is to be measured by means of a galvanometer, why must the galvanometer have a high resistance?

2. How long a piece of No. 30 copper wire will have the same resistance as a meter of No. 30 German silver wire?

3. The resistance of a certain piece of German silver wire is 1 ohm. What will be the resistance of another piece of the same length, but of twice the diameter?

4. The diameter of No. 20 wire is 31.96 mils (1 mil = .001 in.) and that of No. 30 wire 10.025 mils. Compare the resistances of equal lengths of No. 20 and No. 30 German silver wires.

5. What length of No. 30 copper wire will have the same resistance as 20 ft. of No. 20 copper wire?

6. What length of No. 20 German silver wire will have the same resistance as 100 ft. of No. 30 copper wire?

7. How much current will flow between two points whose P.D. is 2 volts, if they are connected by a wire having a resistance of 10 ohms?

8. What P.D. exists between the ends of a wire whose resistance is 100 ohms, when the wire is carrying a current of .3 amperes?

9. If a voltmeter attached across the terminals of an incandescent lamp shows a P.D. of 110 volts, while an ammeter connected in series with the lamp (see Fig. 281) indicates a current of .5 ampere, what is the resistance of the incandescent filament?

10. If a certain Daniell cell has an internal resistance of 2 ohms and an E.M.F. of 1.08 volts, what current will it send through an ammeter whose resistance is negligible? What current will it send through a copper wire of 2 ohms resistance? through a German silver wire of 100 ohms resistance?

11. A Daniell cell indicates a certain current when connected to a galvanometer of negligible resistance. When a piece of No. 20 German silver wire is inserted into the circuit, it is found to require a length of 5 ft. to reduce the current to one half its former value. Find the resistance of the cell in ohms, No. 20 German silver wire having a resistance of 190.2 ohms per 1000 ft.

12. A coil of unknown resistance is inserted in series with a considerable length of No. 20 German silver wire and joined to a Daniell cell. When the terminals of a high-resistance galvanometer are touched to the wire at points 10 ft. apart, the deflection is found to be the same as when they are touched across the terminals of the unknown resistance. What is the resistance of the unknown coil?

13. Find the joint resistance of 10 ft. of No. 30 copper wire and 1 ft. of No. 20 German silver wire connected in series.

14. Ten pieces of wire, each having a resistance of 5 ohms, are connected in parallel (see Fig. 284). If the junction a is connected to one terminal of a Daniell cell and b to the other, what is the total current which will flow through the circuit, when the E.M.F. of the cell is 1 volt and its resistance 1.5 ohms?

15. A voltmeter which has a resistance of 1000 ohms is shunted across the terminals A and B of a wire which has a resistance of 1 ohm. What fraction of the total current flowing from A to B will be carried by the voltmeter?

16. In a given circuit the P.D. across the terminals of a resistance of 19 ohms is found to be 3 volts. What is the P.D. across the terminals of a 3-ohm wire in the same circuit?

Primary Cells

375. Study of the action of a simple cell. If the simple cell already described, i.e. zinc and copper strips in dilute sulphuric acid, is carefully observed, it will be seen that, so long as the plates are not connected by a conductor, fine bubbles of gas are slowly formed at the zinc plate, but none at the copper plate. As soon, however, as the two strips are put into metallic connection, bubbles instantly appear in great numbers about the copper plate (Fig. 286), and at the same time a current manifests itself in the connecting wire. These are bubbles of hydrogen. Their appearance on the zinc may be prevented either

by using a plate of chemically pure zinc, or by amalgamating impure zinc, i.e. by coating it over with a thin film of mercury. But the bubbles on the copper cannot be thus disposed of. They are an invariable accompaniment of the current in the circuit. If the current is allowed to run for a considerable time, it will be found that the zinc wastes away, even though it has been amalgamated, but that the copper plate does not undergo any change.

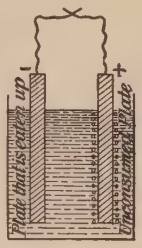


Fig. 286. Chemical actions in the voltaic cell

We learn, therefore, that the electrical current in the simple cell is accompanied with the eating up of the zinc plate by the liquid, and by the evolution of hydrogen bubbles at the copper plate. In every type of galvanic cell actions similar to these two are always found; i.e. *one of the plates is always eaten up, and upon the other plate some element is deposited.* The plate which is eaten is always the one which receives the negative charge, and the other the positive charge, so that in all galvanic cells, when the terminals are connected through a wire, the positive electricity flows through this wire from the uneaten plate to the eaten plate. It will be remembered that the direction in which the *positive* electricity flows is taken for convenience as the direction of the current (see § 341).

376. Local action and amalgamation. The cause of the appearance of the hydrogen bubbles at the surface of impure zinc when dipped in dilute sulphuric acid is that little electrical circuits are set up between the zinc and the small impurities in it, — carbon or iron particles, — in the manner indicated in Fig. 287. If the zinc is pure, these little local currents cannot, of course, be set up, and consequently no hydrogen bubbles appear. Amalgamation stops this so-called *local action*, because the mercury dissolves the zinc, while it does not dissolve the carbon, iron, or other impurities. The zinc-mercury amalgam formed is a homogeneous substance

Fig. 287
Local action

which spreads over the whole surface and covers up the impurities. It is important, therefore, to amalgamate the zinc in a battery in order to prevent the consumption of the zinc when the cell is on open circuit. The zinc is under all circumstances eaten up when the current is flowing, amalgamation serving only to prevent its consumption when the circuit is open.

377. Theory of the action of a simple cell. A simple cell may be made of any two dissimilar metals immersed in a solution of any acid or salt. For simplicity, let us examine the action of a cell composed of plates of zinc and copper immersed in a dilute solution of hydrochloric acid. The chemical formula for hydrochloric acid is HCl. This means that each molecule of the acid consists of one atom of hydrogen combined with one atom of chlorine. In accordance with the theory now in vogue among physicists and chemists, when hydrochloric acid is mixed with water so as to form a dilute solution, the HCl molecules split up into two electrically charged parts, called *ions*, the hydrogen ion carrying a positive charge and the chlorine ion an equal negative charge (Fig. 288). This phenomenon is known as *dissociation.* The solution as a whole is neutral; i.e. it is uncharged, because it contains just as many positive as negative ions.

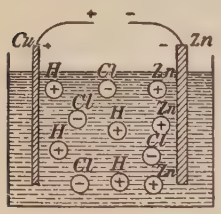


Fig. 288. Showing dissociation of hydrochloric acid molecules in water

When a zinc plate is placed in such a solution the acid attacks it and pulls zinc atoms into solution. Now whenever a metal dissolves in an acid, its atoms, for some unknown reason, go into solution bearing little positive charges. *The corresponding negative charges must be left on the zinc plate* in precisely the same way in which a negative charge is left on silk when positive electrification is produced on a glass rod by rubbing it with the silk. It is in this way, then, that we attempt to account for the

negative charge which we found upon the zinc plate in the experiment which was performed in connection with § 353.

The passage of positively charged zinc ions into solution gives a positive charge to the solution about the zinc plate, so that the hydrogen ions tend to be repelled toward the copper plate. When these repelled hydrogen ions reach the copper plate some of them give up their charges to it and then collect as bubbles of hydrogen gas. It is in this way that we account for the positive charge which we found on the copper plate in the experiment of § 353.

If the zinc and copper plates are not connected by an outside conductor, this passage of positively charged zinc ions into solution continues but a very short time, for the zinc soon becomes so strongly charged negatively that it pulls back on the + zinc ions with as much force as the acid is pulling them into solution. In precisely the same way the copper plate soon ceases to take up any more positive electricity from the hydrogen ions, since it soon acquires a large enough + charge to repel them from itself with a force equal to that with which they are being driven out of solution by the positively charged zinc ions. It is in this way that we account for the fact that on open circuit no chemical action goes on in the simple galvanic cell, the zinc and copper plates simply becoming charged to a definite difference of potential which is called the E.M.F. of the cell.

When, however, the copper and zinc plates are connected by a wire, a current at once flows from the copper to the zinc and the plates thus begin to lose their charges. This allows the acid to pull more zinc into solution at the zinc plate, and allows more hydrogen to go out of solution at the copper plate. These processes, therefore, go on continuously so long as the plates are connected. Hence a continuous current flows through the connecting wire until the zinc is all eaten up or the hydrogen ions have all been driven out of the solution, i.e. until either the plate or the acid has become exhausted.

378. Polarization. If the simple galvanic cell described be connected to a lecture-table ammeter through two or three feet of German silver wire, the deflection of the needle will be found to decrease slowly; but if the hydrogen is removed from the copper plate (this can be done completely only by removing and thoroughly drying the plate), the deflection will be found to return to its first value.

The experiment shows clearly that the observed falling off in current was due to the collection of hydrogen about the copper plate. This phenomenon of the weakening of the current from a galvanic cell is called the *polarization* of the cell.

379. Causes of polarization. The presence of the hydrogen bubbles on the positive plate causes a diminution in the strength of the current for two reasons. First, since hydrogen is a non-conductor, by collecting on the plate it diminishes the effective area of the plate and therefore increases the internal resistance of the cell. Second, the collection of the hydrogen about the copper plate lowers the E.M.F. of the cell, because it virtually substitutes a hydrogen plate for the copper plate, and we have already seen, in § 361, that a change in any of the materials of which a cell is composed changes its E.M.F. That there is a real fall in E.M.F. as well as a rise in internal resistance when a cell polarizes may be directly proved in the following way.

Let the deflection of a lecture-table voltmeter whose terminals are attached to the freshly cleaned plates of a simple cell be noted, then let the cell's terminals be short-circuited through a coarse wire for half a minute. As soon as the wire is removed the E.M.F., indicated by the voltmeter, will be found to be much lower than at first. It will, however, gradually creep back toward its old value as the hydrogen disappears from the plate, but a thorough cleaning and drying of the plate will be necessary to restore completely the original E.M.F.

The different forms of galvanic cells in common use differ chiefly in different devices employed either for disposing of the hydrogen bubbles or for preventing their formation. The most common types of such cells are described in the following sections.

380. The bichromate cell. The bichromate cell consists of a plate of zinc immersed in sulphuric acid between two plates of carbon, carbon being used instead of copper because it gives a greater E.M.F. In the sulphuric acid is dissolved some bichromate of potassium or of sodium, the function of which is to unite chemically with the hydrogen as fast as it is formed at the positive plate, thus preventing its accumulation upon this plate.[1] Such a cell has the high E.M.F. of 2.1 volts. Its internal resistance is low, from .2 to .5 ohms, since the plates are generally large and close together. It will be seen, therefore, that when the external resistance is very small it is capable of furnishing a current of from 5 to 10 amperes, since current is equal to E.M.F. divided by resistance. Since, however, the chromic acid formed by the union of the sulphuric acid with the bichromate attacks the zinc even when the circuit is open, it is necessary to lift the zinc from the liquid when the cell is not in use. In the Grenet form of this cell, shown in Fig. 289, the zinc is attached to a sliding rod *a*, by which it may be raised or lowered at will. Sometimes when several cells are used together the zincs and carbons are all attached to a frame, as in Fig. 290, so that they may all be lifted from the liquid at once. Such an arrangement is known as a *plunge battery*. Bichromate cells are useful where large currents are needed for a short space of time. The disadvantages are that the fluid deteriorates rapidly, and that the E.M.F. of the cell, despite the depolarizing influence of the bichromate, is far from constant.

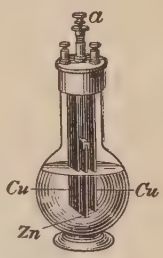


Fig. 289. The Grenet form of bichromate cell

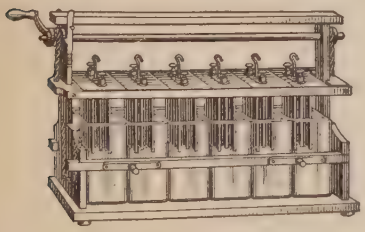


Fig. 290. A plunge battery

381. The Daniell cell. The Daniell cell consists of a zinc plate immersed in zinc sulphate and a copper plate immersed in copper sulphate, the two liquids being kept apart either by means

[1] To set up, dissolve 12 parts, by weight, of sodium bichromate in 180 parts of boiling water. After cooling add 25 parts of commercial sulphuric acid.

of a porous earthen cup, as in the type shown in Fig. 291,[1] or else by gravity, as in the type shown in Fig. 292. This last type, commonly called the gravity, or crowfoot type, is used almost exclusively on telegraph lines. The copper sulphate, being the heavier of the two liquids, remains at the bottom about the copper plate, while the zinc sulphate remains at the top about the zinc plate.

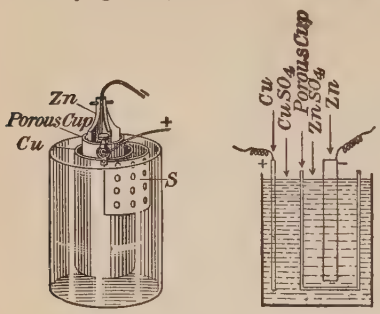


Fig. 291. The Daniell cell

In this cell polarization is almost entirely avoided, for the reason that no opportunity is given for the formation of hydrogen bubbles. For just as the hydrochloric acid solution described in § 377 consists of positive hydrogen ions and negative chlorine ions in water, so the zinc sulphate ($ZnSO_4$) solution consists of positive zinc ions and negative SO_4 ions, and the copper sulphate solution of positive copper ions and negative SO_4 ions. Now the zinc of the zinc plate goes into solution in the zinc sulphate in precisely the same way that it goes into solution in the hydrochloric acid of the simple cell described in § 377. This gives a positive charge to the solution about the zinc plate, and causes a movement of the positive ions between the two plates from the zinc towards the copper, and of negative ions in the opposite direction, both the Zn and the SO_4 ions being able to pass through the porous cup. Since the positive ions about the copper plate consist of atoms of copper, it will be seen that the material which is driven out of solution at the copper

Fig. 292. The gravity cell

[1] To set up, fill the battery jar with a saturated solution of copper sulphate. Fill the porous cup with water and add a handful of zinc sulphate crystals.

plate, instead of being hydrogen, as in the simple cell, is metallic copper. Since, then, the element which is deposited on the copper plate is the same as that of which it already consists, it is clear that neither the E.M.F. nor the resistance of the cell can be changed because of this deposit; i.e. the cause of the polarization of the simple cell has been removed.

The great advantage of the Daniell cell lies in the relatively high degree of constancy in its E.M.F. (1.08 volts). It has a comparatively high internal resistance (one to six ohms) and is therefore incapable of producing very large currents, about one ampere at most. It will furnish a very constant current, however, for a great length of time; in fact, until all of the copper is driven out of the copper sulphate solution. In order to keep a constant supply of the copper ions in the solution, copper sulphate crystals are kept in the compartment *S* of the cell of Fig. 291, or in the bottom of the gravity cell. These dissolve as fast as the solution loses its strength through the deposition of copper on the copper plate.

The Daniell is a so-called "closed-circuit" cell, i.e. its circuit should be left closed (through a resistance of thirty or forty ohms) whenever the cell is not in use. If it is left on open circuit, the copper sulphate diffuses through the porous cup and a brownish muddy deposit of copper or copper oxide is formed upon the zinc. Pure copper is also deposited in the pores of the porous cup. Both of these actions damage the cell. When the circuit is closed, however, since the electrical forces always keep the copper ions moving toward the copper plate, these damaging effects are to a large extent avoided.

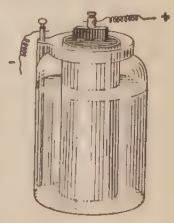


Fig. 293
The Leclanché cell

382. The Leclanché cell. The Leclanché cell (Fig. 293) consists of a zinc rod in a solution of ammonium chloride (150 g. to a liter of water), and a carbon plate placed inside of a porous cup which is packed full of manganese dioxide

and powdered graphite or carbon. As in the simple cell, the zinc dissolves in the liquid and hydrogen is liberated at the carbon, or positive, plate. Here it is slowly attacked by the manganese dioxide. This chemical action is, however, not quick enough to prevent rapid polarization when large currents are taken from the cell. The cell slowly recovers when allowed to stand for a while on open circuit. The E.M.F. of a Leclanché cell is about 1.5 volts and its initial internal resistance is somewhat less than an ohm. It therefore furnishes a momentary current of from one to three amperes. The immense advantage of this type of cell lies in the fact that it is entirely free from local action when on open circuit, and that, therefore, unlike the Daniell or bichromate cells, it can be left for an indefinite time on open circuit without deterioration. Leclanché cells are used almost exclusively where momentary currents only are needed, as, for example, on door-bell circuits. The cell requires no attention for years at a time other than the occasional addition of water to replace loss by evaporation, and the occasional addition of ammonium chloride (NH_4Cl) to keep positive NH_4 and negative Cl ions in the solution.

383. The dry cell. The dry cell is only a modified form of the Leclanché cell. It is not really "dry," since the zinc and carbon plates are imbedded in moist paste which consists usually of one part of crystals of ammonium chloride, three parts of plaster of Paris, one part of zinc oxide, one part of zinc chloride, and two parts of water. The plaster of Paris is used to give the paste rigidity. As in the Leclanché cell, it is the action of the ammonium chloride upon the zinc which produces the current.

384. Combinations of cells. From Ohm's law $C = \dfrac{E}{R_e + R_i}$ it will be seen that if it is desired to increase the current flowing through a circuit, it is necessary either to increase the E.M.F. of the circuit, i.e. the numerator of the fraction, or else to diminish the resistance of the circuit (internal or external), i.e. the denominator of the fraction. Both of these results may be accomplished by means of combinations of cells. There are two different ways in which cells may be connected.

1. The zinc of one cell may be joined to the copper of the second, the zinc of the second to the copper of a third, etc., the copper of the first and the zinc of the last being joined to

the ends of the external resistance. This method is illustrated in Fig. 294. Cells so joined are said to be connected *in series.*

2. All the zincs may be joined to one end of the external resistance and all the coppers to the other end. This method is illustrated in Fig. 295. Cells so joined are said to be joined *in parallel* or *in multiple.*

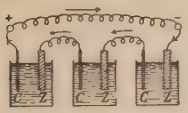


FIG. 294. Cells connected in series

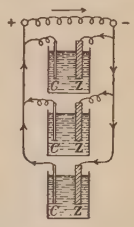


FIG. 295. Cells connected in parallel

385. Advantages of series connections. Let n cells be connected in series and joined to a voltmeter. Their joint E.M.F. will be found to be n times that of a single cell.

The water analogy of this condition is shown in Fig. 296, the difference in level between a and d being three times that between a and b. The internal resistance of the n cells is in this case also n times that of a single cell, since the current must flow through them all in succession, see § 373. Hence the current sent through any external resistance R_e by n cells, each of E.M.F. e and internal resistance R_i, is

$$C = \frac{ne}{R_e + nR_i}. \qquad (5)$$

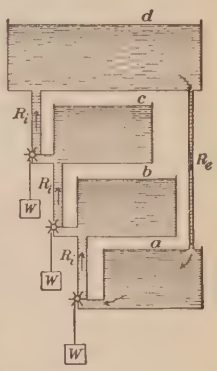


FIG. 296. Water analogy of cells in series

If R_e is very small in comparison with nR_i, e.g. if the external circuit is a piece of thick, short copper wire, no more current will be sent through it by n cells joined in series than would be sent through it by a single cell. If, however, R_e is very large, so that the term nR_i (5) may be neglected in comparison with R_e, then the current furnished is n times that furnished by a single cell. Hence *series connection*

should always be used when the external resistance is large in comparison with the internal resistance of a single cell.

386. Advantages of parallel connections. Let n cells be joined in parallel to a voltmeter. The E.M.F. of the combinations will be found to be precisely the same as the E.M.F. of a single cell.

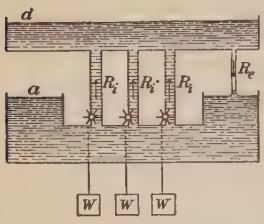


FIG. 297. Water analogy of cells in parallel

The water analogy of this condition is shown in Fig. 297, the difference in water level due to the three pumps being the same as that due to one. It is evident that in this case the joint internal resistance, however, is only $1/n$ that of a single cell, since there are n paths for the current through the cells. Hence, by Ohm's law, for such a combination

$$C = \frac{e}{R_e + \frac{R_i}{n}}. \tag{6}$$

If, therefore, R_e is negligibly small, as in the case of a heavy copper wire, the current flowing through it will be n times as great as that which could be made to flow through it by a single cell. If R_e is very large, the current will be no greater than that furnished by a single cell. *Hence cells should always be joined in parallel when the external resistance is small in comparison with the internal resistance of a single cell.*

QUESTIONS AND PROBLEMS

1. Why is a Daniell cell better than a bichromate cell for telegraphic purposes?

2. Why is a Leclanché cell better than a Daniell cell for ringing door bells?

3. If the internal resistance of a Daniell cell of the gravity type is 4 ohms, and its E.M.F. 1.08 volts, how much current will 40 cells in series

send through a telegraph line having a resistance of 500 ohms? What current will one such cell send through the same circuit? What current will 40 cells joined in parallel send through the same circuit?

4. What current will the 40 cells in parallel send through an ammeter which has a resistance of .1 ohm? What current would the 40 cells in series send through the same ammeter? What current would a single cell send through the same ammeter?

5. Why is the bichromate cell very generally used where large currents are wanted for a short time?

6. Under what conditions will a small cell give practically the same current as a large one of the same type?

7. A Daniell cell is found to yield a current of .5 amperes when connected directly to an ammeter. What is its internal resistance?

8. Why must a galvanometer which is to be used for measuring voltages have a high resistance?

9. Why is it desirable that a galvanometer which is to be used for measuring currents have as small a resistance as possible?

10. Can you prove from a consideration of Ohm's law that when wires of different resistances are inserted in series in a circuit, the P.D.'s between the ends of the various wires are proportional to the resistances of these wires?

CHAPTER XV

CHEMICAL, MAGNETIC, AND HEATING EFFECTS OF THE ELECTRIC CURRENT

Chemical Effects [1]

387. Movement of ions in a conducting liquid. Let an infusion of purple cabbage be prepared by steeping its leaves thoroughly in water. To such an infusion let a few drops of any alkali, such as caustic soda (NaOH), be added. The color of the liquid will at once change from purple to green. To another portion of the infusion let a few drops of any acid, e.g. sulphuric (H_2SO_4), be added. The liquid will at once turn red. This liquid may therefore be used as a test for the presence of either an alkali or an acid. Now let enough of the infusion be added to a solution of sodium sulphate (Na_2SO_4) to give to the latter a decided purple color. Then let the whole be placed in a U-tube (Fig. 298) and platinum electrodes inserted. Then let the current from two bichromate cells joined in series be sent through the arrangement. The solution near the pole at which the current enters the liquid, called the positive electrode or *anode*, will presently be seen to turn red; while that near the pole at which the current leaves the liquid, called the negative electrode or *cathode*, will turn green.[2]

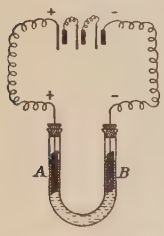


Fig. 298. Showing the appearance of the acid ions at the positive electrode and of the alkali ions at the negative electrode

[1] This subject should be accompanied or followed by a laboratory experiment on electrolysis and the principle of the storage battery. See, for example, Experiment 35 of the authors' manual.

[2] The experiment may be performed equally well with litmus in place of the cabbage infusion, but in this case it must be done in two parts. In the first part the solution is first turned red by a few drops of acid; the passage of the current then causes it to turn *blue* about the *cathode*. In the second part the solution is first turned blue by an excess of alkali (NaOH); the passage of the current then causes it to turn *red* about the *anode*.

These facts are explained in accordance with the theory outlined in § 377, by assuming, as there, that when the salt (Na_2SO_4) goes into solution in water it breaks up into negatively charged SO_4 ions and positively charged Na ions. Hence as soon as the platinum plates *A* and *B* became positively and negatively charged respectively by being attached to the + and − plates of a battery, the negative SO_4 ions were at once attracted toward *A* and the positive Na ions toward *B*. When they reached the plates they gave up their charges to them, and were then in condition to attack the water of the solution, forming sulphuric acid at one plate and sodium hydroxide (NaOH) at the other.

All conduction in liquids, molten metals excepted, is thought to be due to a mechanism similar to that described above, i.e. it is thought to consist in the migration through the liquid of a swarm of positively charged ions in one direction, and of a corresponding swarm of negatively charged ions in the other direction. As soon as the ions give up their charges to the plates they are either deposited upon them, or else act chemically either upon the solution or upon the plate, so as to form new compounds. Strong evidence for the correctness of this view as to the nature of conduction in liquids is found in the fact that *pure* liquids, such as water, alcohol, etc., do not conduct electricity, but are, in every case, rendered conductors by dissolving salts or acids in them; and again, by the further fact that whenever an electric current is passed through such a conducting liquid the two constituents of the substance in solution are found to appear at the two plates. All liquids which conduct in this manner are called *electrolytes*, and the process of separating the two constituents of the substance in solution by means of an electric current is called *electrolysis*.

388. The electrolysis of water. Let two platinum electrodes, sealed into bent glass tubes *e* and *f* (Fig. 299), be inserted into two inverted test tubes, the test tubes and the vessel in which they stand

being filled with a solution of about one part of sulphuric acid to forty parts of water. Let a bichromate battery of two or three cells be attached to the copper terminals *A* and *B*, which make electrical connection with the platinum strips through mercury which is poured into the glass tubing. Gases will at once be seen rising from the two platinum electrodes and collecting in the tops of the tubes. One tube will be found to fill twice as rapidly as the other.[1] When this tube is filled with gas let it be carefully removed from the liquid and a lighted match held beneath its mouth. The gas will burn with a blue flame, thus indicating that it is hydrogen. Let the other tube be removed and a glowing splinter introduced into it. It will take fire and burn vigorously, indicating that the gas is oxygen.

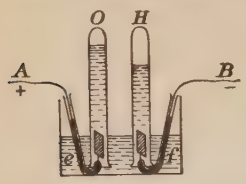


Fig. 299. The electrolysis of water

The water has been decomposed by the electric current into its two elements, hydrogen and oxygen. According to our theory this decomposition has been effected as follows. The positively charged hydrogen ions of the sulphuric acid (H_2SO_4) solution were driven by the electric forces to the negative electrode, where they gave up their charges and appeared at once as hydrogen gas; while the negative SO_4 ions migrated to the positive electrode, where they gave up their charges to it and then acted upon the water (H_2O), forming more sulphuric acid and liberating oxygen.

389. Electroplating. Let copper sulphate ($CuSO_4$) be placed in the U-tube of Fig. 298, the platinum electrodes inserted, and the current started. Presently the negative electrode will be found to be covered with a bright coat of copper.[2] In this case the positively charged Cu ions, after giving up their charges, are deposited as metallic copper, while the SO_4 ions act precisely as they did in the last experiment, i.e. they abstract hydrogen from the water to form sulphuric acid, and

[1] The oxygen will be absorbed by electrodes other than platinum, so that its volume will be much less than one half that of the hydrogen.

[2] The copper may be removed at any time by dipping the electrode into hot nitric acid.

liberate oxygen. If the current through the U-tube is reversed, copper will begin to be deposited on the clean plate, while the coat of copper on the other plate will gradually disappear, for in this case the SO_4 ions, instead of acting on the water, take up the metallic copper and form $CuSO_4$.

The experiment illustrates the whole process of electroplating, — a process which is used very extensively for obtaining gold- and silver-plated ware, for nickel plating iron so as to prevent it from rusting, for copper plating electric-light carbons so as to increase their conductivity, etc. In commercial work the positive plate, i.e. the plate at which the current enters the bath, is always made from the same metal as that which is being deposited from solution; for in this case the SO_4, or other negative ion, dissolves this plate as fast as the metal ions are deposited upon the other. The strength of the solution, therefore, remains unchanged. In effect, the metal is simply taken from one plate and deposited on the other. Fig. 300 represents a silver-plating bath. The bars joined to the anode *A* are of pure silver. The spoons to be plated are connected to the cathode *K*. The solution consists of 500 g. of potassium cyanide and 250 g. of silver cyanide in 10 l. of water.

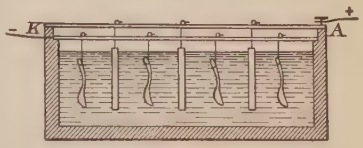


Fig. 300. Electroplating bath

390. Electrotyping. In the process of electrotyping the page is first set up in the form of common type. A mold is then taken in wax or gutta-percha. This mold is then coated with powdered graphite to render it a conductor, after which it is ready to be suspended as the cathode in a copper-plating bath, the anode being a plate of pure copper and the liquid a solution of copper sulphate. When a sheet of copper as thick as a visiting card has been deposited on the mold, the latter is removed and the wax replaced by a type-metal backing, to give rigidity to the copper films. From such a plate as many as a hundred

thousand impressions may be made. Practically all books which run through large editions are printed from such electrotypes.

391. Refining of metals. If the solution consists of pure copper sulphate, it is not necessary that the anode be of chemically pure copper in order to obtain a pure copper deposit on the cathode. *Electrolytic* copper, which is the purest copper on the market, is obtained as follows. The unrefined copper is used as an anode. As it is eaten up the impurities contained in it fall as a residue to the bottom of the tank and pure copper is deposited on the cathode by the current. This method is also extensively used in the refining of metals other than copper.

392. Chemical method of measuring current. In 1834 Faraday found that a given current of electricity flowing for a given time always deposits the same amount of a given element from a solution, whatever be the nature of the solution which contains the element. For example, one ampere always deposits in an hour 4.025 g. of silver, whether the electrolyte is silver nitrate, silver cyanide, or any other silver compound. Similarly, an ampere will deposit in an hour 1.181 g. of copper, 1.203 g. of zinc, etc. This fact is made use of in calibrating fine ammeters, since it is possible to compute with great accuracy the strength of a current which has deposited a given weight of metal in a known length of time. The instrument to be calibrated is connected in series with a silver-plating bath, and the current corresponding to a given deflection is then obtained by dividing the total weight of silver deposited by the product of 4.025 and the number of hours during which the current was flowing.

393. Storage batteries. Let two six- by eight-inch lead plates be screwed to a half-inch strip of some insulating material, as in Fig. 301, and immersed in a solution consisting of one part of sulphuric acid to ten parts of water. Let a current from two bichromate cells C be sent through this arrangement, an ammeter A or any low-resistance galvanometer being inserted in the circuit. As the current flows, hydrogen

bubbles will be seen to rise from the cathode (the plate at which the current leaves the solution), while the positive plate, or anode, will begin to turn dark brown. At the same time the reading of the ammeter will be found to decrease rapidly. The brown coating is a compound of lead and oxygen called lead peroxide (PbO_2), which is formed by the action upon the plate of the oxygen which is liberated precisely as in the experiment on the electrolysis of water (§ 388). Let now the batteries be removed from the circuit by opening the key K_1, and let an electric bell B be inserted in their place by closing the key K_2. The bell will ring and the ammeter A will indicate a current flowing in a direction opposite to the direction of the original current. This current will rapidly decrease as the energy which was stored in the cell by the original current is expended in ringing the bell.

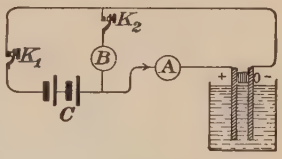


Fig. 301. The principle of the storage battery

This experiment illustrates the principle of the *storage battery.* Properly speaking, there has been no storage of *electricity,* but only a storage of *chemical energy.*

Two similar lead plates have been changed by the action of the current into two dissimilar plates, one of lead and one of lead peroxide. In other words, an ordinary galvanic cell has been formed; for any two dissimilar metals in an electrolyte constitute a primary galvanic cell. In this case the lead peroxide plate corresponds to the copper of an ordinary cell, and the lead plate to the zinc. This cell tends to create a current opposite in direction to that of the charging current; i.e. its E.M.F. pushes back against the E.M.F. of the charging cells. It was for this reason that the ammeter reading fell. When the charging current is removed this *primary* galvanic cell will furnish a current until the thin coating of peroxide is used up. The only important difference between a commercial storage cell (Fig. 302) and the one which we have here used is that the former is provided in the making with a much thicker coat of the "active material" (lead peroxide on the positive plate and

a porous, spongy lead on the negative) than can be formed by a single charging such as we used. This material is pressed into interstices in the plates, as shown in Fig. 302. The E.M.F. of the storage cell is about 2 volts. Since the plates are always very close together and may be given any desired size, the internal resistance is usually small, so that the currents furnished may be very large. They are sometimes as high as several thousand amperes.

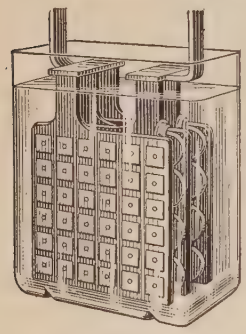


FIG. 302. Lead-plate storage cell

The usual efficiency of the storage cell is about 75%, i.e. but three fourths as much electrical energy can be obtained from it as is put into it.

QUESTIONS AND PROBLEMS

1. Two copper plates are immersed in a bath of copper sulphate and a current sent through the arrangement. How could you tell, from the changes in the weights of the two plates, in which direction the current flowed?

2. If the terminals of a battery are immersed in a glass of acidulated water, how can you tell from the rate of evolution of the gases at the two electrodes which is positive and which negative?

3. How long will it take a current of one ampere to deposit a gram of silver from a solution of silver nitrate?

4. If the same current used in Problem 3 were led through a solution containing a zinc salt, how much zinc would be deposited in the same time?

5. In calibrating an ammeter, the current which produces a certain deflection is found to deposit one half gram of silver in 50 minutes. What is the strength of the current?

6. Why is it possible to get a much larger current from a storage cell than from a Daniell cell?

7. A certain storage cell having an E.M.F. of 2 volts is found to furnish a current of 20 amperes through an ammeter whose resistance is .05 ohm. Find the internal resistance of the cell.

8. Would it be possible to charge a storage cell with a battery of Leclanché cells joined in parallel? Give reason for answer.

Magnetic Properties of Coils

394. Loop of wire carrying a current equivalent to a magnetic disk. Let a single loop of wire be suspended from a thread in the manner shown in Fig. 303, so that its ends dip into two mercury cups. Then let a current from a bichromate cell be sent through the loop. The latter will be found to slowly set itself so that the face of the loop from which the magnetic lines emerge, as given by the right-hand rule (see § 355, p. 266, and also Fig. 304), is toward the north. Let a bar magnet be brought near the loop. The latter will be found to behave toward the magnet in all respects as though it were a flat magnetic disk whose boundary is the wire, the face which turns toward the north being an N pole and the other an S pole.

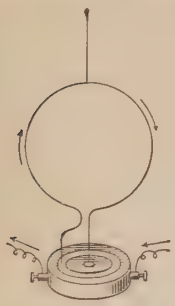


Fig. 303. A loop equivalent to a flat magnetic disk

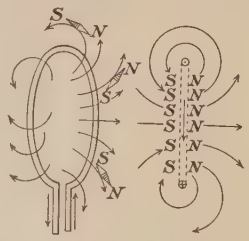


Fig. 304. North pole of disk is face from which magnetic lines emerge; south face is face into which they enter

395. Position assumed by a loop carrying a current in a magnetic field. The experiment of the last paragraph shows what position a loop bearing a current will always tend to assume in a magnetic field; for since a magnet always tends to set itself so that the line connecting its poles is parallel to the direction of the magnetic lines of the field in which it is placed, a coil must set itself so that a line connecting its magnetic poles is parallel to the lines of the magnetic field, i.e. so that *the plane of the coil is perpendicular to the field* (see Fig. 305); or, to state the same thing in slightly

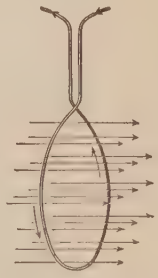


Fig. 305. Position assumed by a coil carrying a current in a magnetic field

different form, *the coil will set itself so as to include as many as possible of the lines of force of the field.* This shows why the coil (Fig. 269, p. 266) tended to turn through 90° when a current was passed through it.

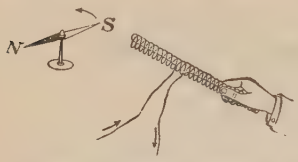


Fig. 306. Magnetic effect of a helix

396. Magnetic properties of a helix. Let a wire bearing a current be wound in the form of a helix and held near a suspended magnet, as in Fig. 306. One end of the helix will be found to attract the north pole of the needle, while the other end repels it. In short, the coil will be found to act in every respect like a magnet, with an N pole at one end and an S pole at the other.

This result might have been predicted from the fact that a single loop is equivalent to a flat-disk magnet. For when a series of such disks is placed side by side, as in the helix, the result must be the same as placing a series of disk magnets in a row, the N pole of one being directly in contact with the S pole of the next, etc. These poles would therefore all neutralize each other except at the two ends. We therefore get a magnetic field of the shape shown in Fig. 307, the direction of the arrows representing as usual the direction in which an N pole tends to move.

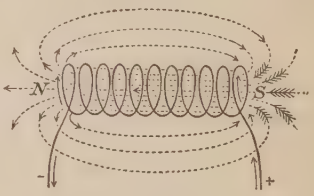


Fig. 307. Magnetic field of helix

397. Rules for north and south poles of a helix. The right-hand rule, as given in § 355, is sufficient in every case to determine which is the N and which the S pole of a helix, i.e. from which end the lines of magnetic force emerge from the helix and at which end they enter it. But it is found convenient, in the consideration of coils, to restate the right-hand rule in a slightly different way, thus: *If the coil is grasped in the right hand in such a way that the fingers point in the direction in which the*

current is flowing in the wires, the thumb will point in the direction of the north pole of the helix (see Fig. 308). Similarly, if the sign of the poles is known, but the direction of the current unknown, it may be determined as follows: If the right hand is placed against the coil with the thumb pointing in the direction of the lines of force (i.e. toward the north pole of the helix), the fingers will pass around the coil in the direction in which the current is flowing.

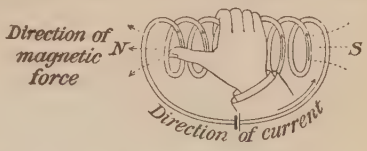


FIG. 308. Rule for poles of helix

398. The electro-magnet. Let a core of soft iron be inserted in the helix (Fig. 309). The poles will be found to be enormously stronger than before. This is because the core is magnetized by induction from the field of the helix in precisely the same way in which it would be magnetized by induction if placed in the field of a permanent magnet. The new field strength about the coil is now the sum of the fields due to the core and that due to the coil. If the current is broken, the core will at once lose the greater part of its magnetism. If the current is reversed, the polarity of the core will be reversed. Such a coil with a soft-iron core is called an *electro-magnet*.

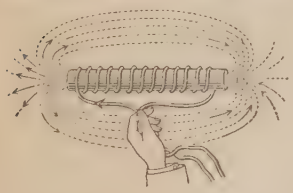


FIG. 309. The bar electro-magnet

399. Strength of an electro-magnet. The strength of an electro-magnet can be very greatly increased by giving it such form that the magnetic lines can remain in iron throughout their entire length instead of emerging into air, as they do in Fig. 309. For this reason electro-magnets are usually built in the horseshoe form and provided with an armature *A* (Fig. 310), through which a complete iron path for the

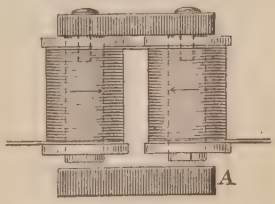


FIG. 310. The horseshoe electro-magnet

lines of force is established, as shown in Fig. 311. The strength of such a magnet depends chiefly upon the number of *ampere turns* which encircle it, the expression "ampere turns" denoting the product of the number of turns of wire about the magnet by the number of amperes flowing in each turn. Thus a current of $\frac{1}{100}$ ampere flowing 1000 times around a core will make an electro-magnet of precisely the same strength as a current of 1 ampere flowing 10 times about the core.

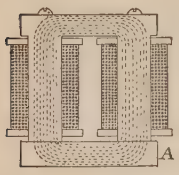


Fig. 311. Magnetic circuit of an electro-magnet

400. The electric bell. The electric bell (Fig. 312) is one of the simplest applications of the electro-magnet. When the button P (Figs. 312 and 313) is pressed the electric circuit of the battery is closed and a current flows in at A, through the magnet, over the closed contact C, and out again at B. But no sooner is this current established than the electro-magnet E pulls over the armature a, and in so doing breaks the contact at C. This stops the current and demagnetizes the magnet E. The armature is then thrown back against C by the elasticity of the spring s which supports it. No sooner is the contact made at C than the current again begins to flow and the former operation is repeated. Thus the circuit is automatically made and broken at C, and the hammer H is in consequence set into rapid vibration against the rim of the bell.

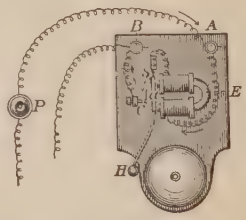


Fig. 312. The electric bell

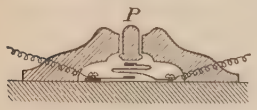


Fig. 313. Cross section of the electric push button

401. The telegraph. The electric telegraph is another simple application of the electro-magnet. The principle is illustrated in Fig. 314. As soon as the key K at Chicago, for example, is closed, the current flows over the line to, we will say, New York. There it passes through the electro-magnet m, and thence back to Chicago through the earth. The armature b is held down by the electro-magnet m as long as the key K is kept closed. As soon as the circuit is broken at K the armature is pulled up by the spring d. By

means of a clockwork device the tape c is drawn along at a uniform rate beneath the pencil or pen carried by the armature b. A very short time of closing of K produces a dot upon the tape, a longer time a dash. As

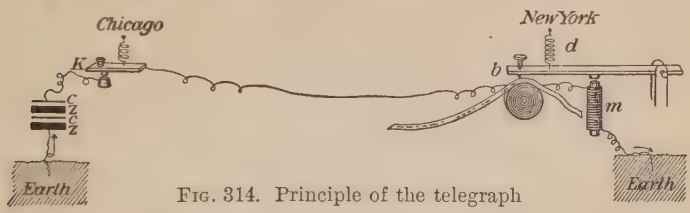


Fig. 314. Principle of the telegraph

the Morse, or telegraphic, alphabet consists of certain combinations of dots and dashes, any desired message may be sent from Chicago and recorded in New York. In modern practice the message is not ordinarily recorded on a tape, for operators have learned to read messages by ear, a very short interval between two clicks being interpreted as a dot, a longer interval as a dash.

The first commercial telegraph line was built by S. F. B. Morse between Baltimore and Washington. It was opened on May 24, 1844, with the now famous message: "What hath God wrought?"

402. The relay and sounder. On account of the great resistance of long lines the current which passes through the electro-magnet is so weak that the armature of this magnet must be made very light in order to respond to the action of the current. The

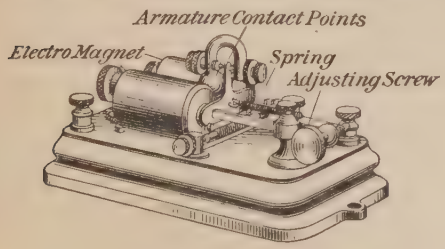


Fig. 315. The relay

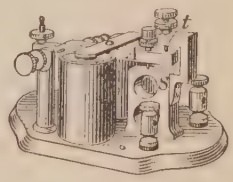


Fig. 316. The sounder

clicks of such an armature are not sufficiently loud to be read easily by an operator. Hence at each station there is introduced a local circuit, which contains a local battery, and a second and heavier electro-magnet which is called a sounder. The electro-magnet on the main line is then called the relay (see Figs. 315, 316, and 317). The sounder has a very

heavy armature (A, Fig. 316), which is so arranged that it clicks both when it is drawn down by its electro-magnet against the stop S and when it is pushed up again by its spring, on breaking the current, against the stop t. The interval which elapses between these two clicks indicates to the operator whether a dot or dash is sent. The current in the main line simply serves to close and open the circuit in the local battery which operates the sounder (see Fig. 317). The electro-magnets of the relay and the sounder differ in that the former consists of many thousand turns of fine wire, usually having a resistance of about 150 ohms, while the latter consists of a few hundred turns of coarse wire having ordinarily a resistance of about 4 ohms.

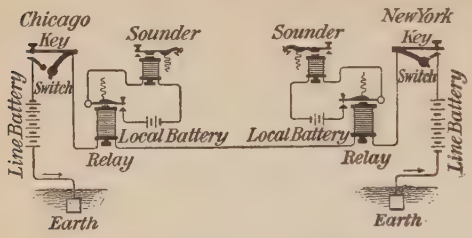


Fig. 317. Telegraphic system

403. Plan of a telegraphic system. The actual arrangement of the various parts of a telegraphic system is shown in Fig. 317. When an operator at Chicago wishes to send a message to New York, he first opens the switch which is connected to his key, and which is always kept closed except when he is sending a message. He then begins to operate his key, thus controlling the clicks of both his own sounder and that at New York. When the Chicago switch is closed and the one at New York open, the New York operator is able to send a message back over the same line. In practice a message is not usually sent as far as from Chicago to New York over a single line, save in the case of trans-oceanic cables. Instead it is automatically transferred at, say, Cleveland to a second line which carries it on to Buffalo, where it is again transferred to a third line which carries it on to New York. The transfer is made in precisely the same way as the transfer from the main circuit to the sounder circuit. If, for example, the sounder circuit at Cleveland is lengthened so as to extend to Buffalo, and if the sounder itself is replaced by a relay (called in this case a repeater), and the local battery by a main battery, then the sounder circuit has been transformed into a repeater circuit and all the conditions are met for an automatic transfer of the message at Cleveland to the Cleveland-Buffalo line. There is, of course, no time lost in this automatic transfer.

QUESTIONS AND PROBLEMS

1. The plane of a suspended loop of wire is east and west. A current is sent through it, passing from east to west on the upper side. What will happen to the loop if it is perfectly free to turn?

2. When a strong current is sent through the coil of a D'Arsonval galvanometer, what position will it assume?

3. If one looks down on the ends of a U-shaped electro-magnet, does the current encircle the two coils in the same or in opposite directions? In which direction does it encircle the *N* pole, clockwise or counter clockwise?

4. Draw a diagram showing the method of operation of an electric bell.

5. Draw a diagram showing how the relay and sounder operate in a telegraphic circuit.

6. Ordinary No. 9 telegraph wire has a resistance of 20 ohms to the mile. What current will 100 Daniell cells send through 100 miles of such wire, if the relays have a resistance of 150 ohms each and the cells an internal resistance of 4 ohms each? (Assume the E.M.F. of each cell to be 1 volt.)

7. If the relays of the preceding problem had each 10,000 turns of wire in their coils, how many ampere turns were effective in magnetizing their electro-magnets?

8. If on the above telegraph line sounders having a resistance of 3 ohms each and 500 turns were to be put in the place of the relays, how many ampere turns would be effective in magnetizing their cores? Why then must a relay be a high-resistance electro-magnet?

9. If the earth's magnetism is due to electric currents flowing about the surface, do these currents flow from east to west or from west to east?

Heating Effects of the Electric Current

404. Heat developed in a wire by an electric current. Let the terminals of a bichromate cell be touched to a piece of No. 40 iron or German silver wire and the length of wire between these terminals shortened to $\frac{1}{4}$ inch or less. The wire will be heated to incandescence and probably melted.

The experiment shows that just as in the charging of a storage battery the energy of the electric current was transformed into the energy of chemical separation, so here in the passage of the current through the wire the energy of the electric current is transformed into heat energy.

405. Amount of electrical energy expended by a current flowing between points of given P.D. Since the P.D. between any two points has been defined as the amount of work required to carry one unit of quantity of electricity from the one point to the other, and since current is defined as the number of units of quantity which pass per second, it follows that the work done, i.e. *the electrical energy expended per second in driving a current C through a conductor whose terminals are at a potential difference P.D., is $C \times$ P.D.* If P.D. is expressed in volts and current in amperes, the product is expressed in joules per second, or watts, since a joule per second is a watt (see § 221, p. 162). This is simply a result of the way in which the units are chosen. We have then, in general,

$$\text{volts} \times \text{amperes} = \text{watts}. \tag{1}$$

406. Calories of heat developed in a wire. The electrical energy expended when a current flows between points of given P.D. may be spent in a variety of ways. For example, it may be spent in producing chemical separation, as in the charging of a storage cell; it may be spent in doing mechanical work, as is the case when the current flows through an electric motor; or it may be spent wholly in heating the wire, as was the case in the experiment of § 404. It will always be expended in this last way when no chemical or mechanical changes are produced by it. The number of calories of heat produced per second in the wire of the last experiment is found, then, by multiplying the number of joules expended by the current per second by the heat equivalent in calories of the joule. This is .24 calories, since by § 249 and § 203, one calorie is 4.2 joules. Therefore when all of the electrical energy of a current is transformed into heat energy, we have

$$\text{calories per second} = \text{volts} \times \text{amperes} \times .24. \tag{2}$$

The total number of calories H developed in t seconds will be given by

$$H = \text{P.D.} \times C \times t \times .24. \tag{3}$$

Thus a current of 10 amperes flowing in a wire whose terminals are at a potential difference of 12 volts will develop in 5 minutes

$10 \times 12 \times 300 \times .24 = 8640$ calories. Since by Ohm's law P.D. $= C \times R$, we have, by substituting CR for P.D. in (3)

$$H = C^2R \times t \times .24\,; \tag{4}$$

or the *heat generated in a conductor is proportional to the time, to the resistance, and to the square of the current.* This is known as Joule's law, having been first announced by him as the result of experimental researches.

407. Incandescent lamps. The ordinary incandescent lamp (Fig. 318) consists of a carbon filament heated to incandescence by an electric current. Since the carbon would burn instantly in air, the filament is placed in a highly exhausted glass bulb. Even then it disintegrates slowly. The normal life of a 16-candle-power lamp filament is from 1000 to 2000 working hours. The filament is made by carbonizing a special form of cotton thread. The ends of the carbonized thread are attached to platinum wires which are sealed into the glass walls of the bulb, and which make contact one with the base of the socket and the other with its rim, these being the electrodes through which the current enters and leaves the lamp.

Fig. 318. The incandescent lamp

The ordinary 16-candle-power lamp is most commonly run on a circuit which maintains a potential difference of either 110 or 220 volts between the terminals of the lamp. In the former case the lamp carries about .5 ampere of current, and in the latter case about .25 ampere. It will be seen from these figures that the rate of consumption of energy is about 3.4 watts per candle power.

A customer usually pays for his light by the "watt hour," a watt hour being the energy furnished in one hour by a current whose rate of expenditure of energy is one watt. Thus the rate at which energy is consumed by a 16-candle-power lamp is $110 \times .5 = 55$ watts. One such lamp running for ten hours would therefore consume 550 watt hours of energy.

408. The arc light. When two carbon rods are placed end to end in the circuit of a powerful electric generator, the carbon about the point of contact is heated red-hot. If then the ends of the carbon rods are separated $\frac{1}{4}$ inch or so, the current will still continue to flow, for a conducting layer of incandescent vapor called an electric arc is produced between the poles. The appearance of the arc is shown in Fig. 319. At the positive pole a hollow, or crater, is formed in the carbon, while the negative carbon becomes cone shaped, as in the figure. These effects are due to the fact that minute particles of carbon are carried across with the current from the positive to the negative carbon. The carbons are consumed at the rate of about an inch an hour, the positive carbon wasting away about twice as fast as the negative. By inclosing the arc in a glass globe, so that oxygen cannot be supplied to it, the rate of consumption of the carbons is reduced to about an eighth of its value in the open air, but at the same time the total candle power is reduced to about one third of its former value.

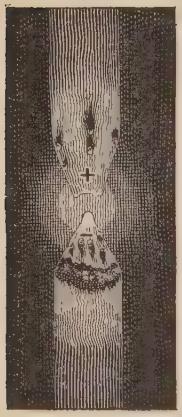


Fig. 319. The arc light

The greater part of the light comes from the crater at the positive terminal. This point has also the highest temperature which can be produced artificially. It is estimated at about 3800° C. All known substances are volatilized in the electric arc.

The ordinary arc requires a current of 10 amperes and a P.D. between its terminals of about 50 volts. Such a lamp produces about 500[1] candle power, and therefore consumes energy at the rate of about 1 watt per candle power. This makes an arc light about 3.5 times as efficient as an incandescent light. The

[1] This is the so-called "mean spherical" candle power. The candle power in the direction of maximum illumination is from 1000 to 1200.

recently invented *flame arc*, produced between carbons which have a composite core consisting of carbon, lime, magnesia, silica, or other light-giving minerals, sometimes reaches an efficiency as high as .27 watt per candle power. This is ten times the efficiency of the ordinary inclosed arc and four times that of the open arc.

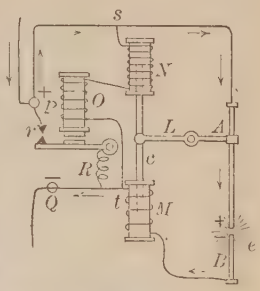


Fig. 320. Feeding device for arc lamp

409. The arc-light automatic feed. Since the two carbons of the arc gradually waste away they would soon become so far separated that the arc could no longer be maintained were it not for an automatic feeding device which keeps the distance between the carbon tips very nearly constant. Fig. 320 shows the essential features of one form of this device. When no current is flowing through the lamp, gravity holds the carbon tips at e together; but as soon as the current is thrown on it energizes the low-resistance electro-magnet M, which is in series with the carbons. This draws down the iron plunger c, which acts upon the lever L and "strikes the arc" at e. But the introduction of the resistance of the arc into the circuit $s.1BMt$ raises the P.D. between s and t and thus causes an appreciable current to flow through the high-resistance magnet N, which is shunted across this circuit. This tends to *raise* the plunger c and thus to shorten the arc. There is thus one particular length of arc for which equilibrium exists between the effects of the series magnet M and the shunt magnet N. This length the lamp automatically maintains. The magnet O is the so-called "cut-out" inserted so that, if the lamp gets out of order and the arc burns out, a current at once flows through N and O of sufficient strength to close the contact points at r and thus permit the main current to flow on to the next lamp over the path $PrRQ$. Without this all the lamps in the series would go out.

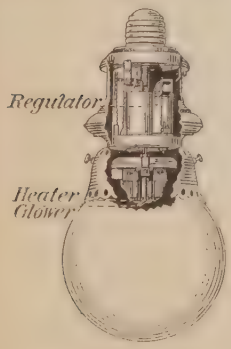


Fig. 321. Nernst lamp

410. The Nernst lamp. The Nernst lamp (Fig. 321) differs from the ordinary incandescent lamp chiefly in that it employs for its filament

a substance which may be raised to a much higher temperature than a carbon filament without disintegrating. This makes it possible to obtain from it about twice as much light as is obtained from the ordinary incandescent lamp with the same expenditure of energy. It produces about one candle for each 2 watts. The substance used for the filament is a compound of rare earths of much the same composition as the Welsbach mantle. Its principal ingredient is the oxide of thorium. Since this does not oxidize even at a very high temperature, it is heated in the open air, or in an unsealed ground-glass globe, instead of in a vacuum bulb. The chief difficulty which the lamp has encountered has arisen from the fact that the material of which the filament is composed is not a conductor at ordinary temperatures, but only becomes conducting when heated quite hot. Hence, in order to start the lamp, it is either necessary to heat the filament with a match, or to surround it with a coil of platinum wire which is first heated to incandescence, and then, as soon as the heat which it radiates has raised the filament to the necessary temperature, to withdraw it from the circuit by an electro-magnetic mechanism similar to that employed in the feed of the ordinary arc lamp. This adds much to the complexity of the lamp, as the figure shows, and therefore to its initial cost.

Fig. 322. The Hewitt mercury vapor lamp

In spite of the fact that the ordinary incandescent and the Nernst lamps are much more expensive to operate per candle power than the arc light, the latter has the great disadvantage that it cannot be used in small units, and is therefore not satisfactory for indoor lighting, except in very large buildings.

411. The Cooper-Hewitt mercury lamp. The Cooper-Hewitt mercury lamp is the most efficient of all electric lights, unless it be the flame arc. It differs from the arc lamp in that the incandescent body is a long column of mercury vapor instead of an incandescent solid (Fig. 322). The lamp consists of an exhausted tube three or four feet long, the positive electrode at the top consisting of a plate of iron, while the negative electrode at the bottom is a small quantity of mercury. Under a sufficient difference of potential between these terminals a long mercury-vapor arc is formed which stretches from terminal to terminal in the tube. This arc emits a very brilliant light, but it is almost entirely wanting in red rays. The efficiency of the

lamp is very high, since it requires but .3 watt per candle power. It is rapidly finding important commercial uses, especially in photography. The chief objection to it arises from the fact that, on account of the absence of red rays, the light gives colored objects an unnatural appearance.

QUESTIONS AND PROBLEMS

1. How many calories of heat are developed per second by a 110-volt incandescent lamp carrying a current of .6 ampere?

2. A current of 10 amperes flows through a conductor having a resistance of 4 ohms. How much heat is generated in the conductor per minute?

3. A 220-volt lamp has a resistance, when hot, of about 750 ohms. How many calories will be developed in it in 10 minutes?

4. If a bichromate cell has an E.M.F. of 2 volts, and furnishes a current of 5 amperes, what is its rate of expenditure of energy in watts?

5. How many cells, working as in Problem 4, would be equivalent to 1 H.P. (see § 221, p. 162).

CHAPTER XVI

INDUCED CURRENTS

The Principle of the Dynamo

412. Current induced by a magnet. Let 400 or 500 turns of No. 22 copper wire be wound into a coil C (Fig. 323) about two and a half inches in diameter. Let this coil be connected into circuit with a simple D'Arsonval galvanometer made in the manner indicated in § 356. The coil should be provided with a pointer p, three or four inches long, and the whole should be protected from air currents on three sides by some sort of a box. Let the coil C be thrust suddenly over the N pole of a strong horseshoe magnet. The deflection of the pointer of the galvanometer will indicate a momentary current flowing through the coil. Let the coil be held stationary over the magnet. The pointer will be found to come to rest in its natural position. Now let the coil be removed suddenly from the pole. The pointer will move in a direction opposite to that of its first deflection, showing that a reverse current is now being generated in the coil.

Fig. 323. Induction of electric currents by magnets

We learn, therefore, that *a current of electricity may be induced in a conductor by causing the latter to move through a magnetic field*, while a magnet has no such influence upon a conductor which is at rest with respect to the field. This discovery, one of the most important in the history of science, was announced by the great Faraday in 1831. From it have sprung directly most of the modern industrial developments of electricity.

MICHAEL FARADAY (1791–1867)

Famous English physicist and chemist; one of the most gifted of experimenters; son of a poor blacksmith; apprenticed at the age of thirteen to a London bookbinder, with whom he worked nine years; applied for a position in Sir Humphry Davy's laboratory at the Royal Institution in 1813; became director of this laboratory in 1825; discovered electro-magnetic induction in 1831 (a discovery made, though not published, in 1830, by the American physicist, Joseph Henry); made the first dynamo; discovered in 1833 the laws of electrolysis, now known as Faraday's laws; the farad, the practical unit of electrical capacity, is named in his honor.

413. Direction of induced current. Lenz's law. In order to find the *direction* of the induced current in the experiment of the last paragraph, let the terminals of a galvanic cell be touched for an instant to the terminals A and B (Fig. 323) of the galvanometer, the cell being short-circuited by a few feet of small copper wire in order that the current sent through the instrument may not be too large. Let the direction in which the pointer moves when the current enters, say at A, be noted. This will at once show in what direction the current was flowing in the coil C when it was being thrust over the N pole in the last experiment. By a simple application to C of the right-hand rule (§ 397, p. 300) we can then tell which was the N and which the S face of the coil when the induced current was flowing through it. In this way it will be found that if the coil was being moved past the N pole of the magnet, the current induced in it was in such a direction as to make the lower face of the coil an N pole during the downward motion and an S pole during the upward motion. In the first case the repulsion of the N pole of the magnet and the N pole of the coil tended to *oppose* the motion of the coil while it was moving from a to b, and the attraction of the N pole of the magnet and the S pole of the coil tended to oppose the motion while it was moving from b to c. In the second case the repulsion of the two N poles tended to oppose the motion between b and c, and the attraction between the N pole of the magnet and the S pole of the coil tended to oppose the upward motion from b to a. *In every case, therefore, the motion is made against an opposing force.*

From these experiments, and others like them, we arrive at the following law: *Whenever a current is induced by the relative motion of a magnetic field and a conductor, the direction of the induced current is always such as to set up a magnetic field which opposes the motion.* This is Lenz's law. This law might have been predicted at once from the principle of the conservation of energy; for this principle tells us that since an electric current possesses energy, such a current can appear only through the expenditure of mechanical work or of some other form of energy.

414. Condition necessary for an induced E.M.F. Let the coil be held in the position shown in Fig. 324, and moved back and forth so that its motion is *parallel* to the magnetic field, i.e. parallel to the line NS. No induced current will be observed.

By experiments of this sort it is found that an E.M.F. is induced in a coil only *when the motion takes place in such a way as to change the total number of magnetic lines of force which are inclosed by the coil.* Or, to state this rule in more general form, *an E.M.F. is induced in any element of a conductor when, and only when, that element is moving in such a way as to cut magnetic lines of force.*[1]

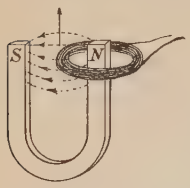


Fig. 324. Currents induced only when conductor *cuts* lines of force

It will be noticed that the first statement of the rule is included in the second, for whenever the number of lines of force which pass through a coil changes, some lines of force must cut across the coil from the inside to the outside or vice versa.

In the preceding statement we have used the expression "induced E.M.F." instead of "induced current" for the reason that whether or not a continuous current flows in a conductor in which an E.M.F. (i.e. a pressure tending to produce a current) exists, depends simply on whether or not the conductor is a portion of a closed electrical circuit. In our experiment the portion of the wire in which the E.M.F. was being generated by its passage across the lines of force running from N to S was a part of such a closed circuit, and hence a current resulted. If we had moved a straight conductor like that shown in Fig. 325, the E.M.F. would have been induced precisely as before; but since the circuit would then have been open, the only effect of this E.M.F. would have been to establish a P.D. between the

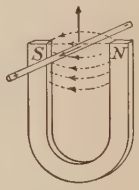


Fig. 325. E.M.F. induced when a straight conductor cuts magnetic lines

[1] If a strong electro-magnet is available these experiments are more instructive if performed, not with a coil, as in Fig. 324, but with a straight rod (Fig. 325) to the ends of which are attached wires leading to a galvanometer. Whenever the rod moves parallel to the lines of magnetic force there will be no deflection, but whenever it moves across the lines the galvanometer needle will move at once.

ends of the wire, i.e. to cause a positive charge to appear at one of its ends and a negative charge at the other, in precisely the same way that the E.M.F. of a battery causes positive and negative charges to appear on the terminals of the battery when it is on open circuit.

415. Strength of the induced E.M.F. The strength of an induced E.M.F. is found to depend simply upon *the number of lines of force cut per second* by the conductor, or, in the case of a coil, upon the *rate of change* in the number of lines of force which pass through the coil. The strength of the current which flows is then given by Ohm's law; i.e. it is equal to the induced E.M.F. divided by the resistance of the circuit. The number of lines of force which the conductor cuts per second may always be determined if we know the velocity of the conductor and the strength of the magnetic field through which it moves. For it will be remembered that according to the convention of § 314, p. 232, a field of unit strength is said to contain one line of force per square centimeter, a field of 1000 units strength, 1000 lines per square centimeter, etc. In a conductor which is cutting lines at the rate of 100,000,000 per second there is an induced E.M.F. of 1 volt. The reason that we used a coil of 500 turns instead of a single turn in the experiment of § 412 was that by thus making the conductor in which the current was to be induced cut the lines of force of the magnet 500 times instead of once, we obtained 500 times as strong an induced E.M.F., and therefore 500 times as strong a current for a given resistance in the circuit.

416. The dynamo rule. Since we found that reversing the direction in which a conductor is cutting lines of force reverses also the direction of the induced E.M.F., we learn that a fixed relation exists between these two directions and the direction of the magnetic lines. What this relation is may be obtained easily from Lenz's law. When the conductor was moving upward (Fig. 324) the current flowed in such a direction as to oppose

the motion, i.e. so as to make the lower face of the coil an S pole. This means that in the portion of the conductor between N and S, where the E.M.F. was being generated, its direction was from back to front, i.e. toward the reader (see arrow, Fig. 325). We therefore set up the following rule, which will be found to apply in every case: *Let the forefinger of the right hand* (Fig. 326) *point in the direction of the magnetic lines, and the thumb in the direction in which the conductor is cutting these lines; then the middle finger, held at right angles to both thumb and forefinger, will point in the direction of the induced current.* This is known as the dynamo rule.

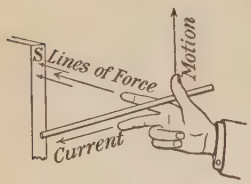


Fig. 326. The dynamo rule

417. Currents induced in rotating coils. Let a 400 or 500 turn coil of No. 28 copper wire be made small enough to rotate between the poles of a horseshoe magnet, and let it be connected into the circuit of a simple D'Arsonval galvanometer, precisely as in Fig. 323. Starting with the coil in the position of Fig. 327, let it be rotated suddenly from left to right through 180°. A strong deflection of the galvanometer will be observed. Let it be rotated through the next 180° back to the starting point. An inverse deflection will be observed.

The arrangement is a *dynamo* in miniature. During the first half of the revolution the wires on the right side of the loop were cutting the lines of force in one direction, while the wires on the left side were cutting them in the opposite direction. Hence a current was being generated down on the right side of the coil and up on the left side (see dynamo rule). It will be seen that both currents flow around the coil in the same direction.

Fig. 327

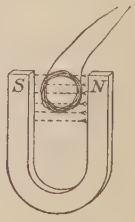


Fig. 328

Direction of currents induced in a coil rotating in a magnetic field

The induced current is strongest when the coil is in the position shown in Fig. 328, because there the lines of force are being cut most rapidly. Just as the coil is moving into or out of the position shown in Fig. 327, it is moving *parallel* to the lines of force, and hence no current is induced, since no lines of force are being cut. As the coil moves through the last 180° of its revolution both sides are cutting the same lines of force as before, but they are cutting them in an opposite direction; hence the current generated during this half is opposite in direction to that of the first half.[1]

QUESTIONS AND PROBLEMS

1. Under what conditions may an electric current be produced by a magnet?

2. A coil is thrust over the *S* pole of a magnet. Is the direction of the induced current clockwise or counter clockwise as you look down upon the pole?

3. State Lenz's law, and show how it follows from the principle of the conservation of energy.

4. If the coil of a D'Arsonval galvanometer is set to swinging while the circuit through the coil is open, it will continue to swing for a long time; but if the coil is short-circuited, it will come to rest after a very few oscillations. Why? (The experiment may easily be tried. Remember that currents are induced in the moving coil. Apply Lenz's law.)

5. A ship having an iron mast is sailing east. In what direction is the E.M.F. induced in the mast by the earth's magnetic field? If a wire is brought from the top of the mast to its bottom, no current will flow through the circuit. Why?

6. When a wire is cutting lines of force at the rate of 100,000,000 per second, there is induced in it an E.M.F. of one volt. A certain dynamo armature has 50 coils of 5 loops each and makes 600 revolutions per minute. Each wire cuts 2,000,000 lines of force twice in a revolution. What is the E.M.F. developed?

7. If a coil of wire is rotated about a vertical axis in the earth's field, an alternating current is set up in it. In what position is the coil when the current changes direction?

[1] A laboratory experiment on the principles of induction should be performed at about this point. See, for example, Experiment 36 of the authors' manual.

Dynamos

418. A simple alternating-current dynamo. The simplest form of commercial dynamo consists of a coil of wire so arranged as to rotate continuously between the poles of a powerful electro-magnet (Fig. 329).

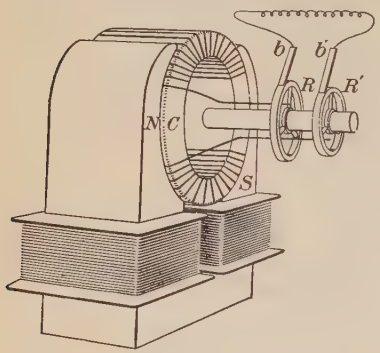


Fig. 329. Ring-wound armature

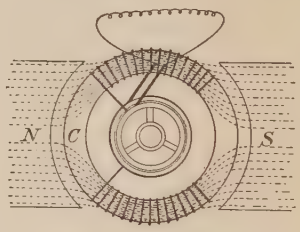


Fig. 330. End view of ring armature

In order to make the magnetic field in which the conductor is moved as strong as possible, the coil is wound upon an iron core C. This greatly increases the total number of lines of magnetic

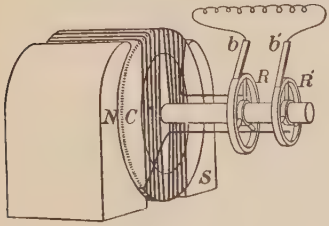


Fig. 331. Drum-wound armature

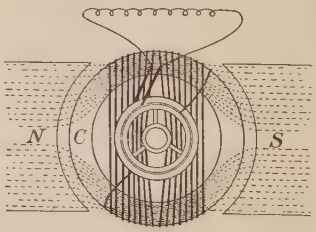


Fig. 332. End view of drum armature

force which pass between N and S, for the core offers an iron path, as shown in Fig. 330, instead of an air path from N to S.

The rotating part, consisting of the coil with its core, is called the *armature*. If the coil is wound in the manner shown in

Figs. 329 and 330, the armature is said to be of the *ring* type; if in the manner shown in Figs. 331 and 332, it is said to be of the *drum* type. The latter form of winding is used almost exclusively in modern machines.

One end of the coil is attached to the insulated metal ring R, which is attached rigidly to the shaft of the armature and therefore rotates with it, while the other end of the coil is attached to a second ring R'. The brushes b and b', which constitute the terminals of the external circuit, are always in contact with these rings.

As the coil rotates an induced alternating current passes through the circuit. This current reverses direction as often as the coil passes through the position shown in Figs. 330 and 332, i.e. the position in which the conductors are moving *parallel* to the lines of force; for at this instant the conductors which have been moving up begin to move down, and those which have been moving down begin to move up. The current reaches its maximum value when the coils are moving through a position 90° farther on than that shown in the figures, for then the lines of force are being cut most rapidly by the conductors on both sides of the coil.

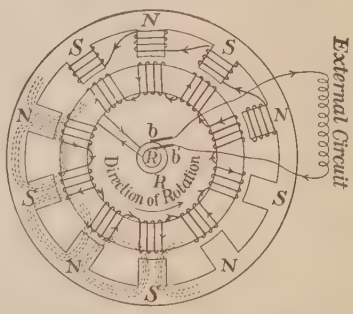


Fig. 333. Diagram of alternating-current dynamo

419. The multipolar alternator. For most commercial purposes it is found desirable to have 120 or more alternations of current per second. This could not be attained easily with two-pole machines like those sketched in Figs. 329 to 332. Hence commercial alternators are usually built with a large number of poles alternately N and S, arranged around the circumference of a circle in the manner shown in Fig. 333. The dotted lines represent the direction of the lines of force through the iron. It will be seen that the coils which are passing beneath north poles have induced currents set up in them the direction of which is opposite to that of the currents which are induced in the conductors

which are passing beneath the south poles. Since, however, the direction of winding of the armature coils changes between each two poles, all the inductive effects of all the poles are added together in the coil and constitute at any instant one single current flowing around the complete circuit in the manner indicated by the arrows in the diagram. This current reverses direction at the instant at which all the coils pass the midway points between the N and S poles. The number of alternations per second is equal to the number of poles multiplied by the number of revolutions per second. The field magnets N and S of such a dynamo are usually excited by a direct current from some other source. Fig. 334 represents a modern commercial alternator.

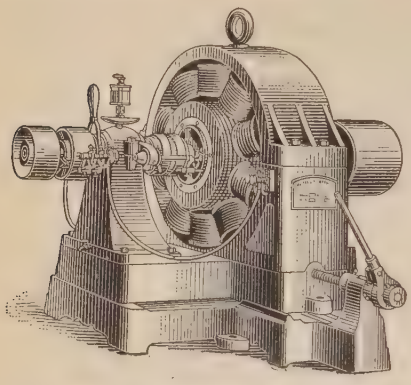


Fig. 334. Alternating-current dynamo

420. The principle of the commutator. By the use of a so-called *commutator* it is possible to transform a current which is alternating in the coils of the armature to one which always flows in the same direction through the external portion of the circuit. The simplest possible form of such a commutator is shown in Fig. 335. It consists of a single metallic ring which is split into two equal insulated semicircular segments a and c. One end of the rotating coil is soldered to one of these semicircles, and the other end to the other semicircle. Brushes b and b' are set in such positions that they lose contact with one semicircle and make contact with the other at the instant at which the current changes direction in

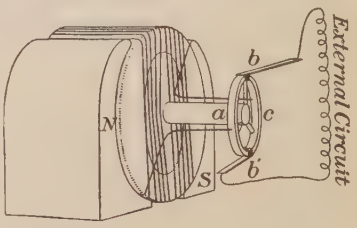


Fig. 335. The simple commutator

the armature. The current therefore always passes out to the external circuit through the same brush. While a current from such a coil and commutator as that shown in the figure would always flow in the same direction through the external circuit, it would be of a pulsating rather than a steady character, for it would rise to a maximum and fall again to zero twice during each complete revolution of the armature. This effect is avoided in the commercial direct-current dynamo by building a commutator of a large number of segments instead of two, and connecting each to a portion of the armature coil in the manner shown in Fig. 336.

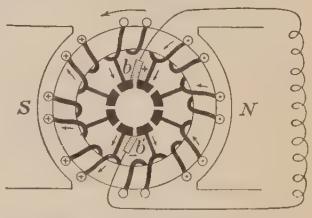


Fig. 336. Two-pole direct-current dynamo with ring armature

421. The ring-armature direct-current dynamo. Fig. 336 is a diagram illustrating the construction of a commercial two-pole direct-current dynamo of the ring-armature type. The figure represents an end view of a core like that shown in Fig. 329. The coil is wound continuously around the core, each segment being connected to a corresponding segment of the commutator, in the manner shown in the figure. At a given instant currents are being induced in the same direction in all the conductors on the outside of the core on the left half of the armature. The cross on these conductors, representing the tail of a retreating arrow, is to indicate that these currents flow away from the reader. No E.M.F.'s are induced in the conductors on the inner side of the ring, since these conductors cut no lines of force (see Fig. 330); nor are currents induced in the conductors at the top and bottom of the ring where the motion is parallel to the magnetic lines. The addition of all these similarly directed currents in the various convolutions of the continuous coil on the left side of the ring constitutes one single current flowing upward through this coil toward the brush *b* (see arrows).

On the right half of the ring, on the other hand, the induced currents are all in the opposite direction, i.e. toward the reader, since the conductors are here all moving up instead of down. The dot in the middle of these conductors represents the head of an approaching arrow. The summation of these currents constitutes one single current also flowing upward in the right half of the coil toward the brush b. These two currents from the two halves of the ring pass out at b through the external circuit and back at b'. This condition always exists, no matter how fast the rotation; for it will be seen that as each loop rotates into the position where the direction of its current reverses, it passes a brush and therefore at once becomes a part of the circuit on the other half of the ring where the currents are all flowing in the opposite direction.

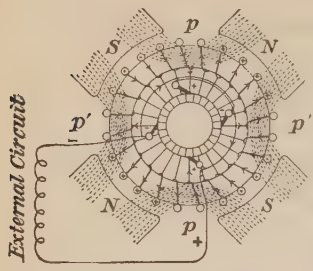


Fig. 337. Four-pole direct-current dynamo, ring-armature type

If the machine is of the four-pole type, like that shown in Fig. 337, the currents flow toward two neutral points, or points of no induction (see p, Fig. 337), instead of toward one, as in two-pole machines. Hence there are four brushes, two positive and two negative, as in the figure. Since the two positive and the two negative brushes are connected as shown, both sets of currents flow off to the external circuit on a single wire. The figure with its arrows will explain completely the generation of currents by a four-pole machine.

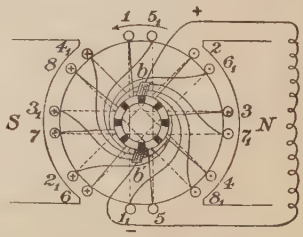


Fig. 338. The direct-current dynamo, drum winding

422. The drum-armature direct-current dynamo. The drum-wound armature, shown in section in Fig. 338, has an advantage over the ring armature in that, while the conductors on

the inside of the latter never cut lines of force and are, therefore, always idle, in the former all of the conductors are cutting lines of force except when they are passing the neutral points. In theory, however, the operation of the drum armature is precisely the same as that of the ring armature. All the conductors on the left side of the line connecting the brushes (see Fig. 338) carry induced currents which flow in one direction, while all the conductors on the right side of this line have opposite currents induced in them. It will be seen, however, in tracing out the connections 1, 1_1, 2, 2_1, 3, 3_1, etc., of Fig. 338 (the dotted lines representing connections at the back of the drum), that the coil is so wound about the drum that the currents in both halves are always flowing toward one brush *b*, from which they are led to the external circuit. Fig. 339 shows a typical modern four-pole generator, and Fig. 340 the corresponding drum-wound armature. Fig. 351, p. 335, illustrates nicely the method of winding such an armature, each coil beginning on one segment of the commutator and ending on the adjacent segment.

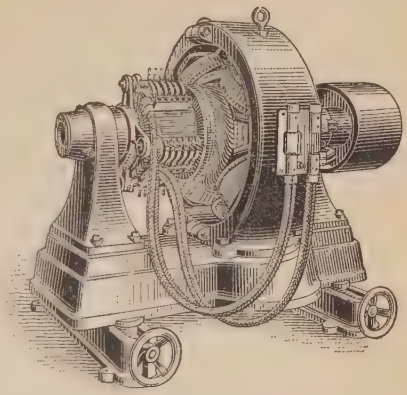


Fig. 339. Holtzer-Cabot four-pole direct-current generator

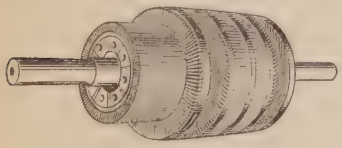


Fig. 340. Holtzer-Cabot armature

423. Series, shunt, and compound-wound dynamos. In direct-current machines the field magnet *NS* is excited by the current which the dynamo itself produces. In the so-called *shunt-wound* machines a small portion of the current is led off from the brushes through a great many turns of fine wire which encircle the core of the magnet, while the rest of the current flows through the external circuit (see Fig. 341). In the so-called *series dynamo* (Fig. 342) the whole of the current is carried through a few turns of coarse

wire which encircle the field magnets. These turns are then in series with the external circuit. In the *compound-wound machine* (Fig. 343) there is both a series and a shunt coil. By this arrangement it is possible to maintain a constant potential difference between the brushes, no matter how much the

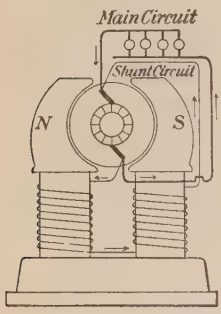


Fig. 341. The shunt-wound dynamo

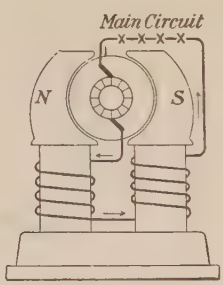


Fig. 342. The series-wound dynamo

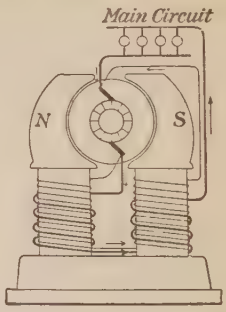


Fig. 343. The compound-wound dynamo

resistance of the external circuit may be varied. Hence, for purposes in which a varying current is demanded, as in incandescent lighting, the operation of street cars, etc., compound-wound dynamos are almost exclusively used.

In all these types of self-exciting machines there is enough residual magnetism left in the iron cores after stopping to start feeble induced currents when started up again. These currents immediately increase the strength of the magnetic field, and so the machine quickly builds up its current until the limit of magnetization is reached.

For incandescent electric lighting it is customary to use a dynamo of the compound type which gives a P.D. between its "mains" of either 110 or 220 volts. The lamps are always arranged in parallel between these mains, as is illustrated in Fig. 343. In arc lighting a series-wound dynamo is usually used, and the lamps are almost invariably arranged in series, as in Fig. 342. About 50 lamps are commonly fed by one

machine. This requires a dynamo capable of producing a voltage of 2500 volts, since each lamp requires a pressure of about 50 volts. Since an arc light usually requires a current of 10 amperes, such a dynamo must furnish 10 amperes at 2500 volts. The power is therefore $10 \times 2500 = 25{,}000$ watts. The dynamo must therefore have an activity of 25 kilowatts, or about 33.5 horse power.

QUESTIONS AND PROBLEMS

1. A multipolar alternator has 20 poles and rotates 200 times per minute. How many alternations per second will be produced in the circuit?

2. Two successive coils on the armature of a multipolar alternator are cutting lines of force which run in opposite directions. How does it happen that the currents generated flow through the wires in the same direction? (Fig. 333.)

3. Explain how an alternating current in the armature is transformed into a unidirectional current in the external circuit.

4. With the aid of the dynamo rule explain why, in Fig. 337, the current in the conductors under the south poles is moving toward the observer, and that in the conductors under the north poles away from the observer. Explain in a similar way the directions of the arrows in Figs. 336 and 338.

5. Explain why the brushes in Fig. 337 touch the commutator in the positions shown rather than at some other points.

6. If a direct-current machine of the same general type as that shown in Fig. 337 had twelve poles, how many brushes would be needed on the commutator?

7. A ring armature which develops the same E.M.F. as a drum armature has nearly twice as much wire and therefore nearly twice as much resistance. Why?

8. If a series-wound dynamo is running at a constant speed, what effect will be produced on the strength of the field magnets by diminishing the external resistance and thus increasing the current? What will be the effect on the E.M.F.? (Remember that the whole current goes around the field magnets.)

9. If a shunt dynamo is run at constant speed, what effect will be produced on the strength of the field magnets by reducing the external resistance? What effect will this have on the E.M.F.? (Remember that reducing the external resistance causes a smaller fraction of the current to flow through the shunt.)

10. In an incandescent-lighting system the lamps are connected in parallel across the mains. Every lamp which is turned on, then, diminishes the

external resistance. Explain from a consideration of Problems 8 and 9 why a compound-wound dynamo keeps the P.D. between the mains constant.

11. Single dynamos often operate as many as 10,000 incandescent lamps at 110 volts. If these lamps are all arranged in parallel and each requires a current of .5 ampere, what is the total current furnished by the dynamo? What is the activity of the machine in kilowatts and in horse power?

12. How many 110-volt lamps can be lighted by a 12,000-kilowatt generator?

The Principle of the Electric Motor

424. Effect of a magnetic field on a wire bearing a current. Let a vertical wire *ab* be rigidly attached to a horizontal wire *gh*, and let the latter be supported by a ring or other metallic support, in the manner shown in Fig. 344, so that *ab* is free to oscillate about *gh* as an axis. Let the lower end of *ab* dip into a trough of mercury. When a magnet is held in the position shown and a current from a Leclanché or bichromate cell is sent through the wire in the direction indicated, the wire will instantly move in the direction indicated by the arrow *f*, viz. at right angles to the direction of the lines of magnetic force. Let the direction of the current in the wire be reversed. The direction of the force acting on the wire will be found to be reversed also.

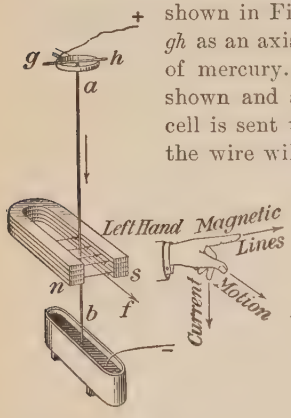


Fig. 344. The principle of the electric motor

We learn, therefore, that *a wire carrying a current in a magnetic field tends to move in a direction at right angles both to the direction of the field and to the direction of the current.* The relation between the direction of the magnetic lines, the direction of the current, and the direction of the force, is often remembered by means of the following rule, known as the *motor rule.* It differs from the dynamo rule only in that it is applied to the fingers of the *left hand* instead of to those of the right. *Let the forefinger of the left hand point in the direction of the magnetic lines of force and the middle finger in the direction of the current sent through*

the wire; the thumb will then point in the direction of the mechanical force acting to move the wire (see Fig. 344).

425. The electric motor. The electric motor is a simple application of the results of the preceding experiment. In construction the motor differs in no essential respect from the dynamo. To analyze the operation as a motor of such a machine as that shown in Fig. 336, suppose a current from an outside source is first sent around the coils of the field magnets and then into the armature at b'. Here it will divide and flow through all the conductors on the left half of the ring in one direction, and through all those on the right half in the opposite direction. Hence, in accordance with the motor rule, all the conductors on the left side are urged upward by the influence of the field, and all those on the right side are urged downward. The armature will therefore begin to rotate, and this rotation will continue so long as the current is sent in at b' and out at b. For as fast as coils pass either b or b', the direction of the current flowing through them changes, and therefore the direction of the force acting on them changes. The left half is therefore always urged up and the right half down. The greater the strength of the current, the greater the force acting to produce rotation.

If the armature is of the drum type (Fig. 338), the conditions are not essentially different. For, as may be seen by following out the windings, the current entering at b' will flow through all the conductors in the left half in one direction and through those on the right half in the opposite direction. The commutator keeps these conditions always fulfilled. The analysis of the operation of a four-pole dynamo (Fig. 337) as a motor is equally simple.

426. Street-car motors. Electric street cars are nearly all operated by direct-current series-wound motors placed under the cars and attached by gears to the axles. Fig. 345 shows a typical four-pole street-car motor. The two upper field poles are raised with the case when the motor is opened for inspection, as in the figure. The current is generally

supplied by compound-wound dynamos which maintain a constant potential of about 500 volts between the trolley, or third rail, and the track which is used as the return circuit. The cars are always operated in parallel, as shown in Fig. 346. In a few instances street cars are operated upon alternating, instead of upon direct-current, circuits. In such cases the motors are essentially the same as direct-current series-wound motors; for since in such a machine the current must reverse in the field magnets at the same time that it reverses in the armature, it will be seen that the armature is always impelled to rotate in one direction, whether it is supplied with a direct or with an alternating current.

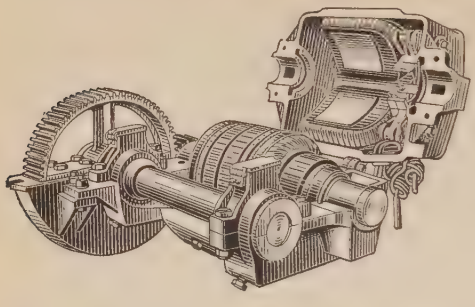


Fig. 345. Railway motor, upper field raised

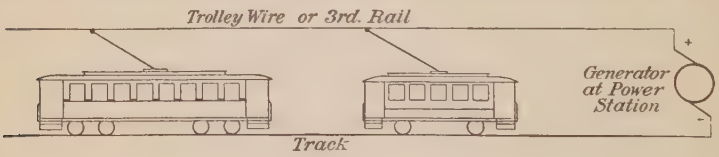


Fig. 346. Street-car circuit

427. Back E.M.F. in motors. When an armature is set into rotation by sending a current from some outside source through it, its coils move through a magnetic field as truly as if the rotation were produced by a steam engine, as is the case in running a dynamo. An induced current, or better, an induced E.M.F., is therefore set up by this rotation. In other words, while the machine is acting as a motor, it is also acting as a dynamo. The direction of the induced E.M.F. due to this dynamo effect will be seen from Lenz's law, or from a consideration of the dynamo and motor rules, to be opposite to the outside P.D. which is causing current to pass through the motor. The faster

the motor rotates, the faster the lines of force are cut, and hence the greater the value of this so-called *back E.M.F.* If the motor were doing no work, the speed of rotation would increase until the back E.M.F. reduced the current to a value simply sufficient to overcome friction. It will be seen, therefore, that in general the faster the motor goes, the less the current which passes through its armature, for this current is always due to the *difference* between the P.D. applied at the brushes,—500 volts in the case of trolley cars,—and the back E.M.F. When the motor is starting the back E.M.F. is zero, and hence, if the full 500 volts were applied to the brushes, the current sent through would be so large as to ruin the armature through overheating. To prevent this each car is furnished with a "starting box," which consists of resistance coils which the motorman throws into series with the motor on starting, and throws out again gradually as the speed increases and the back E.M.F. consequently rises.[1]

QUESTIONS AND PROBLEMS

1. A current is flowing from top to bottom in a vertical wire. In what direction will the wire tend to move on account of the earth's magnetic field?

2. If a current is sent into the armature of Fig. 336 at b', and taken out at b, which way will the armature revolve?

3. When an electric fan is first started the current through it is much greater than it is after the fan has attained its normal speed. Why?

4. If in the machine of Fig. 337 a current is sent in on the wire marked +, what will be the direction of rotation?

5. Would an armature wound on a wooden core be as effective as one made of the same number of turns wound on an iron core?

6. Will it take more work to rotate a dynamo armature when the circuit is closed than when it is open? Why?

7. Show that if the reverse of Lenz's law were true, a motor once started would run of itself and do work, i.e. it would furnish a case of perpetual motion.

[1] This discussion should be followed by a laboratory experiment on the study of a small electric motor or dynamo. See, for example, Experiment No. 37 of the authors' manual.

8. Why does it take twice as much work to keep a dynamo running when 1000 lights are on the circuit as when only 500 are turned on?

9. Explain why a series-wound motor can run either on a direct or an alternating circuit.

10. If the pressure applied at the terminals of a motor is 500 volts, and the back pressure, when running at full speed, is 450 volts, what is the current flowing through the armature, its resistance being 10 ohms?

Principle of the Induction Coil and Transformer

428. Currents induced by varying the strength of a magnetic field. Let about 500 turns of No. 28 copper wire be wound around one end of an iron core, as in Fig. 347, and connected to the circuit of a D'Arsonval galvanometer, like that described in § 356. Let about 500 more turns be wrapped about another portion of the core and connected into the circuit of two bichromate or dry cells. When the key K is closed the deflection of the galvanometer will indicate that a temporary current has been induced in one direction through the coil s, and when it is opened an equal but opposite deflection will indicate an equal current flowing in the opposite direction.

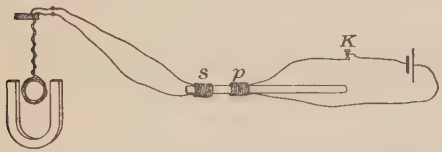


Fig. 347. Induction of current by magnetizing and demagnetizing an iron core

The experiment illustrates the principle of the induction coil and the transformer. The coil p, which is connected to the source of the current, is called the *primary coil*, and the coil s, in which the currents are induced, is called the *secondary coil*. Causing lines of force to spring into existence inside of s—in other words, magnetizing the space inside of s—has caused an induced current to flow in s; and demagnetizing the space inside of s has also induced a current in s in accordance with the general principle stated in § 414, p. 314, that *any change in the number of magnetic lines of force which thread through a coil induces a current in the coil.* We may think of the lines

which suddenly appear within the iron core upon magnetization as springing from without across the loops into the core, and as springing back again upon demagnetization, thus cutting the loops while moving in opposite directions in the two cases.

429. Direction of the induced current. Lenz's law, which, it will be remembered, followed from the principle of conservation of energy, enables us to predict at once the direction of the induced currents in the above experiments; and an observation of the deflections of the galvanometer enables us to verify the correctness of the predictions. Consider first the case in which the primary circuit is *made* and the core thus magnetized. According to Lenz's law, the current induced in the secondary circuit must be in such a direction as to *oppose the change* which is being produced by the primary current, i.e. in such a direction as to tend to magnetize the core oppositely to the direction in which it is being magnetized by the primary. This means, of course, that the induced current in the secondary must encircle the core in a direction opposite to the direction in which the primary current encircles it. We learn, therefore, that *on making the current in the primary the current induced in the secondary is opposite in direction to that in the primary.*

When the current in the primary is *broken*, the magnetic field created by the primary tends to die out. Hence, by Lenz's law, the current induced in the secondary must be in such a direction as to tend to oppose this process of demagnetization, i.e. in such a direction as to magnetize the core in the same direction in which it is magnetized by the decaying current in the primary. Therefore, *at break the current induced in the secondary is in the same direction as that in the primary.*

430. E.M.F. of the secondary. If half of the 500 turns of the secondary *s* (Fig. 347) are unwrapped, the deflection will be found to be just half as great as before. Since the resistance of the circuit has not been changed, we learn from this that *the E.M.F. of the secondary is proportional to the number of turns*

of wire upon it, — a result which followed also from § 415. If, then, we wish to develop a very high E.M.F. in the secondary, we have only to make it of a very large number of turns of fine wire. The wire must not, however, be wrapped so far away from the core as to include the lines of force which are returning through the air (see Fig. 309, p. 301), for when this happens the coils are threaded in both directions by the same lines, and hence have no current induced in them.

431. E.M.F. at make and break. Let the secondary coil *s* (Fig. 347) be replaced by a spool or paper cylinder upon which are wound from 5000 to 10,000 turns of No. 36 or No. 40 copper wire. Let the ends of this coil be attached to metal handles and held in the moistened hands. When the key *K* is closed no shock whatever will be felt, but a very marked one will be observable when the key is opened.

The experiment shows that the E.M.F. developed at the break of the circuit is enormously greater than that at the make. The explanation is found in the fact that the E.M.F. developed in a coil depends upon the *rate* at which the number of lines of force passing through it is made to change (cf. § 415). When the circuit of the primary was *made* the current required an appreciable time, perhaps a tenth of a second, to rise to its full value, just as a current of water, started through a hose, requires an appreciable time to rise to its full height on account of the inertia of the water. An electrical current possesses a property similar to inertia. Hence the magnetic field about the primary also rises equally gradually to its full strength, and therefore its lines pass into the coil comparatively slowly. At *break,* however, by separating the contact points very quickly we can make the current in the primary fall to zero in an exceedingly short time, perhaps not more than .00001 second; i.e. we can make all of its lines pass out of the coil in this time. Hence the *rate* at which lines thread through or cut the secondary is perhaps 10,000 times as great at break as at make, and therefore the E.M.F. is also something like 10,000 times as

great. It should be remembered, however, that in a closed secondary the make current lasts as much longer than the break as its E.M.F. is smaller; hence the total energy of the two is the same, as was indeed indicated by the equal deflections in § 428.

432. The induction coil. The induction coil, as usually made (Fig. 348), consists of (1) a soft iron core *C*, Fig. 348 (1), composed of a bundle of soft iron wires; (2) a primary coil *p* wrapped around this core, and consisting of say 200 turns of coarse copper wire (e.g. No. 16), which is connected into the circuit of a battery through the contact point at the end of

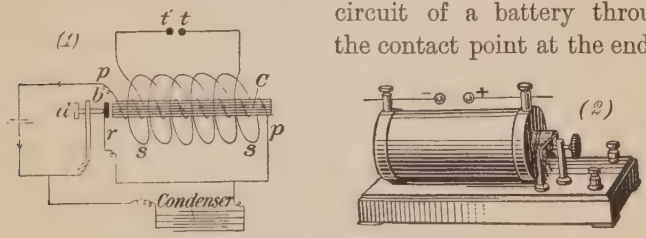


Fig. 348. Induction coil

the screw *d*; (3) a secondary coil *s* surrounding the primary in the manner indicated in the diagram, and consisting generally of between 30,000 and 1,000,000 turns of No. 36 copper wire, the terminals of which are the points *t* and *t'*; and (4) a hammer *b*, or other automatic arrangement for making and breaking the circuit of the primary.

When the current is first started in the primary it magnetizes the core *C*. Thereupon the iron hammer *b* is drawn away from its contact with *d* and the current is thus suddenly stopped. This instantly demagnetizes the core and induces in the secondary *s* an E.M.F. which is usually sufficient to cause a spark to leap between *t* and *t'*. As soon as the core is demagnetized the spring *r* which supports the hammer restores the contact with *d* and the operation is repeated. The condenser, shown in the diagram with its two sets of plates connected to the conductors on either side of the spark gap between *r* and *d*, is not an essential part of a coil, but when it is introduced it is found that

the length of the spark which can be sent across between t and t' is considerably increased. The reason is as follows: When the circuit is broken at b the inertia of the current tends to make a spark jump across from d to b, and if this happens the current continues to flow through this spark (or arc) until the terminals have become separated through a considerable distance. This makes the current die down gradually instead of suddenly, as it ought to do to produce a high E.M.F. But when a condenser is inserted, as soon as b begins to leave d the current begins to flow into the condenser, and this gives the hammer time to get so far away from d that an arc cannot be formed. This means a sudden break and a high E.M.F. Since a spark passes between t and t' only at break (§ 431), it must always pass in the same direction. Coils which give 24-inch sparks (perhaps 500,000 volts) are not uncommon. Such coils usually have hundreds of miles of wire upon their secondaries.

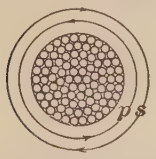


Fig. 349. Core of insulated iron wires

433. Laminated cores. Foucault currents. The core of an induction coil should always be made of a bundle of soft iron wires insulated from one another by means of shellac or varnish (see Fig. 349); for whenever a current is started or stopped in the primary p of a coil furnished with a *solid* iron core (see Fig. 350) the change in the magnetic field of the primary induces a current in the conducting core C for the same reason that it induces one in the secondary s. This current flows around the body of the core in the same direction as the induced current in the secondary, i.e. in the direction of the arrows. The only effect of these so-called *eddy* or *Foucault* currents is to heat the core. This is obviously a waste of energy. If we can prevent the appearance of these currents, all of the energy which they would waste in heating the core may be made to appear in the current of the secondary. The core is therefore built of varnished iron wires, which run parallel to the axis of the coil, i.e. perpendicular to the direction in which the currents would be induced. The induced E.M.F., therefore, finds no closed circuits in which to set up a current (Fig. 349). It is for the same reason that the iron cores of dynamo and motor armatures, instead of being solid, consist of iron disks placed side by side, as shown in Fig. 351, and insulated from one another by films of oxide. A core of this kind is called a

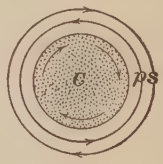


Fig. 350. Diagram showing eddy currents in solid core

laminated core. It will be seen that in all such cores the spaces or slots between the laminæ must run at right angles to the direction of the induced E.M.F., i.e. perpendicular to the conductors upon the core.

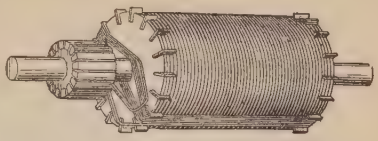


FIG. 351. Laminated drum-armature core with commutator, showing one coil wound on the core

434. The transformer. The commercial transformer is a modified form of the induction coil. The chief difference is that the core *R* (Fig. 352), instead of being straight, is bent into the form of a ring, or is given some other shape such that the magnetic lines of force have a continuous iron path, instead of being obliged to push out into the air, as in the induction coil. Furthermore, it is always an alternating instead of an intermittent current which is sent through the primary *A*. Sending such a current through *A* is equivalent to magnetizing the core first in one direction, then demagnetizing it, then magnetizing it in the opposite direction, etc. The results of these changes in the magnetism of the core is of course an induced alternating current in the secondary *B*.

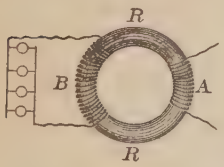


FIG. 352. Diagram of transformer

435. The use of the transformer. The use of the transformer is to convert an alternating current from one voltage to another which for some reason is found to be more convenient. For example, in electric lighting where an alternating current is used, the E.M.F. generated by the dynamo

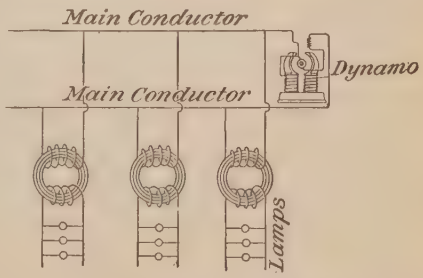


FIG. 353. Alternating current lighting circuit with transformers

is usually either 1100 or 2200 volts, a voltage too high to be introduced safely into private houses. Hence transformers are connected across the main conductors in the manner shown in Fig. 353. The current which passes into the houses to supply the lamps does not come directly from the dynamo. It is an induced current generated in the transformer.

436. Pressure in primary and secondary. If there are a few turns in the primary and a large number in the secondary, the transformer is called a *step-up* transformer, because the P.D. produced at the terminals of the secondary is greater than that applied at the terminals of the primary. Thus, an induction coil is a step-up transformer. In electric lighting, however, transformers are mostly of the *step-down* type; i.e. a high P.D., say 2200 volts, is applied at the terminal of the primary, and a lower P.D., say 110 volts, is obtained at the terminals of the secondary. In such a transformer the primary will have twenty times as many turns as the secondary. In general, *the ratio between the voltages at the terminals of the primary and secondary is the ratio of the number of turns of wire upon the two.*

437. Efficiency of the transformer. In a perfect transformer the efficiency would be unity. This means that the electrical energy put into the primary, i.e. the volts applied to its terminals times the amperes flowing through it, would be exactly equal to the energy taken out in the secondary, i.e. the volts generated in it times the strength of the induced current; and, in fact, in actual transformers the latter product is often more than 97% of the former, — i.e. there is less than 3% loss of energy in the transformation. This lost energy appears as heat in the transformer. This transfer, which goes on in a big transformer, of huge quantities of power from one circuit to another entirely independent circuit, without noise or motion of any sort and almost without loss, is one of the most wonderful phenomena of modern industrial life.

438. Commercial transformers. Fig. 354 illustrates a common type of transformer used in electric lighting. The core is built up of sheet-iron laminæ about $\frac{1}{2}$ mm. thick. Fig. 355 shows a section of the same transformer. The closed

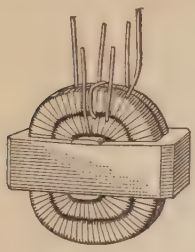


Fig. 354. Commercial transformer

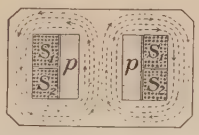


Fig. 355. Cross section of transformer showing shape of magnetic field

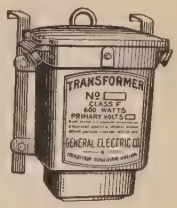


Fig. 356. Transformer case

magnetic circuit of the core is indicated by the arrows. The primary and the two secondaries, which can furnish either 52 or 104 volts, are indicated by the letters p, S_1, and S_2. Fig. 356 is the case in which the transformer is placed. Such cases may be seen attached to poles outside of houses in any district where alternating currents are used for electric-lighting purposes.

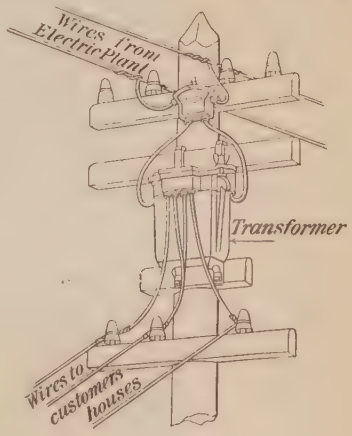


Fig. 357. Transformer on electric-light pole

439. Electrical transmission of power. Since the electrical energy produced by a dynamo is equal to the product of the E.M.F. generated by the current furnished, it is evident that in order to transmit from one point to another a given number of watts, say 10,000, it is possible to have either an E.M.F. of 100 volts and a current of 100 amperes, or an E.M.F. of 1000 volts and a current of 10 amperes. In the two cases, however, the loss of energy in the wire which carries the current from the place where it is generated to the place where it is used will be widely different. If R represents the resistance of this transmitting wire, the so-called "line," and C the current flowing through it, we have seen in § 406, p. 307, that the heat developed in it will be

proportional to C^2R. Hence the energy wasted in heating the line will be but $\frac{1}{100}$ as much in the case of the high-voltage, 10-ampere current, as in the case of the lower-voltage, 100-ampere current. Hence, for long-distance transmission, where line losses are considerable, it is important to use the highest possible voltages.

On account of the difficulty of insulating the commutator segments from one another, voltages higher than 700 or 800 cannot be obtained with direct-current dynamos of the kind which have been described. With alternators, however, the difficulties of insulation are very much less on account of the absence of a commutator. The large 10,000-horse-power alternating-current dynamos on the Canadian side of Niagara Falls generate directly 12,000 volts. This is the highest voltage thus far produced by generators. In all cases where these high pressures are employed they are transformed down at the receiving end of the line to a safe and convenient voltage (from 50 to 500 volts) by means of step-down transformers.

440. Long-distance transmission of power. It will be seen from the above facts that only alternating currents are suitable for long-distance transmission. Plants are now in operation which transmit power as far as 80 miles and use pressures as high as 60,000 volts. In all such cases *step-up* transformers, situated at the power house, transfer the electrical energy developed by the generator to the line, and step-down transformers, situated at the receiving end, transfer it to the motors, or lamps, which are to be supplied. The generators used on the American side of Niagara Falls produce a pressure of 2300 volts. For transmission to Buffalo, twenty miles away, this is transformed up to 22,000 volts. At Buffalo it is transformed down to the voltages suitable for operating the street cars, lights, and factories of the city. On the Canadian side the generators produce currents at 12,000 volts, as stated, and this is transformed up, for long-distance transmission, to 22,000, 40,000, and 60,000 volts.

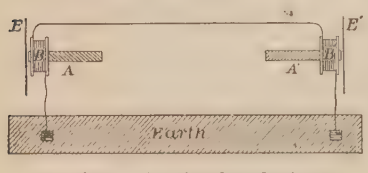


Fig. 358. The simple telephone

441. The simple telephone. The telephone was invented in 1876 by Elisha Gray, of Chicago, and Alexander Graham Bell, of Washington. In its simplest form it consists, at each end, of a permanent bar magnet A (Fig. 358) surrounded by a coil of fine wire B, in series with the line, and an iron disk or

diaphragm E mounted close to one end of the magnet. When a sound is made in front of the diaphragm, the vibrations produced by the sounding body are transmitted by the air to the diaphragm, thus causing the latter to vibrate back and forth in front of the magnet. These vibrations of the diaphragm produce slight backward and forward movements of the lines of force which pass into the disk from the magnet in the manner shown in Fig. 359. Some of these lines of force, therefore, cut across the coil B, first in one direction and then in the other, and in so doing induce currents in it. These induced currents are transmitted by the line to the receiving station, where those in one direction pass around B' in such a way as to *increase* the strength of the magnet A', and thus increase the pull which it exerts upon E'; while the opposite currents pass around B' in the opposite direction, and therefore *weaken* the magnet A' and diminish its pull upon E'. When, therefore, E moves in one direction E' also moves in one direction, and when E reverses its motion the direction of motion of E' is also reversed. In other words, the induced currents, transmitted by the line, force E' to reproduce the motions of E. E' therefore sends out sound waves exactly like those which fell upon E. In exactly the same way a sound made in front of E' is reproduced at E. Telephones of this simple type will work satisfactorily for a distance of several miles. This simple form of instrument is still used at the receiving end of the modern telephone, the only innovation which has been introduced consisting in the substitution of a U-shaped magnet for the bar magnet. The instrument used at the transmitting end has, however, been changed, as explained in the next paragraph,

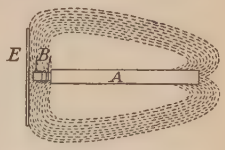


FIG. 359. Magnetic field about a telephone receiver

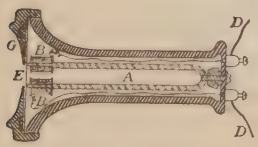


FIG. 360. The modern receiver

and the circuit is now completed through a return wire instead of through the earth. A modern telephone receiver is shown in Fig. 360. G is the mouthpiece, E the diaphragm, A the U-shaped magnet, B the coils, consisting of many turns of fine wire, and D the terminals of the line.

442. The modern transmitter. To increase the distance at which telephoning may be done, it is necessary to increase the strength of the

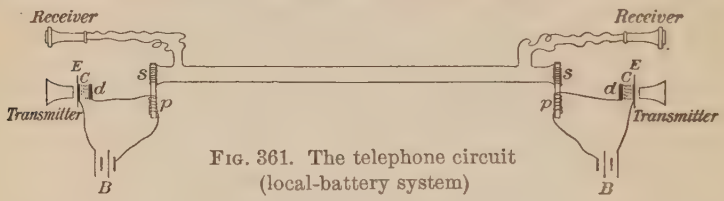


Fig. 361. The telephone circuit (local-battery system)

induced currents. This is done in the modern transmitter by replacing the magnet and coil by an arrangement which is essentially an induction coil, the current in the primary of which is caused to vary by the motion of the diaphragm. This is accomplished as follows. The current from the battery (B, Fig. 361) is led first to the back of the diaphragm E, whence it passes through a little chamber C filled with granular carbon to the conducting back d of the transmitter, and thence through the primary p of the induction coil, and back to the battery. As the diaphragm vibrates it varies the pressure upon the many contact points of the granular carbon through which the primary current flows. This produces considerable variation in the resistance of the primary circuit, so that as the diaphragm moves forward, i.e. toward the carbon, a comparatively large current flows through p, and as it moves back a much smaller current. These changes in the current strength in the primary p produce changes in the magnetism of the soft-iron core of the induction coil. Currents are therefore induced in the secondary s of the induction coil, and these currents pass over the line and affect the receiver at the other end in the manner explained in the preceding paragraph. Fig. 362 shows the cross section of a complete long-distance transmitter.

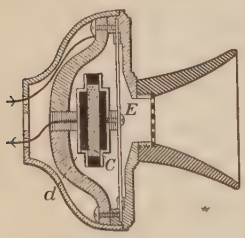


Fig. 362. Cross section of a long-distance telephone transmitter

443. The subscriber's telephone connections. In the most recent practice of the Bell Telephone Company the local battery at the subscriber's end is done away with altogether and the primary current is furnished by a battery at the central station. Fig. 363 shows the essential elements of such a system. A battery B, usually of 25 volts pressure, is always kept connected at "central" to all the lines which enter the exchange. No current flows through these lines, however, so long as the subscribers' receivers R are upon their hooks H; for the line circuit is then open at the contact points t. It would be closed through the bell b were it not for the introduction of the condenser C in series with the bell. This makes it impossible for any *direct* current to pass from one side of the line to the other, so long as the receiver is upon the hook. But if the operator at central wishes to call up the

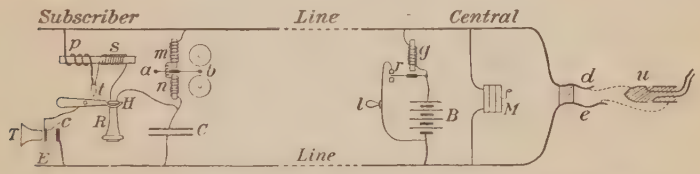


Fig. 363. The modern telephone circuit (central-station system)

subscriber, she has only to throw upon the line an *alternating* current from the magneto M, or from any alternating-current generator whose terminals she can connect to the subscriber's line by turning a switch. This alternating current surges back and forth through the bell into the condenser and out again, first charging the condenser plates in one direction, then in the other. By making the capacity of this condenser sufficiently large this alternating current is made strong enough to pull the armature a first toward the electro-magnet m, then toward n. In this way it rings the bell.

On the other hand, if the subscriber wishes to call up central, he has only to lift the receiver from the hook. This closes the line circuit at t, and the direct current which at once begins to flow from the battery B through the electro-magnet g closes the circuit of B through the glow lamp l and the contact point r. This lights up the lamp l which is upon the switch board in front of the operator. Upon seeing this signal the latter moves a switch which connects her own telephone to the subscriber's line. Then, as the latter talks into the transmitter T, the strength of the direct current from the battery B, through the primary p,

is varied by the varying pressure of the diaphragm E upon the granular carbon c, and these variations induce in the secondary s the talking currents which pass over the line to the receiver of the operator. Although with this arrangement the primary and secondary currents pass simultaneously over the same line, speech is found to be transmitted quite as distinctly as when the two circuits are entirely separate, as is the case with the arrangement of Fig. 361. When the operator finds what number the subscriber wishes, she connects the ends d and e of his line with the ends of the desired line by means of a flexible conducting cord which terminates in a metallic plug u, suitable for making contact with d and e. As soon as the subscriber replaces his receiver upon its hook the lamp l is extinguished and the operator thereupon withdraws u and thus disconnects the two lines.

QUESTIONS AND PROBLEMS

1. Does the spark of an induction coil occur at "make" or at "break"? Why?

2. Explain why an induction coil is able to produce such an enormous E.M.F. Draw a diagram to illustrate the method of operation of the coil.

3. Why could not an armature core be made of coaxial cylinders of iron running the full length of the armature, instead of flat disks, as shown in Fig. 351?

4. What relation must exist between the number of turns on the primary and secondary of a transformer which feeds 110-volt lamps from a main line whose conductors are at 1000 volts P.D.?

5. The same amount of power is to be transmitted over two lines from a power plant to a distant city. If the heat losses in the two lines are to be the same, what must be the ratio of the cross sections of the two lines if one current is transmitted at 100 volts and the other at 10,000 volts?

6. Explain the operation of a simple telephone.

7. What are some of the advantages of the modern transmitter over the original form?

CHAPTER XVII[1]

NATURE AND TRANSMISSION OF SOUND

Speed of Sound

444. Sources of sound. Whenever one investigates the source of a sound he always finds that it can be traced to the motion of some material body. Thus, if he examine a violin string which is giving forth a note, he finds that it looks broader than when at rest, and that it has a hazy outline (Fig. 364). He infers, therefore, that it is in rapid vibration. If he investigates a sounding tuning fork, he finds that if one prong is touched to the surface of a dish of mercury, it sends forth a series of ripples; if it is provided with a stylus and stroked across a smoked-glass plate, it produces a wavy line, as shown in Fig. 365; if a light, suspended ball is brought into contact with it, the latter is thrown off with considerable violence. He infers, therefore, that a sounding tuning fork is in rapid vibration. If he looks about for the source of any sudden noise, he finds that some object has fallen, or some collision has occurred, or some explosion has taken place; in a word, that some violent motion of matter has been set up in some way. From these familiar facts we conclude that *sound arises from the motions of matter.*

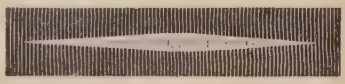


Fig. 364. Appearance of a vibrating string

Fig. 365. Trace made by vibrating fork

[1] This chapter should be accompanied by laboratory experiments on the speed of sound in air, the vibration rate of a fork, and the determination of wave lengths. See, for example, Experiments 38, 39, and 40 of the authors' manual.

445. Media of transmission of sound. Air is ordinarily the medium through which sound comes from its source to the ear of the observer. It is easy to show, however, that substances other than air may also serve to convey it.

Thus, if an ear is placed against one end of a long beam or table, a light scratching at the other end may be heard much more distinctly than if the ear is removed from the wood. Again, most boys are familiar with the fact that the clapping together of two stones may be heard even better when the ear and the stones are under water than when the experiment takes place in air.

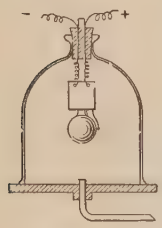


Fig. 366. Sound not transmitted through a vacuum

These experiments show that a gas like air is certainly no more effective in the transmission of sound than a liquid like water or a solid like wood.

Next, let us see whether or not matter is necessary at all for the transmission of sound.

Let an electric bell be suspended inside the receiver of an air pump by means of two fine springs which pass through a rubber stopper in the manner shown in Fig. 366. Let the air be exhausted from the receiver by means of a good air pump. The sound of the bell will be found to become less and less pronounced. Let the air be suddenly readmitted. The volume of sound will at once increase.

Since, then, the nearer we approach a vacuum, the less distinct becomes the sound, we infer that sound cannot be transferred through a vacuum and that therefore *the transmission of sound is effected through the agency of ordinary matter.* In this respect sound differs from heat and light, which evidently pass with perfect readiness through a vacuum, since they reach the earth from the sun and stars.

446. Speed of transmission. In rooms of ordinary dimensions we are not conscious that it requires any time for sound to travel from its source to our ears. That it does, however, have a speed of propagation which is not too fast for easy detection

is proved by the fact of common observation that a thunder-clap follows usually at a considerable interval after the lightning flash, and that this interval is greater, the greater the distance of the observer from the flash; or again, that steam may be seen issuing from the whistle of a distant locomotive or steamboat some seconds before the sound is heard; or again that, in this latter case, the sound is heard for a corresponding interval after the steam has ceased to rise.

The first attempt to measure accurately the speed of sound was made in 1738, when a commission of the French Academy of Sciences stationed two parties about three miles apart and observed the interval between the flash of a cannon and the sound of the report. By taking observations between the two stations, first in one direction and then in the other, the effect of the wind was eliminated. A second commission repeated these experiments in 1832, using a distance of 18.6 km., or a little more than 11.5 mi. The value found was 331.2 m. per second at 0° C. The speed in water is about 1400 m. per second and in iron 5100 m.

447. Speed and temperature. The speed of sound in air is found to increase with an increase in temperature. The amount of this increase is about 60 cm. per degree Centigrade. Hence the speed at 20° C. is about 343.2 m. per second. The above figures are equivalent to 1087 ft. per second at 0° C., or 1126 ft. per second at 20° C.

QUESTIONS AND PROBLEMS

1. A thunder-clap was heard five and one half seconds after the accompanying lightning flash was seen. How far away did the flash occur?

2. On August 26, 1883, a volcanic eruption occurred at Krakatoa, near Java. A great volume of gas was thrown upward and a wave in the atmosphere was thus started around the earth. The existence of this wave was made evident by a sudden rise in the barometers in the regions over which it passed. By this means the progress of the wave from point to point could be traced. It was found to make a complete circuit of the earth in 36 hours. Compute the speed of the wave.

3. A bullet fired from a rifle with a speed of 1200 ft. per second is heard to strike the target 6 seconds afterward. What is the distance to the target, the temperature of the air being 20° C.?

4. A clapper strikes a bell once every two seconds. How far from the bell must a man be in order that the clapper may appear to hit the bell at the exact instant at which each stroke is heard, if the temperature is 20° C.?

5. A stone is dropped into a well 200 m. deep. At 20° C., how much time will elapse before the sound of the splash is heard at the top?

6. A railroad rail was struck a heavy blow with a hammer. How much sooner may a man a mile away hear the blow if his ear is placed against the rail, than if the sound travels all the way through air?

Nature of Sound

448. Mechanism of sound transmission. When a firecracker or toy cap explodes the powder is suddenly changed to a gas, the volume of which is enormously greater than the volume of the powder. The air is therefore suddenly pushed back in all directions from the center of the explosion. This means that the air particles which lie about this center are given violent outward velocities.[1] When these outwardly impelled air particles collide with other particles, they give up their outward motion to these second particles, and these in turn pass it on to others, etc. It is clear, therefore, that the motion started by the explosion must travel on from particle to particle to an indefinite distance from the center of the explosion. Furthermore, it is also clear that, although the motion travels on to great distances, the individual particles do not move far from their original positions; for it is easy to show experimentally that whenever an elastic body in motion collides with another similar body at rest, the colliding body simply transfers its motion to the body at rest, and comes itself to rest.

[1] These outward velocities are simply superposed upon the velocities of agitation which the molecules already have on account of their temperature. For our present purpose we may ignore entirely the existence of these latter velocities and treat the particles as though they were at rest, save for the velocities imparted by the explosion.

Let six or eight equal steel balls be hung from cords in the manner shown in Fig. 367. First, let all of the balls but two adjacent ones be held to one side, and let one of these two be raised and allowed to fall against the other. The first ball will be found to lose its motion in the collision, and the second will be found to rise to practically the same height as that from which the first fell. Next, let all of the balls be placed in line and the end one raised and allowed to fall as before. The motion will be transmitted from ball to ball, each giving up the whole of its motion practically as soon as it receives it, and the last ball will move on alone with the velocity which the first ball originally had.

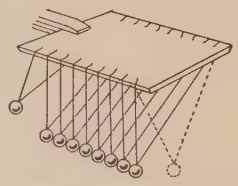


Fig. 367. Illustrating the propagation of sound from particle to particle

The preceding experiment furnishes a very nice mechanical illustration of the manner in which the air particles which receive motions from an exploding firecracker transmit these motions to neighboring layers, these in turn to the next adjoining, etc., until the motion has traveled to very great distances, although the individual particles themselves move only very minute distances. When a motion of this sort, transmitted by air particles, reaches the drum of the ear, it produces the sensation which we call *sound.*

449. Loudness. The *loudness* or *intensity* of the sound perceived by an observer depends simply upon the energy of the impulse which is communicated to the tympanum of the ear; and this in turn depends, first, upon the *energy* of the initial disturbance, and second, upon the *distance* of the ear from it. If, for example, the source of the sound is some particular vibrating rod or string, then the loudness observed at a given distance will depend simply upon the *amplitude of vibration* of the source, since the energy of the initial disturbance depends simply upon this amplitude.

The reason that a given sound grows weaker and weaker as we recede from its source is found in the fact that the original energy which was put into the disturbance gets distributed over

more and more particles the farther the pulse recedes from the origin. Thus, if a sound is free to spread out in all directions from a source O (Fig. 368), when it has reached a distance $2\,a$ from O, the original energy will be distributed over the surface of a sphere of radius $2\,a$, i.e. over four times as large a surface, and therefore over four times as many particles as when the pulse was at a distance a. Hence the intensity, or loudness, at $2\,a$ can be but one fourth as much as it was at a. We see, therefore, that under these ideal conditions *the intensity of a sound pulse varies inversely as the square of the distance from the source.*

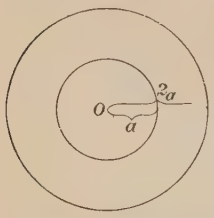


Fig. 368. Geometrical proof that the intensity of sound is inversely proportional to the square of the distance

450. Speaking tubes. The law of the last paragraph is only true when the sound is free to spread equally in all directions. If it is confined within a tube, so that the energy is continually communicated from one layer to another of equal area, instead of from one layer to another of larger area, it can be carried to great distances with no loss in intensity except that due to friction against the walls of the tube. This explains the efficiency of speaking tubes and megaphones.

451. A train of waves; wave length. In the preceding paragraphs we have confined attention to a single pulse traveling out from a center of explosion. Let us next consider the sort of disturbance which is set up in the air by a continuously vibrating body like the prong of Fig. 369. Each time that this prong moves to the right it sends out a pulse which travels through the air at the rate of 1100 ft. per second, in exactly the manner described in the preceding paragraphs. Hence, if the reed is vibrating uniformly, we shall have a continuous succession of

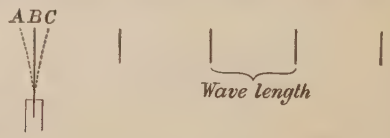


Fig. 369. Vibrating reed sending out a train of equidistant pulses

pulses following each other through the air at exactly equal intervals. Suppose, for example, that the prong makes 110 complete vibrations per second. Then at the end of one second the first pulse sent out will have reached a distance of 1100 ft. Between this point and the prong there will be 110 pulses distributed at equal intervals, i.e. each two adjacent pulses will be just 10 ft. apart. If the prong made 220 vibrations per second, the distance between adjacent pulses would be 5 ft., etc. The *distance between two adjacent pulses in such a train of waves is called a wave length.*

452. Relation between velocity, wave length, and number of vibrations per second. If n represents the number of vibrations per second of a source of sound, l the wave length, and v the velocity with which the sound travels through the medium, it is evident from the example of the preceding paragraph that the following relation exists between these three quantities:

$$l = v/n, \text{ or } v = nl; \qquad (1)$$

i.e. *wave length is equal to velocity divided by the number of vibrations per second, or velocity is equal to number of vibrations per second times wave length.*

453. Condensations and rarefactions. Thus far, for the sake of simplicity, we have considered a train of waves as a series of thin, detached pulses separated by equal intervals of air at rest. In point of fact, however, the air in front of the prong B (Fig. 369) is being pushed forward not at one particular instant only, but during all the time that the prong is moving from A to C, i.e. through the time of one half vibration of the fork; and during all this time this forward motion is being transmitted to layers of air which are farther and farther away from the prong, so that when the latter reaches C all the air between B and some point c (Fig. 370) one half wave length away is crowding forward, and is therefore in a state of compression or *condensation.* Again, as the prong moves back from C to A, since it tends to

leave a vacuum behind it, the adjacent layer of air rushes in to fill up this space, the layer next adjoining follows, etc., etc., so that when the prong reaches *A* all the air between *B* and *c* (Fig. 370) is moving backward and is therefore in a state of

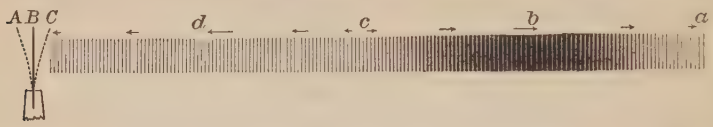


FIG. 370. Illustrating motions of air particles in one complete sound wave consisting of a condensation and a rarefaction

diminished density or *rarefaction.* During all this time the preceding forward motion has advanced one half wave length to the right, so that it now occupies the region between *c* and *a* (Fig. 370). Hence at the end of one complete vibration of the prong we may divide the air between it and a point one wave length away into two portions, one a region of condensation *ac*, and the other a region of rarefaction *cB*. The arrows in Fig. 370 represent the direction and relative magnitudes of the motions of the air particles in various portions of a complete wave.

At the end of *n* vibrations the first disturbance will have reached a distance *n* wave lengths from the fork, and each wave between this point and the fork will consist of a condensation and a rarefaction, so that sound waves may be said to consist of a series of condensations and rarefactions following one another through the air in the manner shown in Fig. 371.

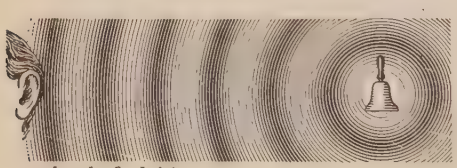


FIG. 371. Illustration of sound waves

Wave length may now be more accurately defined as *the distance between two successive points of maximum condensation* (*b* and *f*, Fig. 371) *or of maximum rarefaction* (*d* and *h*).

454. Water-wave analogy. Condensations and rarefactions of sound waves are exactly analogous to the familiar crests and troughs of water waves. Thus, if Fig. 372 represents a series of ripples passing over the surface of water, the wave length of such a series is defined as the distance *bf* between two crests, or the distance *dh*, or *ae*, or *cg*, or *mn*, between any two points which are in the same condition or *phase* of disturbance. The crests, i.e. the shaded portions, which are above the natural level of the water, correspond exactly to the condensations of sound waves, i.e. to the portions of air which are above the natural density. The troughs, i.e. the dotted portions, correspond to the rarefactions of sound waves, i.e. to the portions of air which are below the natural density.

Fig. 372. Illustrating wave length of water waves

455. Longitudinal and transverse waves. In spite of the analogy mentioned in the last paragraph, water waves differ from sound waves in one very important respect. In the former the particles of water are moving up and down while the wave is traveling along the surface, i.e. horizontally. Hence the motion of the particles is at right angles to the direction in which the wave is traveling. Such a wave is said to be a *transverse* wave, because the motion of the particles is transverse to the direction of propagation. In sound waves, however, as shown in § 453, the particles move back and forth *in* the line of propagation of the wave. Such waves are called *longitudinal waves.* Sound waves are always transmitted through any medium as longitudinal waves.

456. Distinction between musical sounds and noises. Let a current of air from a $\frac{1}{8}$-inch nozzle be directed against a row of forty-eight equidistant $\frac{1}{4}$-inch holes in a metal or cardboard disk, mounted as in Fig. 373 and set into rotation either by hand or by an electric motor. A very distinct musical tone will be produced. Then let the jet of air be

directed against a second row of forty-eight holes, which differs from the first only in that the holes are irregularly instead of regularly spaced about the circumference of the disk. The musical character of the tone will altogether disappear.

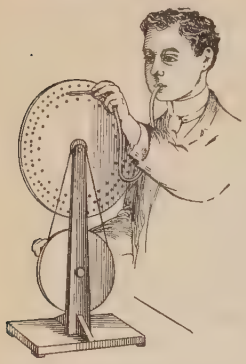


FIG. 373. Regularity of pulses the condition for a musical tone

The experiment furnishes a very striking illustration of the difference between a musical sound and a noise. *Only those sounds possess a musical quality which come from sources capable of sending out pulses, or waves, at absolutely regular intervals.* Therefore it is only sounds possessing a musical quality which may be said to have wave lengths.

457. Pitch. While the apparatus of the preceding experiment is rotating at constant speed let a current of air be directed first against the outside row of regularly spaced holes and then suddenly turned against the inside row, which is also regularly spaced but which contains a smaller number of holes. The note produced in the second case will be found to have a markedly lower pitch than the other one. Again, let the jet of air be directed against one particular row, and let the speed of rotation be changed from very slow to very fast. The note produced will gradually rise in pitch, until at a very high speed it will become shrill and piercing.

We conclude, therefore, that *the pitch of a musical note depends simply upon the number of pulses which strike the ear per second.* If the sound comes from a vibrating body, the pitch of the note depends upon the *rate of vibration* of the body.

458. Doppler's principle. There is another fact of common observation which shows that the pitch of a note is determined by the number of impulses which reach the ear per second. When an express train rushes past an observer he notices a very distinct change in the pitch of the bell as the engine passes him, the pitch being higher as the engine approaches than as it recedes. The explanation is as follows. The bell, of course, sends out pulses at exactly equal intervals of time. As the train is approaching, however, the pulses reach the ear at

shorter intervals than the intervals between emissions, since the train comes toward the observer between two successive emissions. But as the train recedes, the interval between the receipt of pulses by the ear is longer than the interval between emissions, since the train is moving away from the ear during the interval between emissions. Hence the pitch of the bell is higher during the approach of the train than during its recession. This phenomenon of the change in pitch of a note proceeding from an approaching or receding body is known as *Doppler's principle.*

QUESTIONS AND PROBLEMS

1. A telephone which may be used for distances of a quarter of a mile or so may be made by stretching parchment over the ends of two bottomless tin cans and connecting their centers with a thread. Explain in what way the sounds produced at one end are reproduced at the other.

2. The loudness of a sound depends simply on the amount of energy communicated to the drum of the ear. Can you see any reason why, in general, sounds are louder in dense media than in rare ones? (Stones clapped together under water produce an almost deafening sound to an ear placed under water.)

3. A church bell is ringing at a distance of $\frac{1}{2}$ mile from one man and $\frac{1}{4}$ mile from another. How much louder would it appear to the second man than to the first if no reflections of the sound took place?

4. Explain the principle of the ear trumpet.

5. The vibration rate of a fork is 256. Find the wave length of the note given out by it at 20° C.

6. The note from a piano string which makes 300 vibrations per second passes from indoors, where the temperature is 20° C., to outdoors, where it is 5° C. What is the difference in centimeters between the wave lengths indoors and outdoors?

7. As a circular saw cuts into a block of wood the pitch of the note given out falls rapidly. Why?

8. A man riding on an express train moving at the rate of 1 mile per minute hears a bell ringing in a tower in front of him. If the bell makes 300 vibrations per second, how many pulses will strike his ear per second, the velocity of sound being 1130 ft.? (The number of extra impulses received per second by the ear is equal to the number of wave lengths contained in the distance traveled per second by the train.)

9. Since the music of an orchestra reaches a distant hearer without confusion of the parts, what may be inferred as to the relative velocities of the notes of different pitch?

Reflection and Reënforcement of Sound

459. Echo. That a sound wave in hitting a wall suffers reflection is shown by the familiar phenomenon of echo. A sharp sound made, for example, a quarter of a mile in front of a cliff or isolated large building, will be distinctly returned after a lapse of about two and a half seconds. There is a famous spot at Woodstock, England, at which not less than twenty syllables are distinctly returned from a reflecting surface 2300 ft. away.

If the sound is made between two parallel cliffs, the echo may be repeated many times, because of successive reflections. It is then called a *multiple echo.* The dome of the Baptistery in Pisa will prolong a note for several seconds in this way, so that when the three notes *do, mi, sol* are sounded in succession, they come back united by the multiple echo into a full, rich chord. The roll of thunder is due to successive reflections of the original sound from clouds and other surfaces which are at different distances from the observer.

In ordinary rooms the walls are so close that the reflected waves return before the effect of the original sound on the ear has died out. Consequently the echo blends with and strengthens the original sound instead of interfering with it. This is why, in general, a speaker may be heard so much better indoors than in the open air. Since the ear cannot appreciate successive sounds as distinct if they come at intervals shorter than a tenth of a second, it will be seen from the fact that sound travels about 113 ft. in a tenth of a second that a wall which is closer than about 50 ft. cannot possibly produce a perceptible echo. In rooms which are large enough to give rise to troublesome echoes it is customary to hang draperies of some sort, so as to break up the sound waves and prevent regular reflection.

460. Conditions for sound reflection. The conditions under which a reflection of sound must take place may readily be seen from the following experiment.

Let a small steel ball be suspended beside a larger one, as in Fig. 374. Let the first ball be lifted up its arc and allowed to fall against the second. After the blow the first ball, instead of coming to rest as it did in the experiment of § 448, when it struck against a ball of the same size as itself, will be found to rebound from the larger ball with considerable velocity. The larger ball, on the other hand, will move but a comparatively short distance up its arc. Now let the larger ball be raised and allowed to fall against the smaller one. Upon collision the larger ball will not now be brought to rest, nor will it rebound, but it will move on in the direction in which it was originally going.

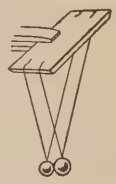


Fig. 374. Illustrating the condition necessary for the reflection of a sound wave

We conclude, therefore, that although an elastic particle which collides with an exactly *similar* particle transfers to the latter all of its energy, an elastic particle which collides with a *heavier* or *lighter* particle transfers only a *part* of its energy in the collision. In precisely the same way, when a sound pulse travels through a medium like air, each layer of which is exactly like that preceding, the whole of the energy is passed on from layer to layer, and no reflection is possible. But as soon as the wave strikes a medium of different density from that in which it has been traveling, only a part of the energy goes on into the new medium, and the remainder is propagated backward through the first medium in the form of a reflected wave in precisely the same way in which the original pulse was propagated forward. *The reflection of sound will then always take place whenever the molecular motion which constitutes a sound wave reaches a medium of different density from that in which it has been traveling.* It is on account of differences in the homogeneity of the atmosphere on different days that sound "carries" so much better at some times than at others. Lack of homogeneity results in a dissipation of the energy of the sound waves by repeated reflections from layers of different density.

461. Nature of the reflected pulse. If the new medium is denser than the old, as when a sound wave traveling in air approaches a wooden wall, the forward-moving particles of an on-coming condensation rebound when they strike the wall, precisely as did the smaller of the two balls in the preceding experiment. This rebound is propagated back through the old medium as a motion in the same direction as that of propagation of the reflected wave, i.e. as a condensation. But if the new medium is rarer than the old, as when sound passes from water into air, when the particles of an oncoming condensation strike the particles of the lighter medium they overshoot their positions of rest, precisely as did the heavier ball in the second experiment with the unequal balls, and thus create a rarefaction behind them in the first medium. This rarefaction is propagated backward through this medium precisely as the rarefaction produced by the backward motion of the prong was shown to be propagated in § 453. We learn, therefore, that *a condensation is reflected from a denser medium as a condensation, from a rarer medium as a rarefaction.* A similar analysis shows that a rarefaction is reflected from a denser medium as a rarefaction, from a rarer medium as a condensation.

462. Sound foci. Let a watch be hung at the focus of a large concave mirror. On account of the reflection from the surface of the mirror a fairly well-defined beam of sound will be thrown out in front of the mirror, so that, if both watch and mirror are hung on a single support and the whole turned in different directions toward a number of observers, the ticking will be distinctly heard by those directly in front of the mirror, but not by those at one side. If a second mirror is held in the path of this beam, as in Fig. 375, the sound may be again brought to a focus, so that if the ear is placed in the focus of this second mirror, or better still, if a small funnel which is connected with the ear by a rubber tube is held in this focus, the ticking of the watch may sometimes be heard hundreds of feet away.

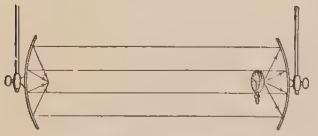


Fig. 375. Sound foci

A whispering gallery is a room so arranged as to contain such sound foci. Any two opposite points a few feet from the walls of a dome, like that of St. Peter's at Rome or St. Paul's at London, are sufficiently near to such sound foci to make very low whispers on one side distinctly audible at the other, although at intermediate points no sound can be heard.

463. Resonance. Resonance is the *reënforcement* or *intensification* of sound because of the union of *direct* and *reflected* waves.

Thus, let one prong of a vibrating tuning fork, which makes, for example, 512 vibrations per second, be held over the mouth of a tube an inch or so in diameter, arranged as in Fig. 376, so that as the vessel *A* is raised or lowered the height of the water in the tube may be adjusted at will. It will be found that, as the position of the water is slowly lowered from the top of the tube, a very marked reënforcement of the sound will occur at a certain point.

Let other forks of different pitch be tried in the same way. It will be found that the lower the pitch of the fork the lower must be the water in the tube in order to get the best reënforcement. This means that the longer the wave length of the note which the fork produces the longer must be the air column in order to obtain resonance.

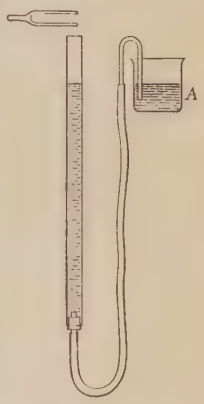


FIG. 376. Illustrating resonance

We conclude, therefore, that a fixed relation exists between the wave length of a note and the length of the air column which will reënforce it.

464. Best resonant length is one fourth wave length. If we calculate the wave length of the note of the fork by dividing the speed of sound by the vibration rate of the fork, we shall find that, in every case, *the length of air column which gives the best response is approximately one fourth wave length.* The reason for this is evident when we consider that the length must be such as to enable the reflected wave to return to the mouth

just in time to unite with the direct wave which is at that instant being sent off by the prong. Thus, when the prong is first starting down from the position A (see Fig. 377) it starts the beginning of a condensation down the tube. If this motion is to return to the mouth just in time to unite with the direct wave sent off by the prong, it must get back at the instant that the prong is first starting up from the position C. In other words, the pulse must go down the tube and back again while the prong is making a half vibration. This means that the path down and back must be a half wave length, and hence that the length of the tube must be a fourth wave length.

FIG. 377. Resonant length of a closed pipe is $\frac{1}{4}$ wave length

From the above analysis it will appear that there should also be resonance if the reflected wave does not return to the mouth till the fork is starting back its second time from C, i.e. at the end of one and a half vibrations instead of a half vibration. The distance from the fork to the water and back would then be one and a half wave lengths; i.e. the water surface would be a half wave length farther down the tube than at first. The tube length would therefore now be three fourths of a wave length.

Let the experiment be tried. A similar response will indeed be found, as predicted, a half wave length farther down the tube. This response will be somewhat weaker than before, as the wave has lost some of its energy in traveling a longer distance through the tube. It may be shown in a similar way that there will be resonance where the tube length is $\frac{5}{4}$, $\frac{7}{4}$, or indeed any odd number of quarter wave lengths.

465. Best resonant length of an open pipe is one half wave length. If the bottom of the tube of Figs. 376 or 377 had been closed by a medium lighter than air, then by § 461 a downward-moving condensation would have been reflected as an upward-moving rarefaction, i.e. as an upward-moving wave in

which the motion of the particles would have been down instead of up. Hence, if this upward-moving wave is to get back to the mouth of the tube just in time to unite with and therefore reënforce a direct wave sent off by the prong, it must return, not now at the instant that the prong is starting *up* from *C*, but rather at the instant at which it is starting *down* again from *A*, i.e. after an interval corresponding to one complete vibration of the prong. Hence the length of the air chamber will need to be, in this case, a half wave length instead of a quarter wave length, if resonance is to be obtained.

Now, as a matter of fact, a pulse traversing a simple tube which is open at the lower end experiences at this end a reflection of precisely the same kind which it would experience if it struck a rarer medium, for within the tube the pulse has been free to push forward only in one direction, but as it reaches the open end it suddenly becomes free to expand in all directions; i.e. it encounters at this point less resistance to its forward motion than it has encountered within the tube, and therefore it acts precisely as it would in going from a denser to a rarer medium. The correctness of this assertion is proved by the following experiment.

Let the same tuning fork which was used in § 463 be held in front of an open pipe (8 or 10 in. long) the length of which is made adjustable by slipping back and forth over it a tightly fitting roll of writing paper (Fig. 378). It will be found that for one particular length this open pipe will respond quite as loudly as did the closed pipe, but the responding length will be found to be just twice as great as before.

Fig. 378. Resonant length of an open pipe is $\frac{1}{2}$ wave length

We learn, then, that the shortest resonant length of an open pipe is one half wave length. It is evident that there must also be resonance when the pipe length is one wave length, for then the first reflected wave will get back just in time to unite with the third forward motion

of the prong. Similarly, there will be resonance when the pipe length is $\frac{3}{2}$, $\frac{4}{2}$, $\frac{5}{2}$, or any number, odd or even, of half wave lengths.

466. Resonators. If the vibrating fork at the mouth of the tubes in the preceding experiments is replaced by a *train of waves* coming from a distant source, precisely the same analysis leads to the conclusion that the waves reflected from the bottom of the tube will reënforce the oncoming waves when the length of the tube is any odd number of quarter wave lengths in the case of a closed pipe, or any number of half wave lengths in the case of an open pipe. It is clear, therefore, that every air chamber will act as a resonator for trains of waves of a *certain* wave length. This is why a conch shell held to the ear is always heard to hum with a particular note. Feeble waves which produce no impression upon the unaided ear gain sufficient strength when reënforced by the shell to become audible. When the air chamber is of irregular form it is not usually possible to calculate to just what wave length it will respond, but it is always easy to determine experimentally what particular wave length it is capable of reënforcing. The resonators on which tuning forks are mounted are air chambers which are of just the right dimensions to respond to the note given out by the fork.

467. Forced vibrations. Sounding-boards. Let a tuning fork be struck and held in the hand. The sound will be entirely inaudible except to those quite near. Let the base of the sounding fork be pressed firmly against the table. The sound will be found to be enormously intensified. Let another fork be held against the same table. Its sound will also be reënforced. In this case, then, the table intensifies the sound of any fork which is placed against it, while an air column of a certain size could intensify only a single note.

The cause of the response in the two cases is wholly different. In the last case the vibrations of the fork are transmitted through its base to the table top and force the latter to vibrate in its own period. The vibrating table top, on account of its large surface, sets a comparatively large mass of air into motion

and therefore sends a wave of great intensity to the ear; while the fork alone, with its narrow prongs, was not able to impart much energy to the air. Vibrations like those of the table top are called *forced* vibrations because they can be produced with any fork, no matter what its period. Sounding-boards in pianos and other stringed instruments act precisely as does the table in this experiment; i.e. they are set into forced vibrations by any note of the instrument, and reënforce it accordingly.

QUESTIONS AND PROBLEMS

1. Why do the echoes which are prominent in empty halls often disappear when the hall is full of people?

2. The report of a gunner was echoed back to him $4\frac{1}{2}$ seconds after he fired the gun. How far away was the reflecting surface, the temperature of the air being 20° C.?

3. Find the number of vibrations per second of a fork which produces resonance in a pipe 1 ft. long. (Take the speed of sound as 1120 ft. per sec.)

4. A fork making 500 vibrations per second is found to produce resonance in an air column like that shown in Fig. 376, first when the water is a certain distance from the top, and again when the water is 34 cm. lower. Find the velocity of sound.

5. Show why an open pipe needs to be twice as long as a closed pipe if it is to respond to the same note.

Interference of Sound

468. Beats. Since two sound waves are able to unite so as to reënforce each other, it ought also to be possible to make them unite so as to interfere with or destroy each other. In other words, under the proper conditions *the union of two sounds ought to produce silence.*

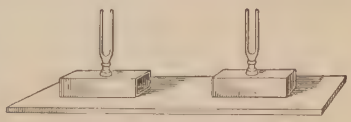


Fig. 379. Arrangement of forks for beats

Let two mounted tuning forks of the same pitch be set side by side, as in Fig. 379. Let the two forks be struck in quick succession with a soft mallet, for example, a rubber

stopper on the end of a rod. The two notes will blend and produce a smooth, even tone. Then let a piece of wax or a small coin be stuck to a prong of one of the forks. This diminishes slightly the number of vibrations which this fork makes per second, since it increases its mass. Again let the two forks be sounded together. The former smooth tone will be replaced by a throbbing or pulsating one. This is due to the alternate destruction and reënforcement of the sounds produced by the two forks. The phenomenon is called the phenomenon of *beats*.

The mechanism of the alternate destruction and reënforcement may be understood from the following. Suppose that one fork makes 256 vibrations per second (see the dotted line AC in Fig. 380), while the other makes 255 (see the heavy line AC in Fig. 380). If, at the beginning of a given second the two forks are swinging *together* so that they simultaneously send out condensations to the observer, these condensations will of course unite so as to produce a double effect upon the ear (see A', Fig. 380). Since now one fork gains one complete vibration per second over the other, at the end of the second considered the two forks will again be vibrating together, i.e. sending out condensations which add their effects as before (see C'). In the middle of this second, however, the two forks are vibrating in opposite directions (see B); i.e. one is sending out rarefactions while the other sends out condensations. At the ear of the observer the union of the rarefaction (backward motion of the air particles) produced by one fork with the condensation (forward motion) produced by the other, results in no motion at all, provided the two motions have the same energy; i.e. in the middle of the second the two sounds have united to produce silence (see B'). If the two sounds are of unequal intensity, the destruction will not be complete, the minimum representing the difference between the two intensities, and the maximum the sum.

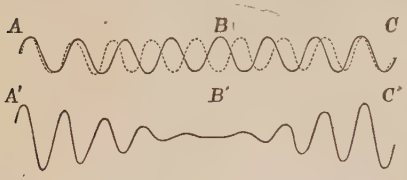


Fig. 380. Graphical illustration of beats

It will be seen from the above that the number of beats per second must be equal to the difference in the vibration numbers of the two forks. To test this conclusion, let more wax or a heavier coin be added to the weighted prong; the number of beats per second will be increased. Diminishing the weight will reduce the number of beats per second.

The experiment, therefore, shows an easy and accurate method of determining the difference in vibration rates of two sounding bodies which have nearly but not quite the same pitch. It is only necessary to sound them together and to note the number of beats per second. This number is the difference in their vibration numbers. If weighting either body increases the number of beats, that body was the slower; if weighting this body diminishes the number of beats, that body was the faster.

469. Interference of sound waves by reflection. Let a thin cork about an inch in diameter be attached to one end of a brass or

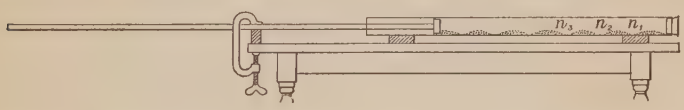


Fig. 381. Interference of advancing and retreating trains of sound waves

glass rod from one to two meters long. Let this rod be clamped firmly in the middle, as in Fig. 381. Let a piece of glass tubing a meter or more long and from an inch to an inch and a half in diameter be closed at one end and slipped over the cork, as shown, care being taken that the cork does not touch the sides of the tube, or touches them only very lightly. Let the end of the rod be gripped firmly with a well-resined cloth and then stroked longitudinally. (A wet cloth will answer better if the rod is of glass.) A loud shrill note will be produced.

This note is due to the fact that the slipping of the resined cloth over the surface of the rod sets the latter into longitudinal vibrations, so that its ends impart alternate condensations and rarefactions to the layers of air in contact with them. As soon as this note is started the cork dust inside the tube will be seen to be intensely agitated. If the effect is not marked at first, a slight slipping of the glass tube forward or back will bring it

out. Upon examination it will be seen that the agitation of the cork dust is not uniform, but at regular intervals throughout the tube there will be regions of complete rest, n_1, n_2, n_3, etc., separated by regions of intense motion. *The points of rest correspond to the positions in which the reflected train of sound waves returning from the end of the tube neutralizes the effect of the advancing train passing down the tube from the vibrating rod.* The points of rest are called *nodes,* the intermediate portions *loops* or *antinodes.*

470. Distance between two nodes equal to one half wave length. The manner in which the advancing and reflected trains of waves unite so as to produce these nodes and loops, or *stationary waves* as they are sometimes called, may be seen from the following. Let a_1, a_2, a_3, a_4 (Fig. 382) represent the fronts of a succession of condensations sent down the tube from the vibrating rod R. At the instant that the first wave front a_1 reaches the end of the tube it is reflected and starts back toward R. Since at this instant the second wave front a_2 is just one wave length to the left of a_1, the two wave fronts must meet each other at a point n_1, just one half wave length from the end of the tube. The exactly equal and opposite motions of the particles in the two wave fronts exactly neutralize each other. Hence the point n_1 is a point of no motion, i.e. a node. Again, at the instant that the reflected wave front a_1 met the advancing wave front a_2 at n_1, the third wave front a_3 was just one wave length to the left of n_1. Hence, as the first wave front a_1 continues to travel back toward R it meets a_3 at n_2, just one half wave length from n_1, and produces there a second node. Similarly a third node is produced at n_3, one half wave length to the left of n_2, etc. Thus *the distance between two nodes must always be just one half the wave length of the train of waves.*

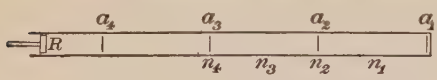


FIG. 382. Distance between nodes is $\frac{1}{2}$ wave length

In this discussion it has been tacitly assumed that the two oppositely moving waves are able to pass through each other without either of them being modified by the presence of the other. That two opposite motions are, in fact, transferred in just this manner through a medium consisting of elastic particles, may be beautifully shown by the following experiment with the row of balls used in § 448, p. 347.

Fig. 383. Nodes and loops in a cord. Black line denotes advancing train; dotted line, reflected train

Let the ball at one end of the row be raised a distance of say 2 inches, and the ball at the other end raised a distance of 4 inches. Then let the balls be dropped simultaneously against the row. The two opposite motions will pass through each other in the row altogether without modification, the larger motion appearing at the end opposite to that at which it started, and the smaller likewise.

Another and more complete analogy to the condition existing within the tube of Fig. 381 may be had by simply vibrating one end of a two- or three-meter rope, as in Fig. 383. The trains of advancing and reflected waves which continuously travel through each other up and down the rope will unite so as to form a series of nodes and loops, as shown. The nodes at c and e are simply the points at which the advancing and reflected waves are always urging the cord in opposite directions. The distance between them is, of course, one half the wave length of the train sent down the rope by the hand.

QUESTIONS AND PROBLEMS

1. One fork makes 260 vibrations per second. When a second fork is sounded with it 4 beats per second are heard. If a small piece of wax is added to one of the prongs of the second fork, 5 beats per second are heard. What is the natural rate (unloaded) of the second fork?

2. The distance between two nodes, as indicated by the cork dust in Fig. 381, was found to be 6 in. Taking the speed of sound as 1130 ft. per sec., find the pitch of the note emitted by the longitudinal vibrations of the rod.

3. If three loops are produced in a 12-ft. cord when the hand makes 4 vibrations per sec., with what speed do waves travel along the cord?

CHAPTER XVIII

PROPERTIES OF MUSICAL SOUNDS

Musical Scales

471. Physical basis of musical intervals. Let a metal or cardboard disk 10 or 12 in. in diameter be provided with four concentric rows of equidistant holes, the successive rows containing respectively 24, 30, 36, and 48 holes (Fig. 384). The holes should be about $\frac{1}{4}$ in. in diameter and the rows should be about $\frac{1}{2}$ in. apart. Let the disk be placed in the rotating apparatus and a constant speed imparted. Then let a jet of air be directed, as in § 456, p. 352, against each row of holes in succession. It will be found that the musical sequence *do*, *mi*, *sol*, *do′* results. If the speed of rotation is increased, each note will rise in pitch, but the sequence will remain unchanged.

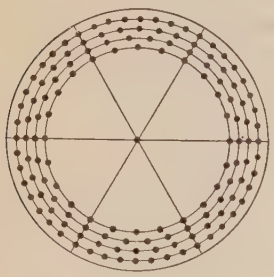


Fig. 384. Disk for producing musical sequence *do*, *mi*, *sol*, *do′*

We learn, therefore, that *the musical sequence do, mi, sol, do′ consists of notes whose vibration numbers have the ratios of 24, 30, 36, and 48, i.e. 4, 5, 6, 8, and that this sequence is independent of the absolute vibration numbers of the tones.*

472. Musical intervals and harmony. All persons who are endowed with even a rudimentary musical sense are aware that two notes an octave apart have certain common characteristics which make them more nearly alike than any other notes. Furthermore, when two notes an octave apart are sounded together, they form the most harmonious combination which it is possible to obtain. These characteristics of notes an octave

apart were recognized in the earliest times, long before anything whatever was known about the ratio of their vibration numbers. The experiment of the last paragraph showed that *this ratio is the simplest possible*, viz. 24 to 48, or 1 to 2. Again, the next easiest musical interval to produce and the next most harmonious combination which can be found corresponds to the two notes commonly designated as *do*, *sol*. Our experiment showed that this interval corresponds to the next simplest possible vibration ratio, viz. 24 to 36, or 2 to 3. When *sol* is sounded with *do'* the vibration ratio is seen to be 36 to 48, or 3 to 4. We see, therefore, that the three simplest possible ratios of vibration numbers, viz. 1 to 2, 2 to 3, and 3 to 4, are used up in the production of the three notes *do*, *sol*, *do'*. Again, our experiment shows that the next most harmonious musical interval, *do*, *mi*, corresponds to the vibration ratio 24 to 30, or 4 to 5. We learn, therefore, that *harmonious musical intervals correspond to very simple vibration ratios, and the more harmonious the combination the simpler the ratio.*

473. The major diatonic scale. When the three notes, *do*, *mi*, *sol*, which, as seen above, have the vibration ratios 4, 5, 6, are all sounded together, they form a remarkably pleasing combination of tones. This combination was picked out and used very early in the musical development of the race. It is now known as the *major chord. The major diatonic scale is built up of three major chords.* The absolute vibration number taken as the starting point is wholly immaterial, but the explanation of the origin of the eight notes of the octave commonly designated by the letters C, D, E, F, G, A, B, C' may be made more simple if we begin, as above, with a note whose vibration number is 24. If this note is designated by the letter C, the two other notes of the first major chord, *do-mi-sol*, are designated by E and G. The second chord is obtained by starting from C', the octave of C, and coming down in the ratios 6, 5, 4. The corresponding vibration numbers are 48, 40, and 32, and the corresponding

notes, known as *do, la, fa,* are designated by the letters C', A, and F. The third chord starts with G as the first note and runs up in the ratios 4, 5, 6. The corresponding notes, known as *sol, si, re,* have the vibration numbers 36, 45, and 54, and are designated by the letters G, B, and D'. It will be seen that the note D' does not fall in the octave between C and C', for its vibration number is above 48. The note D an octave below it falls between C and C' and has a vibration number 27. This completes the eight notes of the diatonic scale. The chord *do-mi-sol* is called the tonic, *sol-si-re* the dominant, and *fa-la-do* the subdominant.

Below is given in tabular form the relations between the notes of an octave.

Syllables	*do*	*re*	*mi*	*fa*	*sol*	*la*	*si*	*do'*
Letters	C	D	E	F	G	A	B	C'
Relative vibration numbers . .	24	27	30	32	36	40	45	48
Vibration ratios in terms of *do*	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
Absolute vibration numbers .	256	288	320	341	384	427	480	512

474. Absolute vibration numbers. Any vibration number whatever may be assigned to the first note C of a series of octaves built up in the manner just outlined. As a matter of fact, there have been and still are several different pitches in common use for the starting point, which is commonly called middle C (lower C of the treble clef). The so-called *international pitch,* which was adopted by the Vienna Congress in 1885 and which is now almost exclusively used, assigns 435 vibrations to middle A. This gives 261 to middle C. In a piano tuned to *concert pitch* middle C has 274 vibrations. Standard middle C forks made for physical laboratories all have the vibration number 256. The term "middle C" then means one of these vibration numbers, and generally either 261 or 256. The successive octaves above C are designated by C', C'', C''', etc., and the successive octaves below C by C_1, C_2, C_3, etc.

QUESTIONS AND PROBLEMS

1. What note has three times as many vibrations per second as middle C?

2. What note has four times as many vibrations per second as G above middle C?

3. What note has five times as many vibrations per second as middle C?

4. What is the wave length of middle C when the speed of sound is 1152 ft. per second?

5. What is the pitch of a note whose wave length is 5.4 inches, the speed being 1152 ft. per second?

6. If middle C had 300 vibrations per second, how many vibrations would F and A have?

Laws of Vibrating Strings[1]

475. Law of lengths. Let two piano wires be stretched over a box, or a board with pulleys attached so as to form a sonometer (Fig. 385). Let the weights A and B be adjusted until the two wires emit exactly the same note. The phenomenon of beats will make it possible to do this with great accuracy. Then let the bridge D be inserted exactly at the middle of one of the wires, and the two wires plucked in succession. The interval will be recognized at once as *do*, *do′*. Next let the bridge be inserted so as to make one wire two thirds as long as the other, and let the two be plucked again. The interval will be recognized as *do*, *sol*.

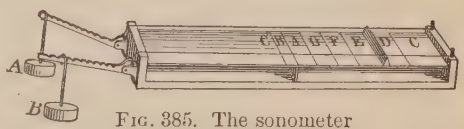


Fig. 385. The sonometer

Now it was shown in § 471 that *do′* has twice as many vibrations as *do*, and *sol* has three halves as many. Hence, since the length corresponding to *do′* is one half as great as the first length, and that corresponding to *sol* two thirds as great, we conclude from this experiment that, other things being equal, *the vibration numbers of strings are inversely proportional to their lengths.* To produce all the notes of an octave on a given string, then, it is only necessary to adjust the bridge so that

[1] This discussion should be followed by a laboratory experiment on the laws of vibrating strings. See, for example, Experiment 41 of the authors' manual.

the lengths are proportional to the reciprocals of the numbers given in the fourth row of the table of § 473.

476. Law of tensions. Again, let the two wires be tuned to unison, and then let the weight A be increased until the pull which it exerts on the wire is exactly four times as great as that exerted by B. The note given out by the A wire will again be found to be an octave above that given out by the B wire.

In this case, therefore, *the vibration numbers*, viz. 2 to 1, *are proportional to the square roots of the stretching weights*, viz. 4 to 1. This relation is found to hold for all notes, so long as the other conditions, such as *diameter*, *material*, and *length* of the wires, are the same. Thus, since the vibration numbers of G and C are in the ratio of 3 to 2, the tensions which will produce them on the same wire are in the ratio 9 to 4.

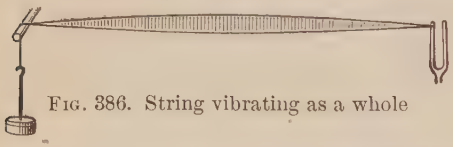


Fig. 386. String vibrating as a whole

477. Nodes and loops in vibrating strings

Let a string a meter long be attached to one of the prongs of a large tuning fork which makes in the neighborhood of 100 vibrations per second. Let the other end be attached as in the figure, and the fork set into vibration. If the fork is not electrically driven, which is much to be preferred, it may be bowed with a violin bow or struck with a soft mallet. By suitably changing the tension of the thread it will be found possible to make it vibrate either as a whole, as in Fig. 386, or in two, or three, or any number of parts (Fig. 387).

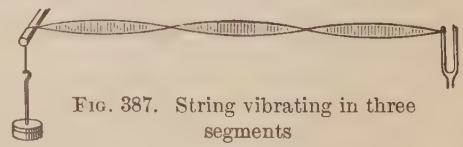


Fig. 387. String vibrating in three segments

This effect is due, as explained in § 470, to the interference of the direct and reflected waves sent down the string from the vibrating fork. But we shall show in § 479 that in considering the effects of the vibrating string on the surrounding air we shall make no mistake if we think of it as clamped at each node, and as actually vibrating in two or three or four separate parts,

as the case may be; so that when, for example, it is vibrating in two parts, the pulses sent out into the air from each vibrating segment will have twice as great a frequency as when it is vibrating in one part, one half as great a frequency as when it is vibrating in four parts, etc.

478. Nodes and loops in the strings of musical instruments. That it is possible to choose the conditions so that the strings of any musical instrument may be made to vibrate, like the cord of the last paragraph, either as a whole, or in any desired number of parts, may be strikingly shown as follows.

Let little paper riders be placed on the sonometer wire at distances apart equal to one eighth the length of the wire. Let the tip of a finger be placed against the wire one fourth of the wire's length from one end (Fig. 388). At a point midway between the finger and the nearest end let the string be bowed with a violin bow. (Picking the wire with the finger will not be found to be satisfactory.) The riders which are at the one-half and three-fourth points will be found to remain at rest, while all the others will be violently agitated and thrown off.

Fig. 388. Nodes in sonometer wires

We conclude, therefore, that touching the finger to the wire one fourth of its length from one end made it vibrate in four segments. Similarly, touching it at one third of its length will be found to make it vibrate in three segments, etc.

Fundamentals and Overtones

479. Fundamentals and overtones defined. The correctness of the assertion made in § 477, that a string which has a node in the middle communicates twice as many pulses to the air per second as a string which vibrates as a whole, may be conclusively shown as follows.

Let the sonometer wire be plucked in the middle and the pitch of the corresponding tone carefully noted. Then let the finger be touched to the middle of the wire, and the latter plucked midway between this point and the end.[1] The octave of the original note will be distinctly heard. Next let the finger be touched at a point one third of the wire length from one end, and the wire again plucked. The note will be recognized as *sol* in the octave above the original note. Since we learned in § 473 that this *sol* has three halves as many vibrations as the *do* next below it, it must have three times as many vibrations as the original note. Hence a wire which is vibrating in three segments sends out three times as many vibrations as when it is vibrating as a whole.

Now when a wire is plucked in the middle it vibrates simply as a whole, and therefore gives forth the lowest note which it is capable of producing. This note is called the *fundamental* of the wire. When the wire is made to vibrate in two parts it gives forth, as has just been shown, a note an octave higher than the fundamental. This is called the *first overtone.* When the wire is made to vibrate in three parts it gives forth a note corresponding to three times the vibration number of the fundamental, viz. *sol'*. This is called the *second overtone.* When the wire vibrates in four parts it gives forth the third overtone, which is a note two octaves above the fundamental. The overtones of wires are often called *harmonics.* They bear the vibration ratios 2, 3, 4, 5, 6, 7, etc., to the fundamental.[2]

480. Simultaneous production of fundamentals and overtones. We have thus far produced overtones only by forcing the wire to remain at rest at certain properly chosen points during the bowing.

Now let the wire be plucked at a point one fourth of its length from one end, *without being touched in the middle.* The tone most distinctly heard will be the fundamental, but if the wire is now touched very lightly exactly in the middle, the sound, instead of ceasing altogether,

[1] It is well to remove the finger almost simultaneously with the plucking.

[2] Some instruments, such as bells, can produce higher tones whose vibration numbers are not exact multiples of the fundamental. These notes are still called overtones, but they are not called harmonics, the latter term being reserved for the multiples. Strings produce only harmonics.

will continue, but the note heard will be an octave higher than the fundamental, showing that in this case there was superposed upon the vibration of the wire as a whole a vibration in two segments also (Fig. 389). By touching the wire in the middle the vibration as a whole was destroyed, but the vibration in two parts remained. Let now the experiment be repeated, with this difference, that the wire is now plucked in the middle instead of one fourth its length from one end. If it is now touched in the middle, the sound will entirely cease, showing that when a wire is plucked in the middle there is no first overtone superposed upon the fundamental. Let the wire be plucked again one fourth of its length from one end, and careful attention given to the compound note emitted. It will be found possible to recognize both the fundamental and the first overtone sounding at the same time. Similarly, by plucking at a point one sixth of the length of the wire from one end, and then touching it at a point one third of its length from the end, the second overtone may be made to appear distinctly, and a trained ear will detect it in the note given off by the wire, even before the fundamental is suppressed by touching at the point indicated.

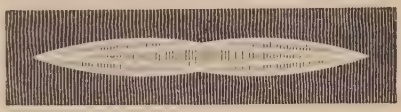


Fig. 389. A wire simultaneously emitting its fundamental and first overtone

These experiments show, therefore, that in general *the note emitted by a string plucked at random is a complex one, consisting of a fundamental and several overtones, and that just what overtones are present in a given case depends on where and how the wire is plucked.*

481. Quality. Let the sonometer wire be plucked first in the middle and then close to one end. The two notes emitted will have exactly the same pitch, and they may have exactly the same loudness, but they will be easily recognized as different in respect to something which we call *quality*. The experiment of the last paragraph shows that the real physical difference in the tones is a difference in the sort of overtones which are mixed with the fundamental in the two cases.

Again, let a mounted C' fork be sounded simultaneously with a mounted C fork. The resultant tone will sound like a rich, full C, which will change into a hollow C when the C' is quenched with the hand.

Every one is familiar with the fact that when notes of the same pitch and loudness are sounded upon a piano, a violin, and a cornet, the three tones can be readily distinguished. The last experiments suggest that the cause of this difference lies in the fact that it is only the *fundamental* which is the same in the three cases, while the *overtones* are different. In other words, the characteristic of a tone which we call its quality is determined simply by *the number and prominence of the overtones*

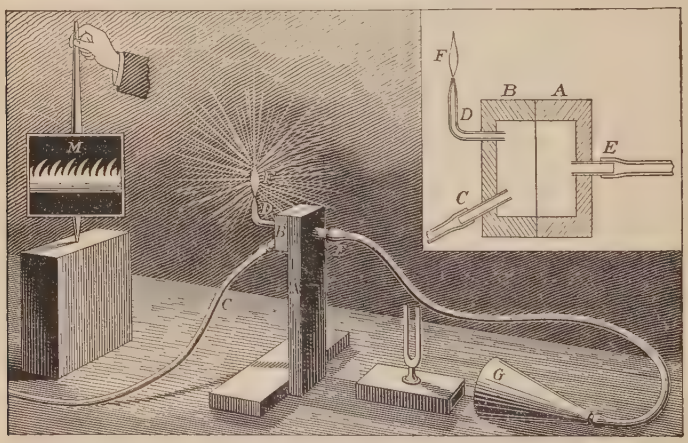


Fig. 390. Analysis of sounds with manometric flames

which are present in the tone. If there are few and weak overtones present, while the fundamental is strong, the tone is, as a rule, soft and mellow, as when a sonometer wire is plucked in the middle, or a closed organ pipe is blown gently, or a tuning fork is struck with a soft mallet. The presence of comparatively strong overtones up to the fifth adds fullness and richness to the resultant tone. This is illustrated by the ordinary tone from a piano or organ, in which several if not all of the first five overtones have a prominent place. When overtones higher than the sixth are present a sharp metallic quality begins to appear. This is illustrated when a tuning fork is struck, or a wire plucked,

with a hard body. It is in order to avoid this quality that the hammers which strike against piano wires are covered with felt.

482. Analysis of tones by the manometric flame. A very simple and beautiful way of showing the complex character of most tones is furnished by the so-called *manometric flames.* This device consists of the following parts: a chamber in the block B (Fig. 390), through which gas is led by way of the tubes C and D to the flame F; a second chamber in the block A, separated from the first chamber by an elastic diaphragm made of very thin sheet rubber or paper, and communicating with the source of sound through the tube E and trumpet G; and a rotating mirror M by which the flame is observed. When a note is produced before the mouthpiece G the vibrations of the diaphragm produce variations in the pressure of the gas coming to the flame through the chamber in B, so that when condensations strike the diaphragm the height of the flame is increased, and when rarefactions strike it the height of the flame is diminished. If these up-and-down motions of the flame are viewed in a rotating mirror, the longer and shorter images of the flame, which correspond to successive intervals of time, appear side by side, as in Fig. 391. If a rotating mirror is not to be had, a piece of ordinary mirror glass held in the hand and oscillated back and forth about a vertical axis will be found to give perfectly satisfactory results.

Fig. 391. Vibration forms shown by manometric flames

First let the mirror be rotated when no note is sounded before the mouthpiece. There will be no fluctuations in the flame, and its image, as seen in the moving mirror, will be a straight band, as shown in 2,

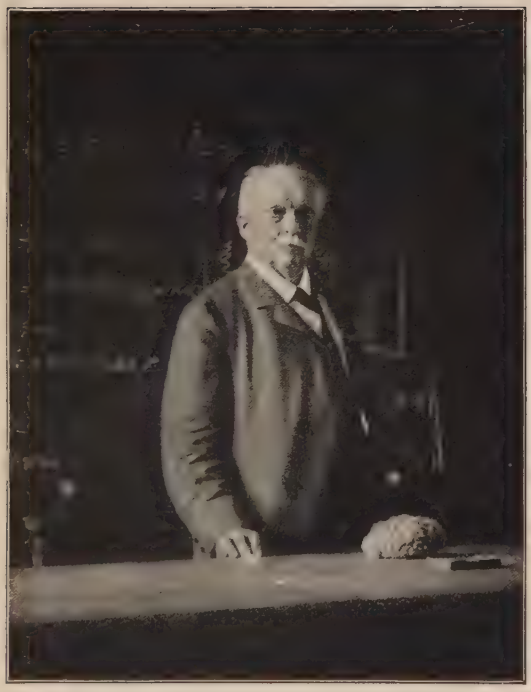
Fig. 391. Next let a mounted C fork be sounded, or some other simple tone produced in front of G. The image in the mirror will be that shown in 3. Then let another fork C' be sounded in place of the C. The image will be that shown in 4. The images of the flame are now twice as close together as before, since the blows strike the diaphragm twice as often. Next let the open ends of the resonance boxes of the two tuning forks C and C' be held together in front of G. The image of the flame will be as shown in 5. If the vowel *o* be sung in the pitch $B\flat$ before the mouthpiece, a figure exactly similar to 5 will be produced, thus showing that this last note is a complex, consisting of a fundamental and its first overtone.

The proof that most other tones are likewise complex lies in the fact that when analyzed by the manometric flame they show figures not like 3 and 4, which correspond to simple tones, but like 5, 6, and 7, which may be produced by sounding combinations of simple tones. No. 6 of the figure is produced by singing the vowel *e* on C'; No. 7 is obtained when *o* is sung on C'.

483. Helmholtz's experiment. If the loud pedal on a piano is held down and the vowel sounds *oo*, *ī*, *ā*, *ah*, *ē* sung loudly into the strings, these vowels will be caught up and returned by the instrument with sufficient fidelity to make the effect almost uncanny.

It was by a method which may be considered as merely a refinement of this experiment that Helmholtz proved conclusively that quality is determined simply by the number and prominence of the overtones which are blended with the fundamental. He first constructed a large number of resonators, like that shown in Fig. 392, each of which would respond to a note of some particular pitch. By holding these resonators in succession to his ear while a musical note was sounding he picked out the constituents of the note, i.e. he found out just what overtones were present and what were their relative intensities. Then he put these constituents together and reproduced the

Fig. 392. Helmholtz's resonator



HERMANN LUDWIG FERDINAND VON HELMHOLTZ (1821-1894)

Noted German physicist and physiologist; professor of physiology and anatomy at Bonn and at Heidelberg from 1855 to 1871; professor of physics at Berlin from 1871 to 1894; published in 1847 a famous paper on the conservation of energy, which was most influential in establishing that doctrine; invented the ophthalmoscope; discovered the physical significance of tone quality, and made other important contributions to physiological acoustics and optics; was preëminent also as a mathematical physicist.

original tone. This was done by sounding simultaneously, with appropriate loudness, two or more of a whole series of tuning forks which had the vibration ratios 1, 2, 3, 4, 5, 6, 7. In this way he succeeded not only in imitating the qualities of different musical instruments, but even in reproducing the various vowel sounds.

484. Sympathetic vibrations. Let two mounted tuning forks of the same pitch be placed with the open ends of their resonators facing each other. Let one be set into vigorous vibration with a soft mallet, and then quickly quenched by grasping the prongs with the hand. The other fork will be found to be sounding loudly enough to be heard over a large room. Next let a penny be waxed to one prong of the second fork and the experiment repeated. When the sound of the first fork is quenched, no sound whatever will be found to be coming from the second fork.

The experiment illustrates the phenomenon of *sympathetic vibrations* and shows what conditions are essential to its appearance. If two bodies capable of emitting musical notes have exactly the same natural period of vibration, the pulses communicated to the air when one alone is sounding beat upon the second at intervals which correspond exactly to its own natural period. Each pulse, therefore, adds its effect to that of the preceding pulses, and though the effect due to a single pulse is very slight, a great number of such pulses produce a large resultant effect. In the same way a large number of very feeble pulls may set a heavy pendulum into vibrations of considerable amplitude if the pulls come at intervals exactly equal to the natural period of the pendulum. On the other hand, if the two sounding bodies have even a slight difference of period, the effect of the first pulses is neutralized by the effect of succeeding pulses as soon as the two bodies, on account of their difference in period, get to swinging in opposite directions.

Let notes of different pitches be sung into a piano when the dampers are lifted. The wire which has the pitch of the note sounded will in every case respond. Sing a little off the key and the response will cease.

485. Sympathetic vibrations produced by overtones. It is not essential, in order that a body may be set into sympathetic vibrations, that it have the same pitch as the sounding body, provided its pitch corresponds exactly with the pitch of one of the *overtones* of that body.

Thus if the damper is lifted from the C string of a piano and C_1 is sounded loudly, C will be heard to sound clearly after C_1 has been quenched by the damper. In this case it is the first overtone of C_1 which is in exact tune with C, and which therefore sets it into sympathetic vibration. Again, if the damper is lifted from the G string while C_1 is sounded, this note will be found to be set into vibration by the second overtone of C_1. A still more interesting case is obtained by removing the damper from E while C_1 is sounded. When C_1 is quenched the note which is heard is not E but an octave above E, i.e. E'. This is because there is no overtone of C_1 which corresponds to the vibration of E, but the fourth overtone of C_1, which has five times the vibration number of C_1, corresponds exactly to the vibration number of E', the first overtone of E. Hence E is set into vibration not as a whole but in halves.

Fig. 393. Illustrating the production of discords

486. Physical significance of harmony and of discord. Let two pieces of glass tubing about an inch in diameter and a foot and a half long be supported vertically, as shown in Fig. 393. Let two gas jets, made by drawing down pieces of one-fourth inch glass tubing until, with full gas pressure, the flame is about an inch long, be thrust inside these tubes to a height of about three or four inches from the bottom. Let the gas be turned down until the tubes begin to sing. Without attempting to discuss the part which the flame plays in the production of the sound, we wish simply to call attention to the fact that the two tones are either quite in unison, or so near it that but a few beats are produced per second. Now let the length of one of the tubes be slightly increased by slipping the paper cylinder S up over its end. The number of beats will be rapidly increased until they will become indistinguishable as separate beats and will merge into a jarring, grating discord.

The experiment teaches that *discord is simply a phenomenon of beats.* If the vibration numbers do not differ by more than five or six, i.e. if there are not more than five or six beats per second, the effect is not particularly unpleasant. From this point on, however, as the difference in the vibration numbers, and therefore in the number of beats per second, increases, the unpleasantness increases, and becomes worst at a difference of about thirty. Thus the notes B and C', which differ by about thirty-two beats per second, produce about the worst possible discord. When the vibration numbers differ by as much as seventy, which is about the difference between C and E, the effect is again pleasing, or harmonious. Moreover, in order that two notes may harmonize well it is necessary not only that the notes themselves shall not produce an unpleasant number of beats, but also that such beats shall not arise from their overtones. Thus C and B are very discordant, although they differ by a large number of vibrations per second. The discord in this case arises between B and C', the first overtone of C.

Again, there are certain classes of instruments, of which bells are a striking example, which produce insufferable discords when even such notes as *do*, *sol*, *do'* are sounded simultaneously upon them. This is because these instruments, unlike strings and pipes, have overtones which are not harmonics, i.e. which are not multiples of the fundamental; and these overtones produce beats either among themselves or with one of the fundamentals. It is for this reason that in playing chimes the bells are struck in succession, not simultaneously.

QUESTIONS AND PROBLEMS

1. At what point must the G string be pressed by the finger of the violinist in order to produce the note C'?

2. If one wire has twice the length of another and is stretched by four times the stretching force, how will their vibration numbers compare?

3. A wire gives out the note G. What is its fourth overtone?

4. What is the fourth overtone of C? the fifth overtone?

5. A wire gives out the note C when the tension on it is 4 kg. What tension will be required to give out the note G?

6. A wire 50 cm. long gives out 400 vibrations per second. How many vibrations will it give when the length is reduced to 10 cm.? What syllable will represent this note if *do* represents the first note?

7. There are seven octaves and two notes on an ordinary piano, the lowest note being A_4 and the highest one C''''. If the vibration number of the lowest note is 27, find the vibration number of the highest.

8. Find the wave length of the lowest note on the piano; the wave length of the highest note. (Take the speed of sound in air as 1130 feet per second.)

9. A violin string is commonly bowed about one seventh of its length from one end. Why is this better than bowing in the middle?

Musical Properties of Air Chambers

487. Fundamentals of closed pipes. Let a tightly fitting rubber stopper be inserted in a glass tube a (Fig. 394), eight or ten inches long and about three fourths of an inch in diameter. Let the stopper be pushed along the tube until when a vibrating C' fork is held before the mouth resonance is obtained as in § 463. (The length will be six or seven inches.) Then let the fork be removed and a stream of air blown across the mouth of the tube through a piece of tubing b, flattened at one end as in the figure.[1] The pipe will be found to emit strongly the note of the fork. Let the pipe length be changed until it responds to a G fork. Blowing through b will now be found to produce the note G.

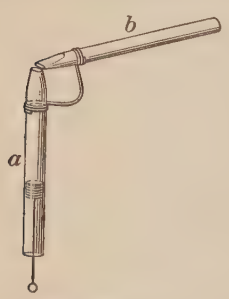


Fig. 394. Musical notes from pipes

In every case it is found that a note which a pipe may be made to emit is always a note to which it is able to respond when used as a resonator. Since, in § 464, the best resonance was found when the wave length given out by the fork was

[1] If the arrangement of Fig. 394 is not at hand, simply blow with the lips across the edge of a piece of ordinary glass tubing within which a rubber stopper may be pushed back and forth.

four times the length of the pipe, we learn that *when a current of air is suitably directed across the mouth of a closed pipe it will emit a note which has a wave length four times the length of the pipe.* This note is called the *fundamental* of the pipe. It is the lowest note which the pipe can be made to produce.

488. Fundamentals of open pipes. Since we found in § 465 that the lowest note to which a pipe open at the lower end can respond is one the wave length of which is twice the pipe length, we infer that an open pipe when suitably blown ought to *emit* a note the wave length of which is twice the pipe length. This means that if the same pipe is blown first when closed at the lower end and then when open, the first note ought to be an octave lower than the second.

Let the pipe *a* (Fig. 394) be closed at the bottom with the hand and blown; then let the hand be removed and the operation repeated. The second note will indeed be found to be an octave higher than the first.

We learn, therefore, that *the fundamental of an open pipe has a wave length equal to twice the pipe length.*

489. Overtones in pipes. It was found in § 464 that there are a whole series of pipe lengths which respond to a given fork, and that these lengths bear to the wave length of the fork the ratios $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, etc. This is equivalent to saying that a closed pipe of *fixed* length can respond to a whole series of notes whose vibration numbers have the ratios 1, 3, 5, 7, etc. Similarly, in § 465, we found that in the case of an open pipe the series of pipe lengths which will respond to a given fork bear to the wave length of the fork the ratios $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, etc. This again is equivalent to saying that an open pipe can respond to a series of notes whose vibration numbers have the ratios 1, 2, 3, 4, 5, etc. Hence we infer that it ought to be possible to cause both open and closed pipes to emit notes of higher pitch than their fundamentals, i.e. overtones, and that the first overtone of an *open*

pipe should have twice the rate of vibration of the fundamental, i.e. that it should be *do′*, the fundamental being considered as *do*; that the second overtone should vibrate three times as fast as the fundamental, i.e. it should be *sol′*; that the third overtone should vibrate four times as fast, i.e. it should be *do″*; that the fourth overtone should vibrate five times as fast, i.e. it should be *mi″*, etc. In the case of the *closed* pipe, however, the first overtone should have a vibration rate three times that of the fundamental, i.e. it should be *sol′*; the second overtone should vibrate five times as fast, i.e. it should be *mi″*, etc. In other words, while an open pipe ought to give forth *all* the harmonics, both odd and even, a closed pipe ought to produce only the *odd* harmonics.

Let the pipe of Fig. 394 be blown so as to produce the fundamental when the lower end is open. Then let the strength of the air blast be increased. The note will be found to spring to *do′*. By blowing still harder it will spring to *sol′*, and a still further increase will probably bring out *do″*. When the lower end is closed, however, the first overtone will be found to be *sol′* and the next one *mi″*, just as our theory demands.

490. Mechanism of emission of notes by pipes. The mechanism by which a musical note is produced when an air current plays across the mouth of a closed pipe may be understood from the following. Suppose that when the air current is first started it is directed against a point *a* (Fig. 395) just inside of the edge of the tube. A condensational pulse is at once started down the tube. This pulse is reflected at the lower end and returns to the mouth as a condensation, i.e. as an upward motion of the air particles. It therefore pushes the jet upward and thus causes it to pass above the edge of the tube instead of striking inside it (see dotted line, Fig. 395).

Fig. 395. Vibrating air jet

This upward push against the jet continues until the rear end of the condensation which the jet had been all the time communicating to the air while it was striking inside the edge, has returned again to the mouth after reflection from the bottom. In other words, the upward push against the jet continues for the time required for a sound wave to travel down the tube and back. Then it ceases and the jet at once begins again to strike against the point *a*. The whole operation then begins over again. It will be seen, therefore, that the reflected pulses returning from the bottom of the pipe force the air jet to vibrate with absolute regularity back and forth into and out of the pipe, the period of one half oscillation, i.e. the time during which the jet remains either inside or outside the tube, being determined solely by the time required for a pulse to travel down and back. Hence the wave length of the pulses sent to the ear of the observer is four times the length of the tube. This is then the fundamental note of the pipe.

By blowing more violently it is possible to create so great and so sudden a compression in the mouth of the pipe that the jet is forced out over the edge before the return of the first reflected pulse. In this case no note will be produced unless the blowing is of just the right intensity to cause the jet to swing out in the period corresponding to an overtone. In this case the reflected pulses will return from the end at just the right intervals to keep the jet swinging in this period. This shows why a current of a particular intensity is required to start any particular overtone.

The theory of open pipes is not essentially different from that of closed pipes. In both the vibrations of the air jet into and out of the pipe are controlled entirely by the return of the reflected pulses. This explains why the periods of all emitted notes are the same as the periods of the notes to which the pipes can respond when used as resonators, in the manner described in §§ 463, 464, and 465.

Wind Instruments

491. Vibrating air-jet instruments. The mechanism of the production of musical tones by the ordinary organ pipe, the flute, the fife, the piccolo, and all whistles is essentially the same as in the case of the pipe of Fig. 394. In all these instruments an air jet is made to play across the edge of an opening in an air chamber, and the reflected pulses returning from the other end of the chamber cause it to vibrate back and forth, first into the chamber and then out again. In this way a series of regularly timed puffs of air is made to pass from the instrument to the ear of the observer precisely as in the case of the rotating disk of § 457. The air chamber may be either open or closed at the remote end. In the flute it is open, in whistles it is usually closed, and in organ pipes it may be either open or closed. Fig. 396 shows a cross section of two types of organ pipes. The jet of air from S vibrates across the lip L in obedience to the pressure exerted on it by waves reflected from O. Pipe organs are provided with a different pipe for each note, but the flute (Fig. 397), piccolo, or fife is made to produce a whole series of notes, either by blowing overtones or by opening holes in the tube, an operation which is equivalent to cutting the tube off at the hole.

Fig. 396. Organ pipes

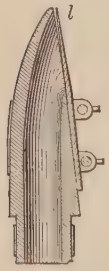


Fig. 399. Mouthpiece of a clarinet, showing the tongue l, which opens and closes the upper end of the pipe

Fig. 397 The flute

Fig. 398 The clarinet

492. Vibrating reed instruments. In reed instruments the vibrating air jet is replaced by a vibrating reed or tongue which opens and closes, at absolutely regular intervals, an opening against which the performer is directing a current of air. In the clarinet (Fig. 398), the oboe, the bassoon, etc., the reed is placed at the upper end of the tube (see l Fig. 399), and the theory of its opening and closing the orifice so as to admit successive puffs of air to the pipe is identical with the theory of the fluctuation of the air jet into and out of the organ pipe. For in these instruments the reed has practically no rigidity, and consequently no natural period. Hence its vibrations are controlled entirely by the reflected pulses.

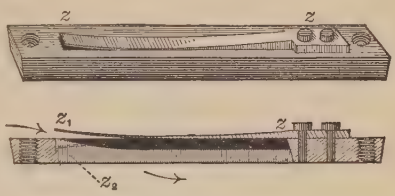


Fig. 400. The vibrating tongue of the mouth organ, accordion, etc.

In other reed instruments, like the mouth organ, the common reed organ, or the accordion, it is the elasticity of the reed alone (see z, Fig. 400) which controls the emission of pulses. In such instruments there is no necessity for air chambers. The arrows of Fig. 400 indicate the direction of the air current which is interrupted as the reed vibrates between the positions z_1 and z_2.

Fig. 401. The reed-organ pipe

In still other reed instruments, like the reed pipes used in large organs (Fig. 401), the period of the pulses is controlled partly by the elasticity of the reed and partly by the return of the reflected waves; in other words, the natural period of the reed is more or less coerced by the period of the reflected pulses. Within certain limits, therefore, such instruments may be tuned by changing the length of the vibrating reed l without changing the length of the pipe. This is done by pushing the wire r up or down.

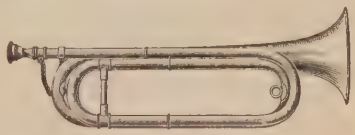


Fig. 402. The bugle

493. Vibrating lip instruments. In instruments of the bugle and cornet type the vibrating reed is replaced by the vibrating lips of the musician, the period of their vibration being controlled, precisely as in

the organ pipe or the clarinet, by the period of the returning pulses. In the bugle (Fig. 402) the pipe length is fixed, and hence the only notes of which such an instrument is capable are the fundamental and about five overtones. In the cornet (Fig. 403) and in most forms of horns valves *a*, *b*, *c* worked by the fingers vary the length of the pipe, and hence such instruments can produce as many series of fundamentals and overtones as there are possible tube lengths. In the trombone (Fig. 404) the variation of pitch is accomplished by blowing overtones and by changing the tube length by a sliding portion *SL*.

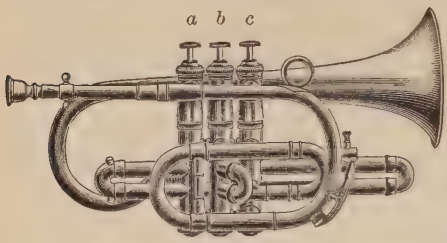


Fig. 403. The cornet

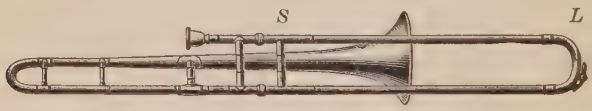


Fig. 404. The trombone

494. The phonograph. In the phonograph (Fig. 405) the sound waves collected by the cone *F* are carried to a thin metallic disk *C*, exactly like a telephone diaphragm, which takes up very nearly the vibration form of the wave which strikes it. This vibration form is permanently impressed on the wax-coated cylinder *M* by means of a stylus *D* (Fig. 406) which is attached to the back of the disk. When the stylus is run a second time over the groove which it first made in the wax, it receives again and imparts to the disk the vibration form which first fell upon it. The membrane imparts its vibrations to the air and thus the original sound is reproduced. This instrument is one of the many inventions of the American inventor Thomas Edison.

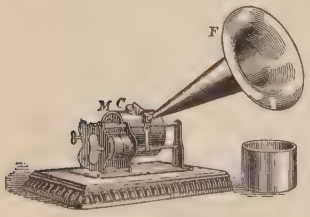


Fig. 405. The phonograph

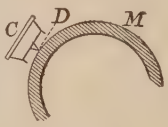


Fig. 406

QUESTIONS AND PROBLEMS

1. What will be the relative lengths of a series of organ pipes which produce the eight notes of the diatonic scale?

2. What must be the length of a closed organ pipe which produces the note *E*? (Take the speed of sound as 340 m. per sec.)

3. Will the pitch of a pipe organ be the same in summer as on a cold day in winter? What could cause a difference?

4. What is the first overtone which can be produced in an open *G* organ pipe?

5. What is the first overtone which can be produced by a closed *C* organ pipe?

6. Explain how an instrument like the bugle, which has an air column of unchanging length, may be made to produce several notes of different pitch.

7. When water is poured into a deep bottle why does the pitch of the sound rise as the bottle fills?

CHAPTER XIX

NATURE AND PROPAGATION OF LIGHT

Transmission of Light

495. Galileo's experiment. That light travels with a speed which is at least much greater than the speed of sound is shown by the facts that the flash of a distant gun is always seen long before the sound of the report is heard, and that lightning always precedes thunder. The first careful experiment upon its speed was made by Galileo, who attempted to measure the time required for the light of a lantern to travel from one hill to another near Florence. He concluded that no time whatever was consumed in its passage between the two hills, and this view was generally accepted until 1675, when Olaf Roemer, a young Danish astronomer, made some observations at the Paris Observatory which proved this theory to be false.

496. Roemer's determination of the speed of light. Roemer was making observations on the largest and brightest of Jupiter's seven moons when he noticed certain phenomena which led him to believe that the time required for light to travel across the diameter of the earth's orbit was equal to 16 min. and 36 sec. The observations were as follows. At every revolution of the satellite M (Fig. 407) about Jupiter J it passes into the shadow which that planet casts in a direction opposite the sun, and thus suffers an eclipse. With the aid of a telescope the instant of this eclipse can be observed with great accuracy. Roemer first determined the interval between two successive eclipses (i.e. the period of revolution of the satellite) when the earth was at E, and found it to be 42 hr. 28 min. and 36 sec.

Using this period as a basis of computation, he could predict the exact instant at which an eclipse should occur six months later when the earth was at E', the point of its orbit farthest from Jupiter, and again a year later when the earth had returned to E. When, however, he observed the eclipse at E' he found that it took place 16 min. and 36 sec. (996 sec.) later than the predicted time, while at the end of a year it occurred exactly at the predicted time. Roemer inferred from this that the 996 sec. delay observed at E' represented the time required for the light to travel across the earth's orbit from E to E', — a distance which was known from astronomical observations to be about 308,000,000 kilometers. Hence Roemer computed the velocity of light to be about

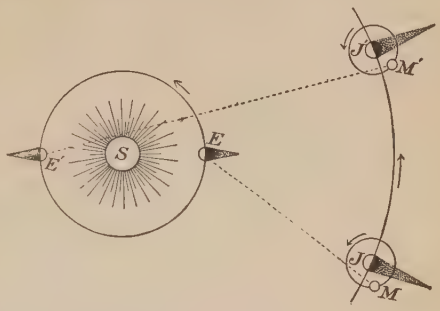


FIG. 407. Illustrating Roemer's determination of the velocity of light

$$\frac{308{,}000{,}000}{996} = 309{,}000 \text{ kilometers per second.}$$

497. Recent determinations. In recent years the speed of light has been directly measured on the earth's surface by several different methods which have yielded an accuracy much greater than that obtained by Roemer. Probably the two most accurate determinations are those made in 1882 by Michelson of the University of Chicago, and in 1902 by Perrotin of the University of Nice, France. Although these observers used wholly different methods their results are almost identical. The former obtained 299,860 km. per second, and the latter 299,880 km. per second. For most purposes it is sufficiently accurate to take the velocity of light in round numbers as

300,000 km. per second, which is equivalent to about 186,000 miles per second. The magnitude of this number will perhaps be better appreciated when we reflect that light could travel about seven and one half times around the earth, at the equator, in one second.

498. Some results of the finite speed of propagation of light. In spite of the enormous value of the speed of light, it is so small in comparison with interstellar distances that the conditions which we see in the heavens are not the conditions which exist *now*, but rather the conditions which *have existed* at some previous time. Thus, since it requires on the average about forty minutes for light to come from Jupiter to the earth, an eclipse of one of Jupiter's moons actually occurs forty minutes before it is seen on the earth, and similarly this moon emerges from eclipse forty minutes before it appears to do so. Again, the light which is now reaching the earth from the nearest fixed star, Alpha Centauri, started 4.4 years ago. If the brightest star in the heavens, Sirius, were suddenly annihilated, it would still shine on, apparently undisturbed, for 8.8 years. If an observer on the pole star had a telescope powerful enough to enable him to see events on the earth, he would not see the battle of Gettysburg (which occurred in July, 1863) until January, 1918.

499. Speed of light in liquids and solids. Both Foucault in France and Michelson in America have measured directly the velocity of light in water and have found it to be only three fourths as great as in air. It will be shown later that the velocity of light in all transparent liquids and solids is less than it is in air. The velocity in air is practically the same as the velocity in a vacuum.

500. Rectilinear propagation of light. Any one who has watched the beam from a search light sweep through the sky on a dark night, or who has noticed a sunbeam tracing its path through a darkened room, or who has reflected upon the sharpness of the shadows cast by trees in the sunlight, or by any objects placed near an electric light, needs no further proof that light travels out from a source in straight lines. In this respect light

seems to differ radically from sound, for the sound of a bell is not cut off when we insert a screen between the bell and ear, while light is completely cut off when such a screen is inserted between a source and the eye. A single line of light, i.e. a beam of light of small cross section, is commonly called a *ray*.

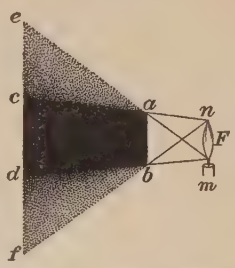


FIG. 408. Shadow from a broad source

501. Shadows. Let any opaque object be held very close to a white screen placed opposite a window or a broad gas flame. So long as the object is very close to the screen the shadow is uniformly dark, but as it is moved toward the source of light (*F*, Fig. 408) two parts to the shadow will be observed, — a very black part *cd* in the middle, from which all the light from the source is excluded, and a lighter part, *ec* and *df*, on either side of *cd*.

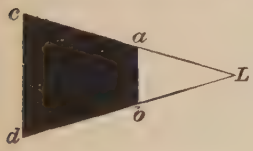


FIG. 409. Shadow from a small source

These effects are easily explained on the basis of the rectilinear propagation of light. The region *abdc*, from which the light from all points of the source *mn* is excluded, is called the *umbra*. The region *ace* and *bdf*, which receives light from some portions of the source but not from all, is called the *penumbra*. It will be seen from the figure that the *penumbra* must decrease as the object approaches the screen, and also as the size of the source diminishes. When the source becomes a mere point there is no penumbra at all (Fig. 409). When the source is larger than the opaque object, as in the case of the sun and earth, the umbra is a cone, as shown in Fig. 410.

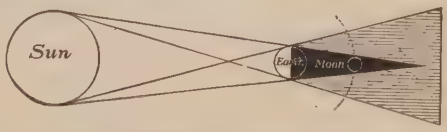


FIG. 410. Illustrating a total eclipse of the moon by passage into the umbra of the earth

QUESTIONS AND PROBLEMS

1. If it takes 54.5 years for light to come from the north star to the earth, how far away is this star?

2. The diameter of the earth is 8000 miles and that of the sun 860,000 miles. The earth is 93,000,000 miles from the sun. What is the length of the earth's umbra?

3. The diameter of the moon is 2000 miles. What is the length of the moon's umbra?

4. Will it ever be possible for the moon to totally eclipse the sun from the whole of the earth's surface at once?

5. If the distance from the center of the earth to the center of the moon were exactly equal to the length of the moon's umbra, over how wide a strip on the earth's surface would the sun be totally eclipsed at any one occasion?

6. If the star Arcturus, distance 600,000,000,000,000 miles from the earth, were to explode suddenly, how long would it be before astronomers could detect the fact?

Intensity of Light

502. Decrease of intensity with distance. Considerations precisely analogous to those which led us to the conclusion

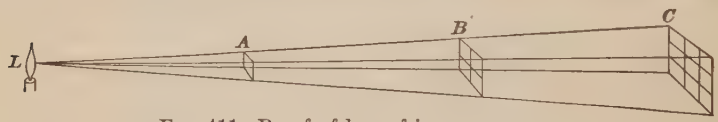


Fig. 411. Proof of law of inverse squares

that the intensity of sound is inversely proportional to the square of the distance from the source show that the same law must hold for light.

Let L (Fig. 411) represent a point source of light and let A be a screen 1 ft. square placed at a distance of 5 ft. from L. Since light travels in straight lines the shadow which the screen casts on a wall B 10 ft. from L will have an area of 4 sq. ft. If now the screen A is removed, the light which will then fall upon the 4 sq. ft. occupied by the shadow must be exactly the same as that which before fell upon the screen 1 ft. square. Since this light is now spread over 4 sq. ft., each square foot can receive but one fourth as much light as fell upon the screen A. If the wall were at C, 15 ft. from L, instead of 10 ft., precisely the

same reasoning would show that each square foot would receive but one ninth of the light which fell upon A. In other words, *the intensity of the illumination due to a given point source must vary inversely as the square of the distance from the source.*

503. Experimental proof of the law of inverse squares. Rumford's photometer. Let four candles be set as close together as possible in such a position B as to cast upon a white screen C, placed in a well-darkened room, a shadow of an opaque object O (Fig. 412). Let one single candle be placed in a position A such that it will cast another shadow of O upon the screen. Since light from A falls on the shadow cast by B, and light from B falls on the shadow cast by A, it is clear that the two shadows will appear equally dark only when light of equal intensity falls on each, i.e. when A and B produce equal illumination upon the screen. Let the positions of A and B be shifted until this condition is fulfilled. Then let the distances from B to C and from A to C be measured. If all five candles are burning with flames of the same size, the first distance will be found to be just twice as great as the second. Hence the illumination produced upon the screen by each one of the candles at B is but one fourth as great as that produced on the screen by one candle at A, one half as far away.

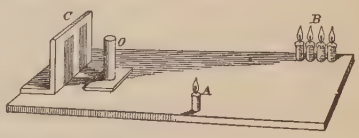


Fig. 412. Rumford's photometer

The experiment, therefore, furnishes direct confirmation of the above theoretical conclusion that the intensity of light varies inversely as the square of the distance from the source.

This method of comparing the intensities of two lights was first used by Count Rumford. The arrangement is therefore called the *Rumford photometer* (light measurer).

504. Candle power. The last experiment furnishes a method of comparing the *light-emitting powers* of various sources of light. For example, suppose that the four candles at B are replaced by a gas flame, and that for the condition of equal illumination upon the screen the two distances BC and AC are the same as above, viz. 2 to 1. We should then know that the gas flame, which is able to produce the same illumination at

a distance of two feet as a candle at a distance of one foot, has a light-emitting power equal to four candles. In general, then, the candle power of any two sources which produce equal illumination on a given screen are *directly proportional to the squares of the distances of the sources from the screen.*

It is customary to express the intensities of all sources of light in terms of candle power, one candle power being defined as the amount of light emitted by a sperm candle $\frac{7}{8}$ in. in diameter and burning 120 grains (7.776 g.) per hour. The candle power of an ordinary gas flame burning 5 cu. ft. per hour is from 16 to 25, depending on the quality of the gas. A Welsbach lamp burning 3 cu. ft. per hour has a candle power of from 50 to 100. Most incandescent electric lamps which are used for domestic purposes are of 16 candle power. The average arc light has a candle power of about 500, although when measured in the direction of greatest intensity the illuminating power may be as great as that of 1000 or 1200 candles.

505. Bunsen's photometer. Let a drop of oil or melted paraffin be placed in the middle of a sheet of unglazed white paper to render it translucent. Let the paper be held near a window and the side *away from* the window observed. The oiled spot will appear *lighter* than the remainder of the paper. Let it be held so that the side of the paper nearest the window may be seen. The oiled spot will appear *darker* than the rest of the paper. We learn, therefore, that *when the paper is viewed from the side of greater illumination the oiled spot appears dark;* but *when it is viewed from the side of lesser illumination the spot appears light.* If, then, the two sides of the paper are equally illuminated, the spot ought to be of the same brightness when viewed from either side. Let the room be darkened and the oiled paper placed between two gas flames, two electric lights, or any two equal sources of light. It will be observed that when the paper is held closer to one than the other, the spot will appear dark when viewed from the side next the closer light; but if it is then moved until it is nearer the other source, the spot will change from dark to light when viewed always from the same side. It is always possible to find some position for the oiled paper at which the spot either disappears altogether or at least appears the same when viewed

from either side. This is the position at which the illuminations from the two sources are equal. Hence, to find the candle power of any unknown source it is only necessary to set up a candle on one side and the unknown source on the other, as in the Fig. 413, and to move the spot A to the position of equal illumination. The candle power of the unknown source C will then be the square of the distance from C to A, divided by the square of the distance from B to A.

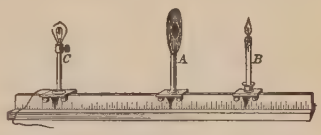


Fig. 413. Bunsen's photometer

This arrangement is known as the *Bunsen photometer*.

QUESTIONS AND PROBLEMS

1. How far from a screen must a 4-candle-power light be placed to give the same illumination as a 16-candle-power electric light 3 m. away?

2. A Bunsen photometer placed between an arc light and an incandescent light of 32 candle power is equally illuminated on both sides when it is 10 ft. from the incandescent light and 36 ft. from the arc light. What is the candle power of the arc?

3. A 5-candle-power and a 30-candle-power source of light are 2 m. apart. Where must the oiled disk of a Bunsen photometer be placed in order to be equally illumined on the two sides by them?

4. If the sun were at the distance of the moon from the earth, instead of at its present distance, how much stronger would sunlight be than at present? The moon is 240,000 miles and the sun 92,000,000 miles from the earth.

5. If a gas flame is 300 cm. from the screen of a Rumford photometer, and a standard candle 50 cm. away gives a shadow of equal intensity, what is the candle power of the gas flame?

Reflection and Refraction of Light

506. Angle of incidence equals angle of reflection.[1] Let a beam of sunlight be admitted to a darkened room through a narrow slit. The straight path of the beam will be rendered visible by the brightly illumined dust particles suspended in the air. Let the beam fall on the surface of a mirror. Its direction will be seen to be sharply changed

[1] An exact laboratory experiment on this law should either precede or follow this discussion. See, for example, Experiment 42 of the authors' manual.

as shown in Fig. 414. Let the mirror be held so that it is perpendicular to the beam. The beam will be seen to be reflected directly back on itself. Let the mirror be turned through an angle of 45°. The reflected beam will move through 90°.

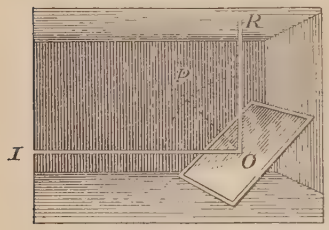


FIG. 414. Illustrating law of reflection of light

The experiment shows roughly, therefore, that the angle *IOP*, between the incident beam and the normal to the mirror, is equal to the angle *POR* between the reflected beam and the normal to the mirror. The first angle *IOP* is called the angle of incidence, and the second *POR* the angle of reflection. Hence the law of the reflection of light may be stated thus: *The angle of reflection is equal to the angle of incidence.*

507. Diffusion of light. In the last experiment the light was reflected by a very smooth plane surface. Let the beam be now allowed to fall upon a rough surface like that of a sheet of unglazed white paper. No reflected beam will be seen; but, instead, the whole room will be brightened appreciably, so that the outline of objects before invisible may be plainly distinguished.

The beam has evidently been scattered in all directions by the innumerable little reflecting surfaces of which the surface of the paper is composed. The effect will be much more noticeable if the beam is allowed to fall alternately on a piece of dead black cloth and on the white paper. The light is largely absorbed by the cloth, while it is scattered or *diffusely reflected* by the paper.

The difference between a smooth reflector and a rough one is illustrated in greatly magnified form in Fig. 415. In both cases the law of reflection for each ray of light is precisely the same, i.e. the angle of incidence is equal to the angle of reflection; but, whereas in the first case all portions of the reflecting surface are parallel to one another, and therefore reflect in the same

direction all the rays which fall upon them from a given direction, in the second case the little elements of the surface are turned in a great variety of ways, and hence the reflected rays pass off in every conceivable direction. Even the smoothest surfaces which can be made diffuse light to a slight extent.

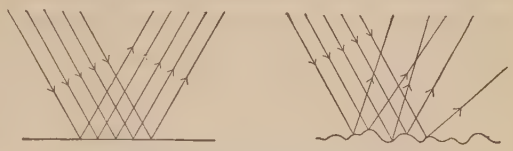


Fig. 415. Regular and irregular reflection

508. Visibility of nonluminous bodies. Every one is familiar with the fact that certain classes of bodies, such as the sun, a gas flame, etc., are self-luminous, i.e. visible on their own account; while other bodies, like books, chairs, tables, etc., can be seen only when they are in the presence of luminous bodies. The above experiment shows how such nonluminous, diffusing bodies become visible in the presence of luminous bodies. For, since a diffusing surface scatters in all directions the light which falls upon it, each small element of such a surface is sending out light in a great many directions, in much the same way in which each point on a luminous surface is sending out light in all directions. Hence we always see the *outline* of a diffusing surface as we do that of an emitting surface, no matter where the eye is placed. On the other hand, when light comes to the eye from a polished reflecting surface, since the form of the beam is wholly undisturbed by the reflection, we see the outline, not of the mirror, but rather of the source from which the light came to the mirror, whether this source is itself self-luminous, or is only acting, because of its light-scattering power, like a self-luminous source. Points on the mirror which are not in line with this source can send no light whatever to the eye. Hence the mirror itself must be invisible. The fact that one often runs into a large mirror or plate-glass window is sufficient confirmation of the truth of the statement that neither a perfect reflector

nor a perfectly transparent body is itself visible. All bodies other than self-luminous ones are visible only by the light which they diffuse. Perfectly black bodies send no light to the eye, but their outlines can be distinguished because of the light which is sent to the eye from the background. Any object *which can be seen*, therefore, may be regarded as itself sending rays to the eye, i.e. it may be treated as a luminous body.

509. Conditions for the reflection of light. Let a candle flame, a gas flame, or an incandescent lamp be viewed by reflection in a piece of red glass. Two distinct images of the source of light will be seen, one image being white and the other red. When the light is viewed *through* the glass it will appear red. Hence we conclude that the red image obtained by reflection must be formed by light which traveled through the glass and was reflected from the *farther* side, while the white image was produced by light reflected from the *nearer* surface of the glass.

The experiment shows, therefore, that light undergoes a reflection as well when it is passing from glass into air as when it is passing from air into glass. In general, then, *light, like sound, suffers reflection whenever it meets a medium in which its speed is different from that in the medium in which it has been traveling.* It shows further that whenever a beam strikes a new, transparent medium, it divides into two portions, one of which is transmitted and the other reflected.

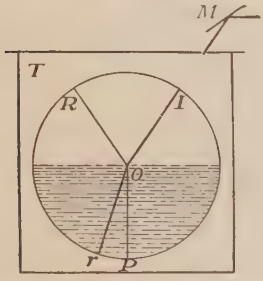


Fig. 416. Refraction of light passing from air to water

510. Refraction. Let a ray of sunlight be admitted to a darkened room and reflected so as to fall on the surface of the water in a tank[1] T in the manner shown in Fig. 416. The division of the beam into a

[1] All of these experiments on reflection and refraction may be done almost as effectively and even more conveniently by substituting for the water tank disks of glass, like those used with the "Hartl Optical Disk," through which the beam can be traced.

reflected part OR and a transmitted part Or will be plainly seen, particularly if smoke or chalk dust is blown into the tank. In addition, it will be observed that the transmitted portion has suffered a change in direction. When light is *bent in this way in passing from one medium to another, it is said to undergo refraction.* Let the mirror M be rotated so as to cause the beam of light to strike the water surface at different angles. It will be found that when it strikes the surface normally it undergoes no bending, but that bending occurs at all other angles. It will further be seen that the *greater the angle of incidence the greater the bending.* Again, the ray within the water will always be seen to be bent *toward* the perpendicular OP drawn from the surface into the water at the point where the light strikes it. Next, let the refracted beam fall upon a mirror in the bottom of the tank, so that it may be reflected and brought again to the surface in the manner shown in Fig. 417. As it emerges again into the air it will be seen to suffer a second bending, this time being turned away from the perpendicular $O'P'$ drawn into the air from the surface of the water at the point where the ray leaves it.

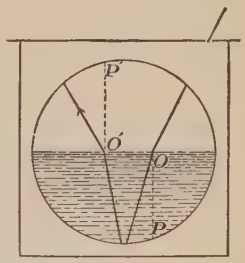


FIG. 417. Refraction of light passing from water to air

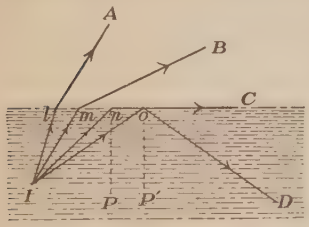


FIG. 418. Rays coming from a source I under water to the boundary between air and water at different angles of incidence

Similar experiments made with other substances have brought out the general law that *whenever light travels obliquely from one medium into another in which the speed is less, it is bent toward the perpendicular, and when it passes from one medium to another in which the speed is greater, it is bent away from the perpendicular drawn into the second medium.*

511. Total reflection. Since the rays emerging from water into air are always bent *from* the perpendicular (see IlA, ImB etc., Fig. 418), it is clear that if the angle of incidence on the

under surface of the water is made larger and larger, a point must be reached at which the refracted ray is *parallel to the surface* (see *InC*, Fig. 418). It is interesting to inquire what will happen to a ray *Io* which strikes the surface at a still greater angle of incidence *IoP'*. It will not be unnatural to suppose that since the ray *nC* just grazed the surface, the ray *Io* will not be able to emerge at all. An experiment with the tank of Fig. 416 will show that this is indeed the case.

Let a beam be made to enter the glass end of the tank in the manner shown in Fig. 419. If the angle of incidence *IOP* is small, the beam will be seen to divide at the point *O*, and one portion will be reflected back into the water while the other portion passes out into the air. Let the mirror *M* then be turned so as to gradually increase the angle *IOP*. A point will be reached at which the emerging beam disappears completely, and at the same time the brightness of the reflected beam will be seen to increase.

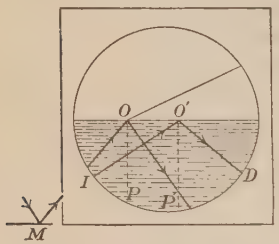


Fig. 419. Total reflection

This phenomenon is called *total reflection*, because the intensity of the reflected beam *O'D* is the same as that of the incident beam *IO'*. It will be seen, too, from the above discussion, that *total reflection can take place only when light traveling in any medium meets another medium in which the speed is greater.*

512. Critical angle. The angle *IO'P'* (Fig. 419), i.e. the angle between the *incident* ray and the perpendicular drawn to the surface in the medium of smaller velocity at the point at which total reflection first begins to occur, is called the *critical angle.* This angle varies with the nature of the substance. Thus the critical angle for water and air is about 48.5°, for flint glass 38.6°, for crown glass 42.5°, for diamond 23.7°.

Since the critical angle for water is 48.5°, to an eye at *E* (Fig. 420) placed under water, all outside objects will appear to lie

within a cone whose angle is $2 \times 48.5° = 97°$ (Fig. 420). As the eye looks toward the surface at an angle greater than 48.5°, it can see nothing but the reflection from the bottom of the body of water.

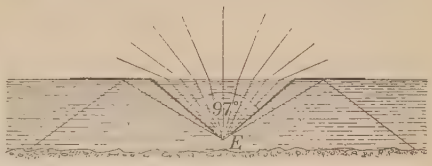


Fig. 420. To an eye under water all external objects appear to lie within a cone whose angle is 97°

513. Total reflection within a glass prism. Let a prism with three polished faces be held in the path of a beam of sunlight in the position shown in Fig. 421. Of the two parts into which the beam splits when it reaches the surface AB, one part will be transmitted and produce a spot of light on the wall at S (neglect for the present the color), while the other will be reflected and produce a spot on the wall at S'. Let the prism be rotated slowly in the direction of the arrow. A position will be reached at which the spot at S wholly disappears, while, at the same time, the spot at S' shows an appreciable increase in brightness. This is the position at which the angle of incidence IOP upon the face AB has become equal to the critical angle of the glass, viz. 42.5°, and hence it is the position at which total reflection begins to take place. Let the spot S' be observed carefully at the instant at which S begins to disappear. It will be seen to be divided into two parts by a bluish line, the part on one side of this line having a considerably greater brightness than the part on the other side. The brighter half represents light which has been totally reflected at AB; the darker half represents only the reflected portion of a beam which has been partially transmitted at AB.

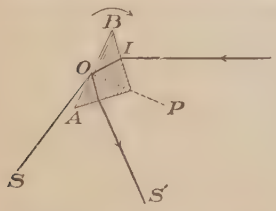


Fig. 421. Transmission and reflection of light at surface AB of a right-angled prism

The phenomenon is due to the fact that the original beam of light which fell upon AB came from all parts of the sun, so that the rays coming from one edge of the sun's disk struck the surface AB at a slightly different angle from those coming from the other edge. Hence the total reflection of these

first rays began to take place before the critical angle had been reached for the second rays.

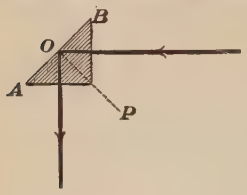


FIG. 422. Total reflection of light from surface AB of a right-angled prism

A prism placed in the position shown in Fig. 422 is the most perfect reflector known. Such prisms are frequently used in optical instruments. They are called *total reflecting prisms.* It is preferable to make them with one right angle, as shown, for then the beam may both enter and emerge from the prism in a direction at right angles to the face.

514. Path of a ray through a prism. When a ray of light does not suffer total reflection within a prism, the last experiment shows that its path is that shown in Fig. 423, i.e. light in passing through a prism is always bent around the base ab, — *never* around the apex c. This result could have been foreseen, for we learned in § 510 that the ray must be bent toward the perpendicular OP on entering at O, and away from the perpendicular $O'P'$ on emerging at O'.

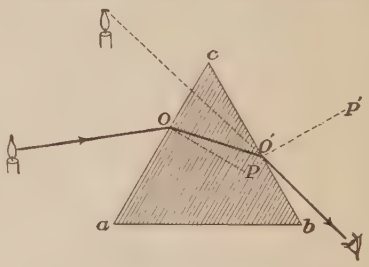


FIG. 423. Path of a ray through a prism

515. Path of a ray of light through a plate of glass with parallel faces. Let a ray of sunlight be sent obliquely through a piece of plate glass. Its path, as traced by the dust particles of the air and the diffusing particles in the glass, will be seen to be that shown in Fig. 424. The emerging beam will be strictly parallel to the incident beam.

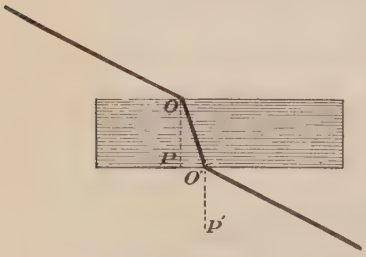


FIG. 424. Path of a ray through a medium bounded by parallel faces

We learn, therefore, that *when light passes obliquely through a medium bounded by parallel planes the refractions at the two surfaces are equal and opposite*, i.e. the beam is bent toward the perpendicular on entering the glass, and an equal amount away from it on emerging. The beam, therefore, suffers only a lateral displacement and not a change in direction. The amount of this lateral displacement depends on the nature of the medium, the thickness of the medium, and the obliquity of the rays.

QUESTIONS AND PROBLEMS

1. Why is a room with white walls much lighter than a similar room with black walls?

2. If the word "white" be painted across the face of a mirror and held in the path of a beam of sunlight in a darkened room, in the middle of the spot on the wall which receives the reflected beam the word "white" will appear in black letters. Explain.

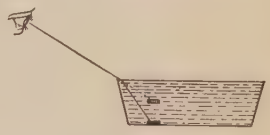


Fig. 425

3. Explain how the phases of the moon show that it shines only by reflected light.

4. Draw a diagram showing what must be the relative positions of the earth, sun, and moon at new moon; at half moon; at full moon.

5. Explain why it does not become dark as soon as the sun sets.

6. The earth reflects sixteen times as much light to the moon as the moon does to the earth. Trace from the sun to the eye of the observer the light by which he is able to see the dark part of the new moon. Why can we not see the dark part of a three-quarter moon?

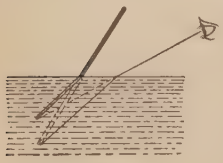


Fig. 426

7. If a penny is placed in the bottom of a vessel in such a position that the edge just hides it from view (Fig. 425), it will become visible as soon as water is poured into the vessel. Explain.

8. A stick held in water appears bent, as shown in Fig. 426. Explain.

9. Should a man who wishes to spear a fish aim a little high or a little low? Why?

10. The speed of light in air is slightly less than it is in a vacuum, and the denser the air the less the speed. In consequence of this fact a ray of sunlight at sunset or sunrise has the shape shown in the Fig. 427, i.e. the sun appears to be at S' when it is actually at S. Explain why the ray is

curved instead of being bent at a sharp angle. (In the figure the bending is enormously exaggerated; it is, in fact, only sufficient to make the sun appear about one diameter higher than it really is when at the horizon. We see, therefore, that when the sun appears to us to be rising it is actually still a full diameter below the horizon, and when it appears to be setting it has in fact already sunk about a diameter (accurately 35′) beneath the horizon.

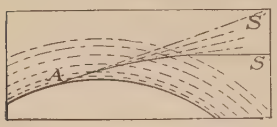


Fig. 427

11. Explain why a straight wire seen obliquely through a piece of glass appears broken, as in Fig. 428.

12. In what direction must a fish look in order to see the setting sun?

13. In what respect is a right-angled prism (Fig. 422) a better mirror than one of the ordinary kind?

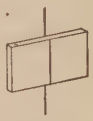


Fig. 428

14. What is the principal reflecting medium in an ordinary mirror?

15. Fig. 429 represents a section of a plate of Luxfer prism glass. Explain how glass of this sort is so much more efficient than ordinary window glass in illuminating the rears of dark stores on the ground floor in narrow streets.

Fig. 429
Luxfer prism glass

The Nature of Light

516. The corpuscular theory of light. All of the properties of light which have so far been discussed are perhaps most easily accounted for on the hypothesis that light consists of streams of very minute particles, or *corpuscles,* projected with the enormous velocity of 300,000 km. per second from all luminous bodies. The facts of straight-line propagation and reflection are exactly as we should expect them to be if this were the nature of light. The facts of refraction can also be accounted for, although somewhat less simply, on this hypothesis. As a matter of fact, this theory of the nature of light, known as the *corpuscular theory,* was the one most generally accepted up to about 1800.

517. The wave theory of light. A rival hypothesis, which was first completely formulated by the great Dutch physicist

CHRISTIAN HUYGENS (1629–1695)

Great Dutch physicist, mathematician, and astronomer; discovered the rings of Saturn; made important improvements in the telescope; invented the pendulum clock (1656); developed with marvelous insight the wave theory of light; discovered in 1690 the "polarization" of light. (The fact of double refraction was discovered by Erasmus Bartholinus in 1669, but Huygens first noticed the polarization of the doubly refracted beams, and offered an explanation of double refraction from the standpoint of the wave theory.)

Huygens (1629–1695), regarded light, like sound, as a *form of wave motion.* This hypothesis met at the start with two very serious difficulties. In the first place, light, unlike sound, not only travels with perfect readiness through the best vacuum which can be obtained with an air pump, but it travels without any apparent difficulty through the great interstellar spaces which are probably infinitely better vacua than can be obtained by artificial means. If, therefore, light is a wave motion, it must be a wave motion of some medium which fills all space and yet which does not hinder the motion of the stars and planets. Huygens assumed such a medium to exist, and called it *the ether.*

The second difficulty in the way of the wave theory of light was that it seemed to fail to account for the fact of straight-line propagation. Sound waves, water waves, and all other forms of waves with which we are most familiar bend readily around corners, while light apparently does not. It was this difficulty chiefly which led many of the most famous of the early philosophers, including the great Sir Isaac Newton, to reject the wave theory and to support the projected particle theory. Within the last hundred years, however, this difficulty has been completely removed and in addition other properties of light have been discovered for which the wave theory offers the only satisfactory explanation. The most important of these properties will be treated in the next paragraph.

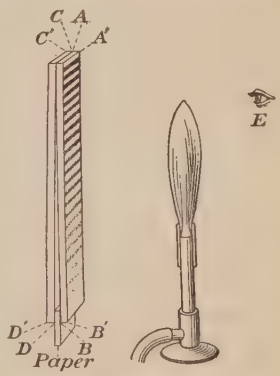


Fig. 430. Interference of light waves

518. Interference of light. Let two pieces of plate glass about half an inch wide and four or five inches long be separated at one end by a thin sheet of paper in the manner shown in Fig. 430, while the other end is clamped or held firmly together, so that a very thin wedge

of air exists between the plates. Let a piece of asbestus or blotting paper be soaked in a solution of common salt (sodium chloride) and placed over the tube of a Bunsen burner so as to touch the flame in the manner shown. The flame will be colored a bright yellow by the sodium in the salt. When the eye looks at the reflection of the flame from the glass surfaces, a series of fine black and yellow lines will be seen to cross the plate.

The wave theory offers the following explanation of these effects. Each point of the flame sends out light waves which travel to the glass plate and are in part reflected and in part transmitted at all the surfaces of the glass, i.e. at $A'B'$, at AB, at CD, and at $C'D'$ (Fig. 430). We will consider, however, only those reflections which take place at the two faces of the air wedge, viz. at AB and CD. Let Fig. 431 represent a greatly magnified section of these two surfaces. Let the wavy line as represent a light wave reflected from the surface AB at the point a, and returning from there to the eye. Let the dotted wavy line ir represent a light wave reflected from the surface CD at the point i, and returning from there to the eye. Similarly, let all the continuous wavy lines of the figure represent light waves reflected from different points on AB to the eye, and let all the dotted wavy lines represent waves reflected from corresponding points on CD to the eye. Now, in precisely the same way in which two trains of sound waves from two tuning forks were found, in the experiment

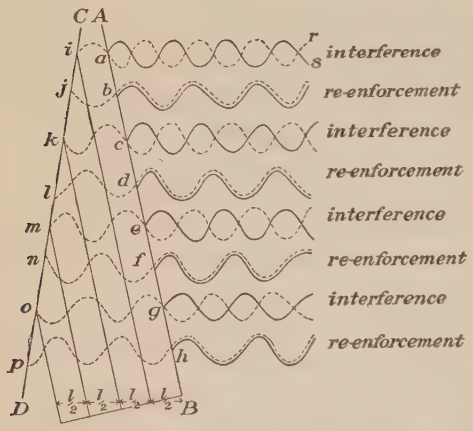


FIG. 431. Explanation of formation of dark and light bands by interference of light waves

illustrating beats (see p. 361), to interfere with each other so as to produce silence whenever the two waves corresponded to motions of the air particles in opposite directions, so in this experiment the two sets of light waves from *AB* and *CD* interfere with each other so as to produce darkness wherever these two waves correspond to motions of the light-transmitting medium in opposite directions. The dark bands, then, of our experiment are simply the places at which the two beams reflected from the two surfaces of the air film neutralize or destroy each other, while the light bands correspond to the places at which the two beams reënforce each other and thus produce illumination of double intensity.

Now the condition for destructive interference is obviously that the wave which passes through the film and is reflected from any point on *CD*, such, for example as *i*, shall return to *AB*, after its double passage through the film, in the condition, *or phase*, of vibration which is exactly opposite to that of the wave which is being reflected at that instant from the corresponding point on *AB*, viz. from *a*. If this condition occurs at *a*, it must occur again at a point *c* enough farther down the wedge to make the double path through the film just one wave length more, and again at *e* where the double path is two wave lengths more, etc. In other words, the dark bands ought to follow one another at equal intervals down the wedge, precisely as we observed them to do. Between each two successive points of interference there must, of course, be a point like *b*, *d*, *f*, or *h*, at which the waves reflected from the two surfaces unite in *like* phases and therefore reënforce each other. This phenomenon of the interference of light is met with in many different forms, and in every case the wave theory furnishes at once a wholly satisfactory explanation of the observed effects; while the corpuscular theory, on the other hand, is unable to account for any of these interference effects without the most fantastic and violent assumptions. Hence *the corpuscular theory*

is now practically abandoned, and light is universally regarded by physicists as a form of wave motion.

519. The ether. We have already indicated that if the wave theory is to be accepted, we must conceive, with Huygens, that all space is filled with a medium, called *the ether*, in which the waves can travel. This medium cannot be like any of the ordinary forms of matter; for if any of these forms existed in interplanetary space, the planets and the other heavenly bodies would certainly be retarded in their motions. As a matter of fact, in all the hundreds of years during which astronomers have been making accurate observations of the motions of heavenly bodies no such retardation has ever been observed. The medium which transmits light waves must, therefore, have a density which is infinitely small even in comparison with that of our lightest gases. The existence of such a medium is now universally assumed by physicists.

Further, in order to account for the transmission of light through transparent bodies it is necessary to assume that the ether penetrates not only all interstellar spaces but all intermolecular spaces as well.

520. Wave length of yellow light. Although light, like sound, is a form of wave motion, light waves differ from sound waves in several important respects. In the first place, an analysis of the preceding experiment, which seems to establish so conclusively the correctness of the wave theory, shows that the wave length of the light waves used in that experiment is extremely minute in comparison with that of ordinary sound waves. Thus, suppose the air wedge was 10 cm. long, and that the upper edges of the glass strips were in contact, while the lower edges were held apart by a sheet of paper .03 mm. thick. Suppose, further, that the black bands were found to be 1 mm. apart. Now, since the air wedge is 100 mm. long, the difference in its thickness at two points such as *a* and *c* (see Fig. 431), 1 mm. apart, must be .01 of its thickness at the base, i.e. $\frac{1}{100}$ of .03 mm. or .0003 mm. Since it is the *double* path through the air wedge at *c* which must be exactly one wave length longer than the double path at *a* (see § 518), we see that the difference in the thicknesses of the wedge at *c* and at *a*

must be $\frac{1}{2}$ wave length. Hence the wave length of yellow light must be $2 \times .0003 = .0006$ mm. Careful measurements by better methods give .000589 mm. as the correct value.

The *number of vibrations per second* made by the little particles which send out the light waves may be found, as in the case of sound, by dividing the velocity by the wave length. Since the velocity of light is 30,000,000,000 cm. per second and the wave length is .00006 cm., the number of vibrations per second of the particles which emit yellow light has the enormous value 500,000,000,000,000.

521. Sources of light waves. Just as sound waves are disturbances set up in the air by the vibrations of bodies of ordinary dimensions, so light waves are disturbances set up in the ether probably by the vibrations of the minute corpuscles, or *electrons*, of which the atoms of ordinary matter are supposed to be built up. Since these corpuscles are extremely minute in comparison with ordinary bodies, it is not surprising that their rates of vibration, as calculated in § 520, are enormously larger than the vibration rates of tuning forks or other bodies which send out sound waves. Just how these corpuscles are set into vibration, and in just what manner they vibrate, we cannot say as yet with certainty; but since we do know that an increase in the temperature of all bodies means an increase in the agitation of the molecules and atoms of which these bodies are composed, it is not surprising that the vibrations which communicate light waves to the ether take place, in general, in bodies which have a high temperature, and that the hotter the body becomes the more intense become the light waves which it emits. It is not improbable that, just as tuning forks are set into vibration by blows struck upon them by a mallet, so the corpuscles whose vibrations give rise to light waves are set into vibration by the blows which the atoms strike against one another in their collisions.

522. Why light travels in straight lines. We have already pointed out that the chief objection raised by Newton and his contemporaries to the wave theory of light was that light,

unlike sound, travels in straight lines, i.e. casts sharp shadows. Since Newton's day it has been shown that all waves, of whatever nature, cast sharper and sharper shadows the shorter the wave length becomes. Thus it is easy to show that sound shadows are very much sharper if the sounding body is making 20,000 vibrations per second than if it is making only 100 or 200 vibrations per second. Since, then, the property of casting sharp shadows is determined by wave length, it is to be expected that light waves, which, as we have seen, have a wave length of only a few ten thousandths of a millimeter, will cast very much sharper shadows than are cast by any sound waves. Ether waves exactly like light waves in all respects, except that their wave lengths are as long, or longer, than those of sound, have been produced artificially in recent years, and are found to bend around corners as readily as sound waves. The reason that short waves form sharp shadows, i.e. travel in straight lines, while long waves do not, is that the former interfere with and destroy one another outside of the limits of the geometrical shadow, while the latter do so to a much less degree.

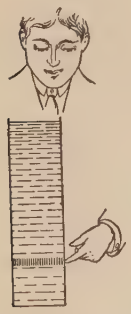


Fig. 432. Apparent elevation of the bottom of a body of water

523. Experiment showing that light travels slower in water than in air. Let one look vertically down upon a glass or tall jar full of water and place his finger on the side of the glass at the point at which the bottom appears to be, as seen through the water (Fig. 432). In every case it will be found that the point touched by the finger will be about one fourth of the depth of the water above the bottom.

According to the wave theory this effect is due to the fact that the speed of light is less in water than in air. Thus, consider a wave which originates at any point P (Fig. 433) beneath a surface of water and spreads from that point with equal speed in all directions. At the instant at which the front of this wave

first touches the surface mn it will, of course, be of spherical form, having P as its center. Let aob be a section of this sphere. An instant later, if the speed had not changed in passing into air, the wave would have still had P as its center, and its form would have coincided with the line dotted co_1d, so drawn that ac, oo_1, and bd are all equal. But if the velocity in air is *greater* than in water, then at the instant considered the disturbance will have reached some point o_2 instead of o_1, and hence the emerging wave will actually have the form of the heavy line co_2d instead of the dotted line co_1d. Now this wave co_2d is more curved than the old wave aob, and hence it has its center at some point P' above P. In other words, the wave has bulged upward in passing from water into air. Therefore, when a section of this wave enters the eye at E the wave appears to originate not at P but at P', for the light actually comes to the eye from P' as a center rather than from P. We conclude, therefore, *that if light travels slower in water than in air, all objects beneath the surface of water ought to appear nearer to the eye than they actually are.* This is precisely what we found in our experiment to be the case.

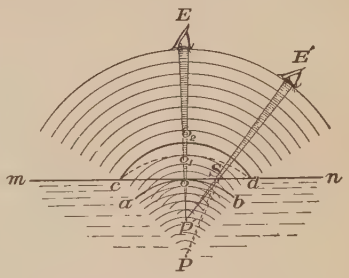


Fig. 433. Representing a wave emerging from water into air

524. Ratio of the speeds of light in air and water. The last experiment not only indicates qualitatively that the speed of light is greater in air than in water, but it furnishes a simple means of determining the precise ratio of the two speeds. Thus, in Fig. 433, the line oo_2 represents just how far the wave travels in air while it is traveling the distance ac $(= oo_1)$ in water. Hence $\frac{oo_2}{oo_1}$ is the ratio of the speeds of light in air and in water. Now it may be shown that $\frac{oo_2}{oo_1}$ is equal to $\frac{oP}{oP'}$. But in our

experiment we found that the bottom was raised one fourth of the depth, i.e. that $\frac{oP}{oP'} = \frac{4}{3}$. We conclude, therefore, that light travels three fourths as fast in water as in air.

The fact that the value of this ratio, as determined by this indirect method, is precisely the same as the value found by Foucault and Michelson, by direct measurement (§ 499), furnishes one of the strongest evidences of the correctness of the wave theory.

525. Index of refraction. The *ratio of the speed of light in air to its speed in any medium is called the index of refraction of that medium.* It is evident that the method employed in the last paragraph for determining the index of refraction of water can be easily applied to any transparent medium whether liquid or solid. If an object within such a medium appears nearer to the eye than it actually is, we know that the speed of light in the medium is less than in air, and the greater the apparent displacement of the object toward the eye the greater must be the change in velocity in passing into air, i.e. the greater the index of refraction of the medium. If the object should appear farther from the eye than it actually is, then we should know that the speed of light in the medium was greater than the speed in air. As a matter of fact, in all transparent liquids and solids there is apparent *approach* rather than *recession.* Hence we know that light travels slower in all transparent liquids and solids than it does in air.

The refractive indices of some of the commoner substances are as follows:

Water	1.33	Crown glass	1.53
Alcohol	1.36	Flint glass	1.67
Turpentine	1.47	Diamond	2.47

526. Explanation of refraction. It will be readily seen from Fig. 433 that, although the object P is never seen in its true

position, it is nevertheless seen in its true *direction* when, and only when, the eye looks normally down upon the surface from the position E. If P is viewed from any other position E', it must appear in a direction above its true direction, for the rays enter the eye from the direction $P'E$ instead of PE. Since, however, the light originates at P its actual path in coming from this point to the eye is the broken line PSE'. We see, therefore, *that in passing obliquely from one medium to another, in which the speed is greater, light rays must always be bent away from the perpendicular drawn into the second medium from the point at which the rays strike it.* This is exactly the law discovered by direct experiment in § 510. If the light had passed from a medium of greater to one of lesser speed, then the point P would evidently have appeared depressed below its natural position; and hence the oblique rays would have been bent *toward* the perpendicular drawn into the second medium, as we found in § 510 to be precisely the case.[1]

527. Light waves are transverse. Thus far we have discovered but two differences between light waves and sound waves; namely, the former are disturbances in the ether and are of very short wave length, while the latter are disturbances in ordinary matter and are of relatively large wave length. There exists, however, a further radical difference which follows from a capital discovery made by Huygens in the year 1690. It is this. While sound waves consist, as we have already seen, of *longitudinal* vibrations of the particles of the transmitting medium, i.e. vibrations back and forth in the line of propagation of the wave, light waves are like the water waves of Fig. 372, p. 351, in that they consist of *transverse* vibrations, i.e. vibrations of the medium at right angles to the direction of the line of propagation.

[1] Laboratory experiments on the ratio of speeds of light in air and water, or in air and glass, and on critical angle, should follow the above discussion. See, for example, Experiments 43 and 44 of the authors' manual.

In order to appreciate the difference between the behavior of waves of these two types under certain conditions, conceive of transverse waves in a rope to be made to pass through two gratings in succession, as in Fig. 434. So long as the slits in both gratings are parallel to the plane of vibration of the hand, as in Fig. 434, 1, the waves can pass through them with perfect ease; but if the slits in the first grating P are parallel to the direction of vibration, while those of the second grating Q are turned at right angles to this direction, as in Fig. 434, 2, it is evident that the waves will pass readily through P but will be stopped completely by Q, as shown in the figure. In other words, these gratings P and Q will let through only such vibrations as are parallel to the direction of their slits.

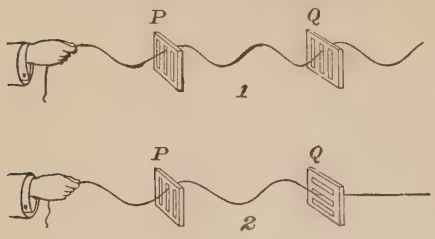


Fig. 434. Transverse waves passing through slits

If, on the other hand, a longitudinal instead of a transverse wave — such, for example, as a sound wave — had approached such a grating, it would have been as much transmitted in one position of the grating as in another, since a *to-and-fro* motion of the particles can evidently pass through the slits with exactly the same ease, whatever be the direction in which the slits are turned.

Fig. 435. Tourmaline tongs

Now two crystals of tourmaline are found to behave with respect to light waves precisely as the two gratings behave with respect to the waves on the rope.

Let one such crystal a (Fig. 436) be held in front of a small hole in a screen through which a beam of sunlight is passing to a neighboring wall; or, if the sun is not shining, simply let the crystal be held between the eye and a source of light. The light will be readily transmitted,

although somewhat diminished in intensity. Then let a second crystal b be held in line with the first. The light will still be transmitted, *provided the axes of the crystals are parallel*, as shown in Fig. 436. When, however, one of the crystals is rotated in its ring through 90° (Fig. 437), the light *is cut off*. This shows that a crystal of tourmaline is capable of transmitting only light which is vibrating in one particular plane.

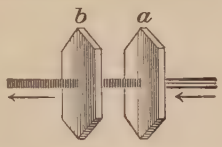


FIG. 436. Light passing through tourmaline crystals

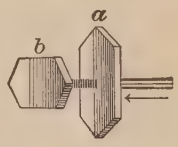


FIG. 437. Light cut off by crossed tourmaline crystals

From this experiment, therefore, we are forced to conclude that *light waves are transverse rather than longitudinal vibrations.*

The above experiment illustrates what is technically known as the *polarization of light*, and the beam which, after passage through a, is unable to pass through b if the axes of a and b are crossed, is known as a *polarized beam.* It is, then, the phenomenon of the *polarization of light* upon which we base the conclusion that light waves are transverse.

QUESTIONS AND PROBLEMS

1. What is the speed of light in water? (Index of refraction is 1.33.)

2. Will a beam of light going from water into flint glass be bent toward or away from the perpendicular drawn into the glass?

3. If the wedge-shaped film of air in Fig. 430 were replaced by water, would the distance between successive fringes be greater or less than in air? Why?

4. When light passes obliquely from air into carbon bisulphide it is bent more than when it passes from air into water at the same angle. Is the speed of light in carbon bisulphide greater or less than in water?

5. Does a man above the surface of water appear to a fish below it farther from or nearer to the surface than he actually is?

CHAPTER XX

FORMATION OF IMAGES

Images in Plane Mirrors

528. Image of a point in a plane mirror. When a light-emitting point appears to the eye to be in any position in space other than that at which it actually is, this second point is called the *image* of the first point.

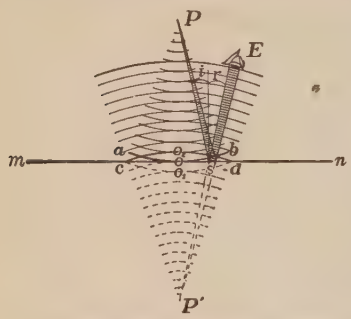


Fig. 438. Wave reflected from a plane surface

When a pencil point is held in contact with a reflecting surface the image of the point is seen in actual contact with the point itself. If now the point be drawn farther and farther away from the surface, the image will be seen to recede farther and farther behind the surface.

To find what must be the exact location of the image with respect to the point and the mirror, consider a light wave which originates in a point P (Fig. 438) and spreads in all directions. Let aob be a section of the wave at the instant at which it reaches the reflecting surface mn. An instant later, if there were no reflecting surface, the wave would have reached the position of the dotted line co_1d. Since, however, reflection took place at mn, and since the reflected wave is propagated backward with exactly the same velocity with which the original wave would have been propagated forward, at the proper instant, the reflected wave must have reached the position of the line

co_2d, so drawn that oo_1 is equal to oo_2. Now the wave co_2d has its center at some point P', and it will be seen that P' must lie just as far below mn as P lies above it, for co_1d and co_2d are arcs of equal circles having the common chord cd. For the same reason, also, P' must lie on the perpendicular drawn from P through mn. When, then, a section of this reflected wave co_2d enters the eye at E, the wave appears to have originated at P' and not at P, for the light actually comes to the eye from P' as a center rather than from P. Hence P' is the image of P. We learn, therefore, that *the image of a point in a plane mirror lies on the perpendicular drawn from the point to the mirror, and is as far back of the mirror as the point is in front of it.*

529. Why angle of reflection is equal to angle of incidence. Since the light rays which enter the eye at E come from the direction $P'E$, while they actually originate at P, they must travel over the path PsE. Since oP has just been proved equal to oP', it may be seen at once from Fig. 438 that the angle of incidence i must be equal to the angle of reflection r; for $i = sPo$, and $r = sP'o$, and $sPo = sP'o$. This is precisely the law discovered experimentally in § 506, p. 395.

530. Construction of image of object in a plane mirror. The image of an extended *object* in a plane mirror may be completely located by applying the law proved above for each of its points, i.e. *by drawing from each point a perpendicular to the reflecting surface, and extending it an equal distance on the other side.* In the case of a straight, thin object, like the arrow AB (Fig. 439), it is only necessary to locate in this way the positions of the images A' and B' of the two ends, since the images of intermediate points must fall on the line connecting A' and B'.

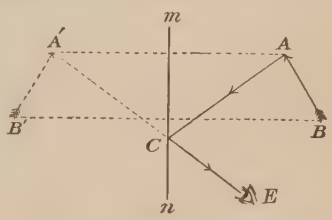


FIG. 439. Construction of image of object in plane mirror

The figure shows why a plane mirror never reveals our features or other objects to us exactly as they are; for left always appears as right if the mirror is vertical, and up as down if the mirror is horizontal. An image of this sort is called a *reversed* image. How much it differs from a correct image will be realized when one attempts to read a printed page in a mirror.

To find the path of the rays which come to an eye placed at E from any point such as A of the object, we have only to draw a line from the image A' of this point to the eye and connect the point of intersection of this line with the mirror, namely C, with the original point A. ACE is then the path of the ray.

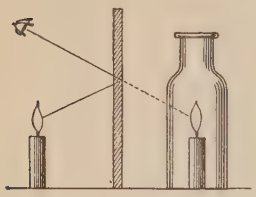


Fig. 440. Position of image in a plane mirror

531. Experimental illustration of law of position of image in plane mirror. Let a candle (Fig. 440) be placed exactly as far in front of a pane of window glass as a bottle full of water is behind it, both objects being on a perpendicular drawn through the glass. The candle will appear to be burning inside the water. This not only furnishes an experimental proof of the law in question, but it explains a large class of familiar optical illusions, such as "the figure suspended in mid-air," the "bust of a person without a trunk," the "stage ghost," etc. In the last case the illusion is produced by causing the audience to look at the actors obliquely through a sheet of very clear plate glass, the edges of which are concealed by draperies. Images of strongly illuminated figures at one side then appear to the audience to be in the midst of the actors.

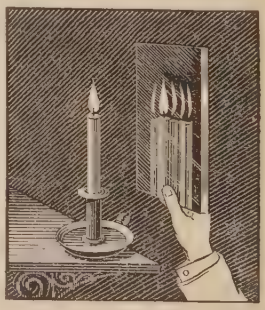


Fig. 441. Multiple image in ordinary mirror

532. Multiple images from an ordinary mirror. Let the flame of a candle be observed very obliquely in an ordinary mirror. From four to ten images of the flame may be seen arranged in a row in the manner shown in Fig. 441. The second image, however, will be by far the most brilliant.

Fig. 442 shows the several reflections which a ray of light undergoes at the unsilvered front face and the silvered back face of such a mirror, hence the succession of images. It is because the front face reflects only 8% or 10% of the light which falls upon it, while the back face reflects nearly all the light which falls on it, that the second image is so much brighter than any of the others.

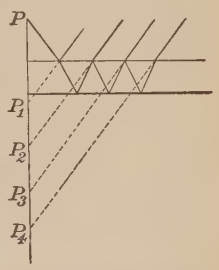


FIG. 442. Reflections in an ordinary mirror

QUESTIONS AND PROBLEMS

1. A man is standing squarely in front of a plane mirror which is very much taller than himself. The mirror is tipped toward him until it makes an angle of 45° with the horizontal. He still sees his full length. What position does his image occupy?

2. The angle between an incident and a reflected ray on a plane mirror is 60°. What is the angle between the incident ray and the mirror?

3. A man runs toward a plane mirror at the rate of 12 ft. per second. How fast does he approach his image?

4. Show from a construction of the image that a man cannot see his entire length in a vertical mirror unless the mirror is half as tall as he is. Decide from a study of the figure whether or not the distance of the man from the mirror affects the case.

IMAGES IN CONVEX MIRRORS

533. Image of a point in a convex mirror. Let a small object like a lighted candle be held close to a convex mirror. The image will be seen close behind the reflecting surface, as in the case of a plane mirror. Then let the candle be gradually removed to greater and greater distances. The image, instead of moving back as fast as the candle moves forward, as it did in the plane mirror, will be seen to recede a short distance only and then remain practically stationary, no matter how far the candle is carried away.

The reason for this behavior will be made clear by a consideration of the change which the mirror makes in the curvature of the waves which strike it. (The explanation of the continual

diminution in the *size* of the image will be reserved to § 537.) Thus, suppose the point P, at which the light originates, has been removed so far that when the light wave first reaches the mirror *cod* (Fig. 443) it is practically a plane surface of which *aob* is a section. Theoretically, of course, this could only happen when P was at an infinite distance, for the wave front at o is always a circle having P as a center. Practically, however, *aob* would be a straight line if P were a few rods away. If there were no reflecting surface, an instant after the wave reached the position *aob* it would have passed on to the position co_1d. Because, however, of the fact that the reflection has caused the center of the wave to travel back with the same speed with which the sides are traveling forward, the actual position of the wave at the instant considered will be co_2d instead of co_1d, where oo_1 is equal to oo_2. Therefore to an eye placed anywhere to the left of the mirror the source of the wave appears to be the center of the circle co_2d, i.e. the point F; for the light actually comes to the eye from this point as a center.

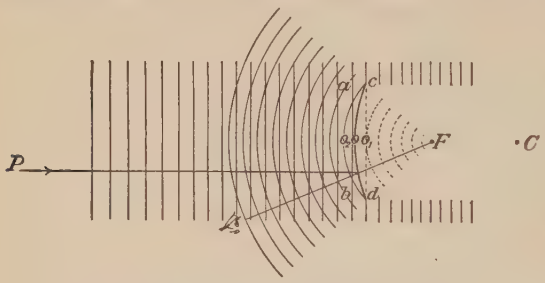


Fig. 443. Reflection of a plane wave from a convex mirror

Hence we learn that as the candle was moved forward from the point of contact with the surface of the mirror, the image could move backward from contact with the surface only as far as the point F. It can never go farther back than F, since, theoretically, it is only when the source P has receded to an infinite distance that the wave *aob* becomes plane.

534. Focal length of a convex mirror. The distance from the mirror to the point at which the image of an infinitely

distant object is formed, i.e. the distance oF, is called the *focal length* of the mirror. We shall denote this distance by f.

To determine f experimentally, let a beam of sunlight pass through a circular hole np in a sheet of cardboard tu and be received upon a convex mirror as shown in Fig. 444. By shaking a little chalk dust into the space in front of the mirror the path of the reflected beam as it diverges from the mirror will be made easily visible. Let the mirror be moved back and forth in front of the hole until the outer edge rq of the reflected light upon the screen is a circle of exactly twice the diameter of the hole. The distance from the screen to the mirror is then the focal length, since by similar triangles, mn being equal to nr, mo must equal oF.

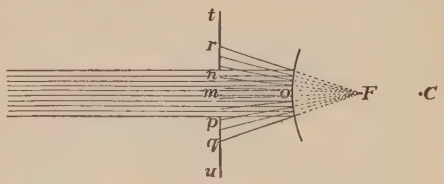


FIG. 444. Determination of focal length of a convex mirror

535. Focal length of a convex mirror equal to one half its radius. The curvature of a mirror cod (Fig. 443) is rigorously defined as the reciprocal of its radius, i.e. as $\frac{1}{oC}$; but, so long as the arc cod is small, the curvature may without appreciable error be considered as measured by the distance o_1o, i.e. by the amount of departure of the curved line cod from the straight line co_1d. Since $o_1o = oo_2$, we have $o_1o_2 = 2o_1o$; i.e. the curvature of the reflected wave co_2d is equal to twice the curvature of the mirror. We learn, therefore, since the incident wave aob had zero curvature, that *a convex mirror impresses upon an incident wave twice its own curvature.* This means that the center of the reflected plane wave is one half as far from the mirror as is the center of the mirror C, i.e. $oF = \frac{1}{2}\, oC$. In other words, *the focal length of a convex mirror is equal to one half the radius of the sphere of which the mirror is a part.*

536. Construction of image of object in convex mirror. To locate geometrically the image P' of a point P in any position

whatever in front of the mirror mn (Fig. 445), we proceed precisely as in § 533. Thus we draw from P as a center an arc co_1d, which represents the position a wave from P would occupy at the instant we choose to consider, if there had been no mirror. We then draw the actual form of the reflected wave co_2d by taking oo_2 equal to o_1o. The center of the circle of which co_2d is an arc is then P', the image of P.

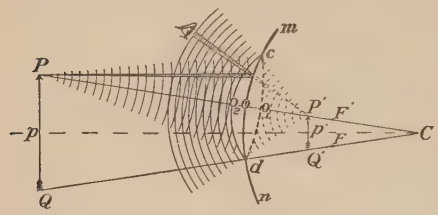


FIG. 445. Construction of image in a convex mirror

Since cd is a chord common to all the arcs cod, co_1d, and co_2d, it will be seen that, precisely as in the case of a plane mirror, the image of any point P must lie upon the perpendicular drawn from P to the mirror, i.e. upon a line drawn from P through C, the center of the mirror. Hence, to locate the complete image of a straight object PQ it is sufficient to locate by the above method the image P' of one extremity P, and then to draw $P'Q'$ between the lines CP and CQ.

537. Size of image. It is evident at once from Fig. 445 why a convex mirror always forms an erect, diminished image; for the perpendiculars PC and QC, upon which the images of the extremities of the object must lie, are now *converging* lines instead of *parallel* lines, as in the case of the plane mirror (§ 530).

The figure also shows what ratio exists between the size of the image and the size of the object. For since the triangles PCQ and $P'CQ'$ are similar, $\frac{PQ}{P'Q'} = \frac{PC}{P'C}$; i.e. *the ratio of the size of image and object is the ratio of their respective distances from the center of curvature C of the mirror.*[1]

[1] It may be shown, by analysis which will not be introduced here, that *the ratio of the size of image and object is also the ratio of their respective distances from the surface, as well as from the center of curvature, of the mirror.*

Images in Concave Mirrors

538. Image of object nearer to a concave mirror than its principal focus. Let a pencil point be placed in contact with a concave mirror. Its image will be seen in contact with the point, and of practically the same size as the point itself, just as was the case with both the plane and the convex mirror. Let the point then be slowly withdrawn. The image will recede faster than the point, and at the same time will become magnified. When the point has receded to about half the distance between the mirror and its center, the image will appear very large, and will lie very far behind the mirror. If the point is still farther withdrawn the image will disappear completely.

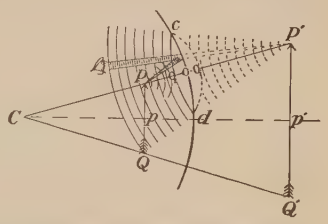


Fig. 446. Virtual image in a concave mirror

The cause of all these effects will be perfectly clear from a consideration of Fig. 446, in which the image $P'Q'$ is constructed precisely as in Fig. 445, oo_2 being made equal to o_1o. *Since a convex surface adds twice its own curvature to that of the incident wave, a concave surface must subtract the same amount,* as indeed the figure shows that it does. Hence the reflected wave co_2d has a *smaller* curvature, i.e. a larger radius than the incident wave, so that P' is farther behind the mirror than P is in front of it.

Further, since the images of the two extremities of the object lie upon the perpendiculars to the mirror, they are upon the diverging lines CP' and CQ'; hence the image is necessarily larger than the object. As P retreated from the surface the curvature of the incident wave became less and less, and hence a position was soon reached in which the incident wave had all of its curvature removed by the reflection; in other words, the reflected wave co_2d was plane. The image $P'Q'$ then appeared very distant, since only distant objects send plane waves to the eye.

The distance from P to the mirror when the reflected wave is plane is the focal length. For the same reason presented in the case of convex mirrors it is one half the radius of the mirror.

The reason that the image disappeared on further withdrawal of P is that the eye can accommodate itself only to waves which are either convex toward it or else plane. When P was withdrawn farther than the focal distance the waves became concave toward the eye (see § 541).

539. Virtual images. All of the images thus far treated are called *virtual images* because they are formed behind the mirror at points where no light disturbance actually exists. In other words, light only *appears* to emanate from them as centers. Such images can obviously never be cast upon a screen.

540. Formation of real images in concave mirrors. Let a lighted candle or an electric-light bulb be held just outside of the principal focus of a concave mirror. Then let a white screen be moved back and forth at a considerable distance from the mirror. A position will be found at which a sharply defined, inverted, and magnified image of the flame will be seen upon the screen (Fig. 447). If the screen is removed and the eye placed, as shown in Fig. 448, behind the position which was occupied by the screen, an enlarged and inverted image of the candle will be seen suspended in mid-air at P'.

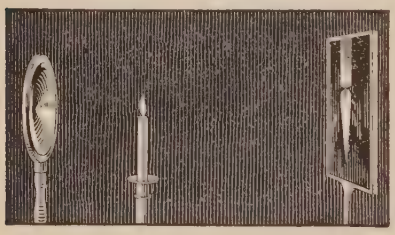


Fig. 447. Real image of candle formed by a concave mirror

This image differs from all which have so far been studied in that it can be thrown upon a screen, i.e. in that light actually emanates from P' as a center. For this reason it is called a *real image.*

541. Explanation of the formation of real images. The method of formation of a real image may be seen readily from Fig. 448, which is constructed precisely like Figs. 445 and 446.

So long as P was inside the focal distance the curvature of the incident wave was greater than the curvature subtracted by the mirror; but as soon as P was moved farther out than the focal distance, the initial curvature of the wave was more than neutralized by the curvature impressed by the mirror, and hence after reflection the wave was concave toward C, as the figure shows. This means, of course, that the reflected waves now actually converge to a center at P', and then spread out again in the manner shown. P and P' are called *conjugate points*

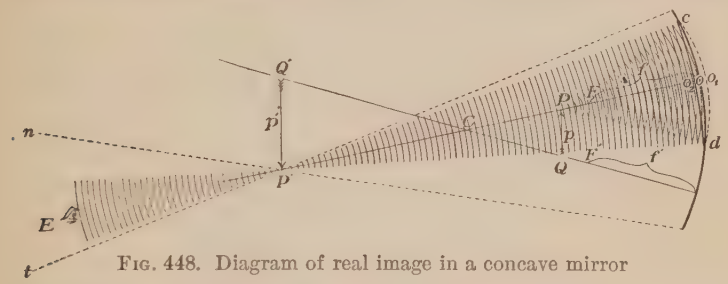


FIG. 448. Diagram of real image in a concave mirror

because an object at P has its image at P', and an object at P' has its image at P.

Without the aid of a screen the image P' can be seen only if the eye is placed within the cone $nP't$, since the total beam which comes from the mirror is bounded by this cone. When, however, a screen is placed at P, the image becomes visible from all directions because of the diffusing power of the screen.

542. Relative size of object and image. It will be seen from the construction that the image of each point of the object must be formed, as in the case of all mirrors, on the perpendicular drawn from that point to the mirror. Hence the complete image of any object PQ must be included between the perpendiculars drawn through C and each of the points P and Q. To locate completely the image, therefore, we locate one of the points, for example P', and then draw $P'Q'$ between the perpendiculars to the mirror through PC and QC. The construction shows that a real

image is necessarily inverted, and it shows also that *the ratio of the size of the object and image is equal to the ratio of their respective distances from C.*

543. Determination of focal length of concave mirror. Since a wave emitted by a point whose distance from the mirror is equal to the focal length is reflected as a plane wave, it is evident that an incident plane wave must be brought to a focus at a distance from the mirror equal to the focal length. In other words, a real image of a distant object PQ will be formed at the focal distance in front of a mirror.

Hence, to find the focal length of a concave mirror experimentally, let the edge of a sheet of cardboard b (see Fig. 449) be moved back

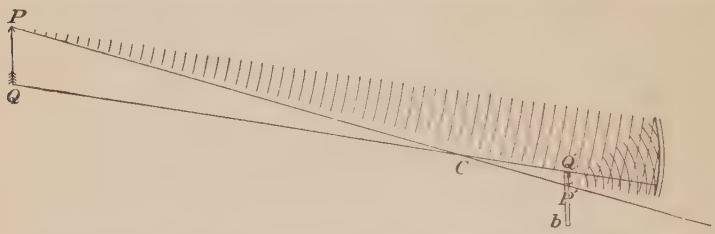


Fig. 449. Method of determining focal length of a concave mirror

and forth in front of one half of a mirror until the image $P'Q'$ of a distant house or tree PQ is in sharp focus on the screen. The distance from the image to the mirror is then the focal length.

Let the image of the sun be formed in the same way on the screen. Its distance will be found to be the same. If the sunlight enters through a round hole into a darkened room, the screen is unnecessary, for a complete outline of the beam, as it converges to a focus at $P'Q'$ and then diverges again, will be visible, provided the air in front of the mirror is filled with chalk dust.

544. Determination of center of curvature of mirror. When the object is at a distance from the mirror equal to the radius of the latter, the incident wave evidently has the same curvature as the mirror. Hence, since the mirror subtracts twice its own curvature, the reflected wave will be returned with a curvature

equal and opposite to that of the incident wave. In other words, the image P' will be formed at a distance from the mirror equal to its radius R.

Hence, to find R let a lighted candle be moved back and forth in front of a mirror until its inverted image is in sharp focus upon a card placed immediately beside the candle. The distance from the card or candle to the mirror is then the radius of the mirror. A measurement will show that this distance is twice the focal length, as the theory requires.

A very pretty modification of this experiment is the following. Let a rose R be pinned upside down inside a box, the interior of which is strongly illumined by some source of light L (Fig. 450). Let a glass of water W be placed on top of the box. Let a concave mirror be placed at C, at a distance equal to its radius from the rose. When the eye is placed at E the rose will appear to be upright in the water above the box.

FIG. 450. Image of object at center of curvature

545. Series of images possible with a concave mirror. As a candle is moved steadily back from the face of a concave mirror the following series of images is observed.

1. So long as the candle is nearer to the mirror than its principal focus only virtual, erect, and magnified images are formed.

2. When the candle is at the focal distance the reflected waves are plane, and hence, in the strict sense, no image is formed by the mirror. Since, however, the eye can bring plane waves to a focus, if it receives these plane waves it will see an erect magnified image behind the mirror.

3. As soon as the candle moves farther from the mirror than the focal distance a real, magnified, inverted image is formed at a great distance in front of the mirror. As the candle moves from F toward C (Fig. 448) this image approaches C, at the same time diminishing in size. When P reaches C the inverted image is

at the same distance from the mirror as the object and is also of the same size.

4. As the candle moves from C to an infinite distance the image moves from C to F, always becoming smaller and smaller.

Images formed by Lenses

546. Focal length of a convex lens. Let a convex lens be held in the path of a beam of sunlight which enters a darkened room through a circular aperture. It will be seen to be brought to a focus and then to diverge in a cone-shaped bundle, just as when reflected from a concave mirror.

The explanation of this is easily seen from a consideration of the effect of the lens on the curvature of the waves which fall upon it. Since the sun is so distant the waves from it which fall on the lens are practically plane; and since the speed of light in glass is less than its speed in air, these plane waves must become curved, in the manner shown in Fig. 451, as they pass through the lens, so that the waves which were plane on entering are convergent on leaving. Hence the emergent waves converge to a real focus at F, from which they again spread out as shown in the figure. *The distance from the center of the lens to the point at which incident plane waves are brought to a focus is called the focal length of the lens.*

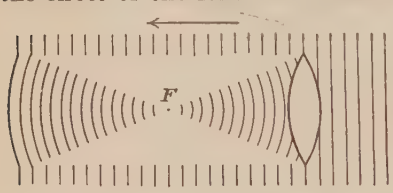


Fig. 451. Focus of a convex lens

547. Series of possible images produced by a lens. Since, then, a convex lens has the same effect upon light waves which pass through it as has a concave mirror on waves which are reflected from it, we may expect to find convex lenses producing the same series of images as were found to be produced by concave mirrors. This inference may be tested as follows.

1. Let a lens be moved back and forth near a screen which is placed opposite a distant window or landscape. A real inverted image of the object will fall upon the screen at the focus precisely as we found to be the case with the concave mirror.

2. Let a candle be placed just outside of F (Figs. 452 and 453). A real, enlarged, inverted image will be thrown upon a distant screen (Fig. 452).

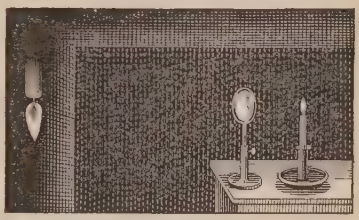


Fig. 452. Image of object placed just outside the focus of a convex lens

3. Let the candle be moved farther away from the lens. The image will draw nearer and decrease in size.

4. When the candle has reached a distance from the lens equal to twice its focal length, the image will be formed at exactly the same distance from the lens on one side as the object is on the other, and the two will have exactly the same size. This is completely analogous to the condition which exists when a candle is at the center of a concave mirror, i.e. at a distance equal to twice the focal length (§ 544). These two points S (Fig. 453) whose distances from the lens on either side are

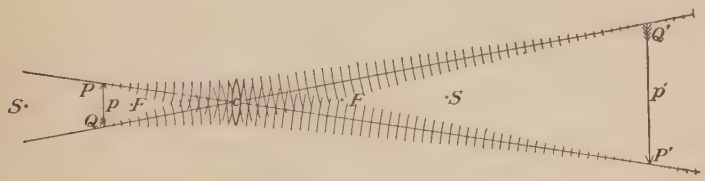


Fig. 453. Method of formation of a real image by a lens

twice its focal length, are called *secondary foci*, to distinguish them from any other pair of conjugate foci.

5. Let the candle be moved from S out to a great distance to the left of the lens (Fig. 453). The image will move from S on the right side up to F, and will become smaller and smaller in so doing.

6. Let the candle be moved nearer to the lens than its focal length. No real image will be formed. But if the eye is placed on the side of the lens opposite the candle, an enlarged virtual image will be seen.

We learn, then, that *a convex lens forms a series of images which is identical with the series formed by a concave mirror.*

548. Relative size of object and image. Fig. 453 shows how the lens changes the curvature of the waves coming from the extremities P and Q of an object, so as to cause real images of these points to be formed at P' and Q'. That P' and Q' must lie on lines drawn from P and Q respectively through the center c of the lens is evident from the fact that any ray which passes through c enters and leaves the lens through surfaces which are parallel to each other. Hence its direction after emerging from the lens must be the same as it was upon entering (§ 515). It follows, then, from the similar triangles PcQ and $P'cQ'$ that $\frac{PQ}{P'Q'} = \frac{pc}{p'c}$; i.e. *the ratio of the size of object and image is the ratio of their respective distances from the center of the lens.*

549. Virtual images by a convex lens. Since a convex lens brings plane waves together at its focus, it is evident that waves emerging from an object at the focus will be plane after passing through the lens; i.e. in this case the curvature which the lens subtracts from the incident wave is the whole curvature which it possesses. When, however, the object is placed closer to the lens than its principal focus, the waves which it sends to the lens have a greater curvature than that which the lens is able to subtract. Hence, after emergence, the waves are still concave toward the lens, but their center (P', Fig. 454) is farther away than the object (P). The image is therefore virtual; hence it can be seen only by placing the eye at some point such as E (Fig. 454). For the same reason discussed in the last paragraph the images P' and Q' of the extremities P and Q must lie in the prolongation of the lines PC and QC. The figure shows the way in which the image of any point P is formed. It also shows why the virtual image in a convex lens, as in a concave mirror, is always erect and magnified.

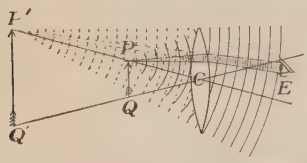


Fig. 454. Virtual image formed by a convex lens

550. Images formed by concave lenses. When a plane wave strikes a concave lens it must emerge as a divergent wave, since the middle of the wave is retarded by the glass less than the edges (Fig. 455). The point F, from which plane waves appear to come after passing through such a lens, is called the principal focus of the lens. Such lenses, like convex mirrors, can give rise only to erect, virtual images smaller than the object (Fig. 456).

Fig. 455. Virtual focus of a concave lens

For the same reason as in the case of convex lenses, the centers of the transmitted waves from P and Q, i.e. the images P' and Q', must lie upon the lines PC and QC, and since the curvature is increased by the lens they must lie closer to the lens than P and Q. Fig. 456 shows the way in which such a lens forms an image.

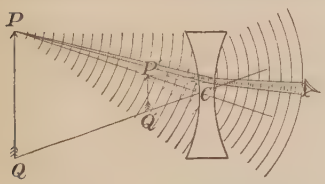


Fig. 456. Image in a concave lens

551. Formulæ for mirrors and lenses. The curvature which a mirror or lens impresses upon a plane wave is, by definition, the reciprocal of the radius of curvature of the wave front as it leaves the mirror or lens (cf. § 535); i.e. it is $1/f$, f being the focal length (Figs. 443 and 451). If the incident wave is not plane, i.e. if it comes from an object at a finite distance u, then by definition the curvature of the waves incident upon the mirror is $1/u$. If v represents the distance from the mirror at which the image is formed, then the curvature of the wave as it leaves the mirror or lens is $1/v$. Now in the case of *convex mirrors* and *concave lenses*, the final curvature $1/v$ is simply equal to the *sum* of the initial curvature $1/u$ and the curvature $1/f$ impressed by the mirror or lens; i.e.

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}, \quad \text{or} \quad \frac{1}{v}-\frac{1}{u}=\frac{1}{f}. \tag{1}$$

In the case of concave mirrors and convex lenses (Figs. 448 and 453) the final curvature is the difference between the initial curvature and the impressed curvature, i.e.

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}, \quad \text{or} \quad \frac{1}{u}+\frac{1}{v}=\frac{1}{f}. \tag{2}$$

Thus, for example, suppose that an object is 2 m. from a concave mirror or a convex lens, and its image is found to be 50 cm. from the mirror or lens. To obtain the focal length we substitute in (2), thus: $1/200 + 1/50 = 1/f$. Therefore $f = 40$ cm.[1]

QUESTIONS AND PROBLEMS

1. How tall is a tree 500 ft. away, if the image of it formed by a lens of focal length 4 in. is 1 in. long?

2. How long an image of the same tree will be formed at the focus of a lens having a focal length of 9 in.?

3. Why does the image of an object become smaller and smaller the farther the latter is removed from a convex mirror (Fig. 445)?

4. Why does the nose appear relatively large in comparison with the ears when the face is viewed in a convex mirror? (See answer to Problem 3.)

5. Can a convex mirror ever form an inverted image? Give reason for your answer.

6. When does a convex lens form a real, and when a virtual image? When an enlarged, and when a diminished image? When an erect, and when an inverted one?

7. Describe the image formed by a concave lens. Why can it never be larger than the object?

8. What is the difference between a real and a virtual image?

9. A candle placed 20 cm. in front of a concave mirror has its image formed 50 cm. in front of the mirror. Find the radius of the mirror.

10. Find the relative sizes of image and object in Problem 9.

11. An object is 15 cm. in front of a convex lens of 12 cm. focal length. What will be the nature of the image, its size, and its distance from the lens?

12. By making u and v equal in formula (2), show that the secondary foci are twice as far from the lens as the principal focus.

13. What is the focal length of a lens if the image of an object 10 ft. away is 3 ft. from the lens?

14. If the object in Problem 13 is 6 in. long, how long will the image be?

15. An object is 150 cm. in front of a convex mirror of 40 cm. focal length. Where is the image and what is its size?

16. An object is 30 cm. in front of a concave mirror of 40 cm. focal length. Find the distance of the image from the mirror and the size of the image.

[1] Laboratory experiments on the formation of images by concave mirrors and by lenses should follow this discussion. See, for example, Experiments 45 and 46 of the authors' manual.

Optical Instruments

552. The pin-hole camera. Let a large-sized pin hole be made in a card held a few feet from a gas flame. A very fair inverted image of the flame may be obtained on a screen held a few inches behind the hole. The experiment may be performed on a larger scale in either of the following ways. On a sunny day let a hole about two inches in diameter be made in the curtain of a darkened room. A fairly distinct picture of opposite houses and trees may be seen on the wall opposite the hole. As the hole is made smaller and smaller the image becomes more and more distinct in outline, but less and less bright. If the size of the hole is increased, after it reaches certain dimensions all semblance of an image will vanish and only a brightly illuminated screen will be seen.

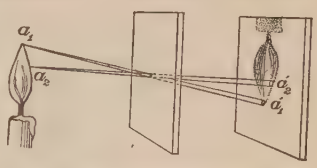


Fig. 457. Image formed by a small opening

The above experiments illustrate how fairly distinct pictures may be taken with a pin-hole camera. So long as the hole is very small a narrow pencil of light passes from each point of the object in a straight line through the hole and illuminates but a very small area of the screen on which it falls (Fig. 457); hence the outline of the illuminated portion of the screen must be a distinct duplicate of the outline of the source. As the size of the opening is increased, however, the narrow pencil from each point becomes a *cone* of considerable size, and the bases a'_1 and a'_2 of these cones from adjacent points such as a_1 and a_2 overlap and thus destroy the distinctness of the outline. If, on the other hand, the hole is too small, not enough light gets through it to illuminate the screen perceptibly.

553. The photographic camera. It is possible to gain the increased brightness due to the larger hole, without sacrificing distinctness of outline, by placing a lens in the hole (Fig. 458); for then, if the screen is properly placed, the whole cone of rays which comes to the lens from any particular point is brought

together at a single point on the screen. But while, with the pin-hole camera, the screen may be placed at any distance from the hole, it is clear that when a lens is used it must be in the position which is the conjugate focus of the position occupied by the object.

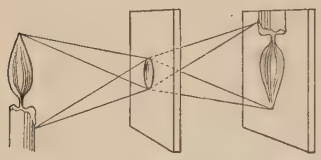


Fig. 458. Principle of the photographic camera

In order to test these conclusions let a lens be placed in front of a hole in a darkened room and a screen properly placed behind it. A perfect reproduction of the landscape will be seen. If the image is to be visible to a large number of persons, the lens should have a focal length of three feet or more. If in the above arrangement we should replace the screen by a sensitive plate, we should have the essential elements of a photographic camera (Fig. 459).

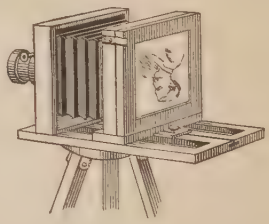


Fig. 459. The photographic camera

554. The projecting lantern. The projecting lantern is essentially a camera in which the position of object and image have been interchanged; for in the use of the camera the object is at a considerable distance and a small inverted image is formed on a plate placed somewhat farther from the lens than the focal distance. In the use of the projecting lantern the object P (Fig. 460) is placed a trifle farther from the lens L' than its focal length, and an enlarged inverted image is formed on a distant screen S. In both instruments the optical part is simply a convex lens or a combination of lenses which is equivalent to a convex lens.

The object P, whose image is formed on the screen, is usually a transparent slide which is illuminated by a powerful light A. The image is as many times larger than the object as the distance from L' to S is greater than the distance from L' to P. The light A is usually either a calcium light or an electric arc.

The above are the only essential parts of a projecting lantern. In order, however, that the slide may be illuminated as brilliantly as possible, a so-called condensing lens L is always used. The object of L is to concentrate as much light as possible upon the transparency.

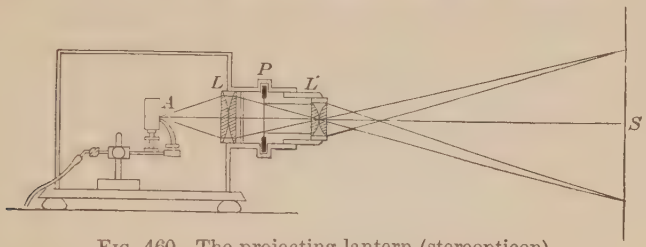


FIG. 460. The projecting lantern (stereopticon)

In order to illustrate the principle of the instrument let a beam of sunlight be reflected into the room and fall upon a lantern slide. When a lens is placed a trifle more than its focal distance in front of the slide a brilliant picture will be formed on the opposite wall.

555. The eye. The eye is essentially a camera in which the cornea C (Fig. 461), the aqueous humor l, and the crystalline lens o act as one single lens which forms an inverted image $P'Q'$ on the retina, an expansion of the optic nerve covering the inside of the back of the eyeball.

In the case of the camera the images of objects at different distances are obtained by placing a plate nearer to or farther from the lens. In the eye, however, the distance from the retina to the lens remains constant, and the adjustment for different distances is effected by changing the focal length of the lens itself in such a way as always to keep the image upon the retina. Thus, when the normal eye is perfectly relaxed, the lens has just the proper curvature to focus plane waves upon the retina, i.e. to make distant objects distinctly visible. But by directing attention upon near objects

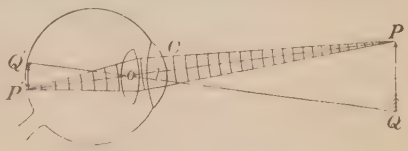


FIG. 461. The human eye

we cause the muscles which hold the lens in place to contract in such a way as to make the lens more convex, and thus bring into distinct focus objects which may be as close as eight or ten inches. This power of adjustment, however, varies greatly in different individuals.

556. The apparent size of a body. The apparent size of a body depends simply upon the size of the image formed upon the retina by the lens of the eye, and hence upon the size of *the visual angle* pCq (Fig. 462). The size of this angle evidently increases as the object is brought nearer to the eye (see PCQ). Thus the image formed on the retina when a man is 100 ft. from the eye is in reality only one tenth as large as the image formed of the same man when he is but 10 ft. away. We do not actually interpret the larger image as representing a larger man simply because we have been taught by lifelong experience to take account of the known distance of an object in forming our estimate of its actual size. To an infant who has not yet formed ideas of distance the man 10 ft. away doubtless appears ten times as large as the man 100 ft. away.

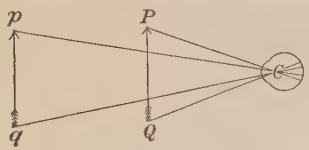


FIG. 462. The visual angle

557. Distance of most distinct vision. When we wish to examine an object minutely we bring it as close to the eye as possible in order to increase the size of the image on the retina. But there is a limit to the size of the image which can be produced in this way. For we have already seen that when the object is brought nearer to the normal eye than about ten inches, the curvature of the incident wave becomes so great that the eye lens is no longer able, without too much strain, to thicken sufficiently to bring the image into sharp focus upon the retina. Hence a person with normal eyes holds an object which he wishes to see as distinctly as possible at a distance of about 10 inches.

Although this so-called *distance of most distinct vision* varies somewhat with different people, for the sake of having a standard of comparison in the determination of the magnifying powers of optical instruments, some exact distance had to be chosen. The distance so chosen is 10 in., or 25 cm.

558. Magnifying power of a convex lens. If a convex lens is placed immediately before the eye, the object may be brought much closer than 25 cm. without loss of distinctness, for the curvature of the wave is partly, or even wholly, overcome by the lens before the light enters the eye.

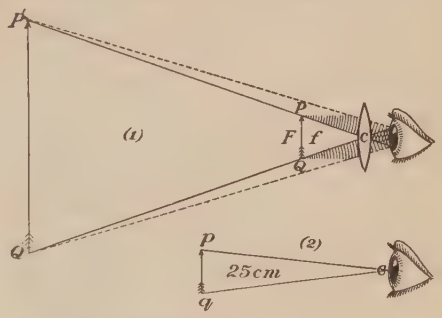


Fig. 463. Magnifying power of a lens

If we wish to use a lens as a magnifying glass to the best advantage, we place the eye as close to it as we can, so as to gather as large a cone of rays as possible, and then place the object at a distance from the lens equal to its focal length, so that the waves after passing through it are plane. They are then focused by the eye with the least possible effort. The visual angle in such a case is PcQ [Fig. 463, (1)], for, since the emergent waves are plane, the rays which pass through the center of the eye from P and Q are parallel to the lines through Pc and Qc. But if the lens were not present and if the object were 25 cm. from the eye, the visual angle would be pcq [Fig. 463, (2)]. The ratio of these two angles is approximately $25/f$, where f is the focal length of the lens expressed in centimeters. Now *the magnifying power of a lens or microscope is defined as the ratio of the angle actually subtended by the image when viewed through the instrument, to the angle subtended by the object when viewed with the unaided eye at a distance of 25 cm.* Therefore the

magnifying power of a simple lens is $25/f$. Thus if a lens has a focal length of 2.5 cm., it produces a magnification of 10 diameters when the object is placed at its principal focus. If the lens has a focal length of 1 cm., its magnifying power is 25, etc.

559. Magnifying power of an astronomical telescope. In the astronomical telescope the *objective*, or forward lens, forms at its focus an image

FIG. 464. Magnifying power of a telescope objective is $F/25$

of a distant object. Suppose that this image were viewed directly by an eye 25 cm. from the image, as in Fig. 464. The angle subtended by the image at the eye would then be $P'EQ'$; but the angle subtended by the object is PEQ, which is practically the same as $P'cQ'$; for $P'cQ' = PcQ$, and, since the object is very distant, $PcQ = PEQ$ approximately. But $P'cQ'$ divided by $P'EQ'$ is equal to $F/25$, F being the focal length of the objective measured in centimeters. Hence the forward lens alone enables us to increase the visual angle of the object $F/25$ times.

In practice, however, the image is not viewed with the unaided eye, but with a simple magnifying glass called an eyepiece (Fig. 465) placed so that the image is at its focus. Since we have seen in § 558 that the simple magnifying glass increases the visual angle $25/f$ times, f being the focal length of the eyepiece, it is clear that the total magnification produced by both lenses, used as above, is $F/25 \times 25/f = F/f$. *The magnifying power of an astronomical telescope is therefore the focal length of the objective divided by the focal length of the eyepiece.* It will be seen, therefore, that to get a high magnifying power it is necessary to use an objective of as great focal length as possible and an eyepiece of as short focal length

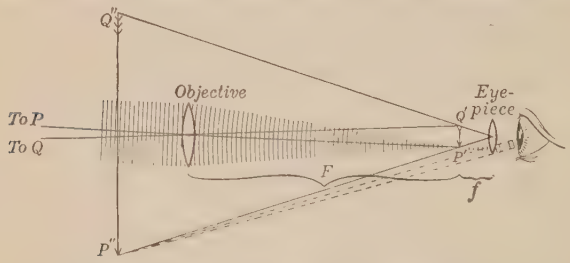


FIG. 465. Magnifying power of a telescope is F/f

as possible. The focal length of the great lens at the Yerkes Observatory is about 62 feet and its diameter 40 inches. The object of the great diameter is to enable it to collect a very large amount of light.

Eyepieces often have focal lengths as small as $\frac{1}{4}$ in. Thus the Yerkes telescope when used with a $\frac{1}{4}$ in. eyepiece has a magnifying power of 2976.

560. The magnifying power of the compound microscope. The compound microscope is like the telescope in that the front lens, or *objective*, forms a real image of the object at the focus of the eyepiece. The size of the image $P'Q'$ (Fig. 466) formed by the objective is as many times the size of the object PQ as v, the distance from the objective to the image, is times u, the distance from the objective to the object (see § 548). Since the eyepiece magnifies this image $25/f$ times, the total magnifying power of a compound microscope is $\frac{v}{u}\frac{25}{f}$. Ordinarily v is practically the length L of the microscope tube, and u is the focal length F of the objective. Wherever this is the case, then, the magnifying power of the compound microscope is $\frac{25L}{Ff}$.

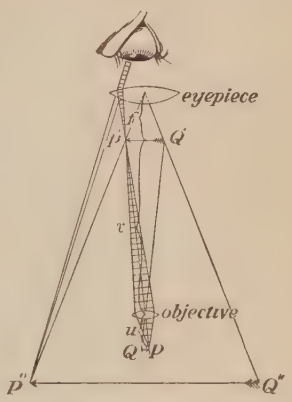


FIG. 466. The compound microscope

The relation shows that in order to get a high magnifying power with a compound microscope the focal length of both eyepiece and objective should be as short as possible, while the tube length should be as long as possible. Thus if a microscope has both an eyepiece and an objective of 6 mm. focal length and a tube 15 cm. long, its magnifying power will be $\frac{25 \times 15}{.6 \times .6} = 1042$. Magnifications as high as 2500 or 3000 are sometimes used, but it is impossible to go much farther for the reason that the image becomes too faint to be seen when it is spread over so large an area.

561. The terrestrial telescope. In both the microscope and the telescope, since the image formed by the objective is a real image, it is inverted. Since the eyepiece forms a virtual image of this real image, the object as seen by the eye will appear upside down. This is a serious objection when it is desired to use the telescope as a field glass. Hence the *terrestrial telescope* is constructed with an objective exactly like

that of the astronomical telescope, but with an eyepiece which is essentially a compound microscope. Since, then, the image is twice inverted, once by the objective O (Fig. 467), and once by O', it appears erect.

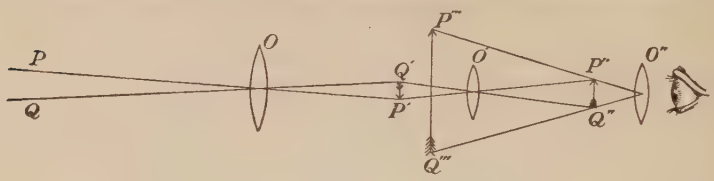


Fig. 467. The terrestrial telescope

562. The opera glass. On account of the large number of lenses which must be used in the terrestrial telescope, it is too bulky and awkward for many purposes, and hence it is often replaced by the opera glass (Fig. 468). This instrument consists of an objective like that of the telescope, and an eyepiece which is a concave lens of the same focal length as the eye of the observer. The effect of the eyepiece is, therefore, to just neutralize the lens of the eye. Hence the objective, in effect, forms its image directly upon the retina. It will be seen that the size of the image formed upon the retina by the objective of the opera glass is as much larger than the size of the image formed by the naked eye as the focal length CR of the objective is greater than the focal length cR of the eye. Since the focal length of the eye is the same as that of the eyepiece, *the magnifying power of the opera glass, like that of the astronomical telescope, is the ratio of the focal lengths of the*

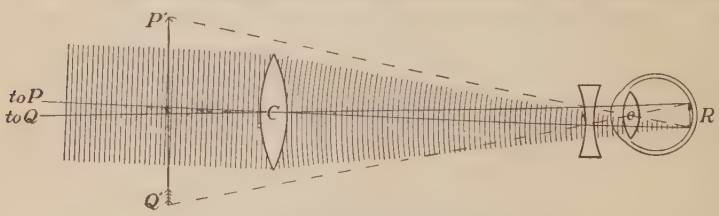


Fig. 468. The opera glass

objective and eyepiece. Objects seen with an opera glass appear erect, since the image formed on the retina is inverted, as is the case with images formed by the lens of the eye unaided.

563. The stereoscope. Binocular vision. When an object is seen with both eyes the images formed on the two retinas differ slightly, because

of the fact that the two eyes, on account of their lateral separation, are viewing the object from slightly different angles. It is this difference in the two images which gives to an object or landscape viewed with two eyes an appearance of depth, or solidity, which is wholly wanting when one eye is closed. The stereoscope is an instrument which reproduces in photographs this effect of binocular vision. Two photographs of the same object are taken from slightly different points of view. These photographs are mounted at A and B (Fig. 469), where they are simultaneously viewed by the two eyes through the two prismatic lenses m and n. These two lenses superpose the two images at C because of their action as prisms, and at the same time magnify them because of their action as simple magnifying lenses. The result is that the observer is conscious of viewing but one photograph; but this differs from ordinary photographs in that, instead of being flat, it has all of the characteristics of an object actually seen with both eyes.

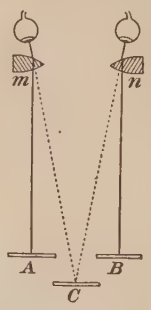


Fig. 469. Principle of the stereoscope

The opera glass has the advantage over the terrestrial telescope of affording the benefit of binocular vision; for while telescopes are usually constructed with but one tube, opera glasses always have two, one for each eye.

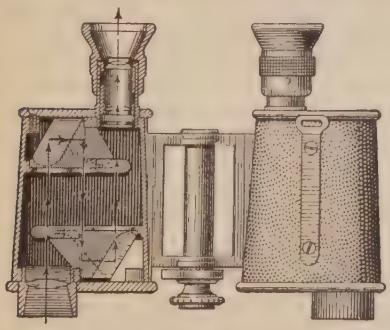


Fig. 470. The Zeiss binocular

564. The Zeiss binocular. The greatest disadvantage of the opera glass is that the field of view is very small. The terrestrial telescope has a larger field but is of inconvenient length. An instrument called the Zeiss binocular (Fig. 470) has recently come into use, which combines the compactness of the opera glass with the wide field of view of the terrestrial telescope. The compactness is gained by causing the light to pass back and forth through total reflecting prisms, as in the figure. These reflections also perform the function of reinverting the image, so that the real image which is formed at the focus of the eyepiece is erect. It will be seen, therefore, that the instrument is

essentially an astronomical telescope in which the image is reinverted by reflection, and in which the tube is shortened by letting the light pass back and forth between the prisms.

A further advantage which is gained by the Zeiss binocular is due to the fact that the two objectives are separated by a distance which is greater than the distance between the eyes, so that the stereoscopic effect is more prominent than with the unaided eye or with the ordinary opera glass.[1]

QUESTIONS AND PROBLEMS

1. If a photographer wishes to obtain the full figure on a plate of cabinet size, does he place the subject nearer to or farther from the camera than if he wishes to take only the head? Why?

2. The image on the retina of a book held a foot from the eye is larger than that of a house on the opposite side of the street. Why do we not judge that the book is actually larger than the house?

3. What is the magnifying power of a $\frac{1}{4}$-in. lens used as a simple magnifier?

4. A telescope has an objective of 30 ft. focal length and an eyepiece of 1 in. focal length. What is its magnifying power?

5. A stereopticon is provided with two lenses, one of 6-in. and the other of 12-in. focal length. Which lens should be used if it is desired to get as large an image as possible on a distant screen?

6. A compound microscope has a tube length of 8 in., an objective of focal length $\frac{1}{2}$ in., and an eyepiece of focal length 1 in. What is its magnifying power?

7. Explain why a terrestrial telescope shows objects erect rather than inverted.

8. If the focal length of the eye is 1 in., what is the magnifying power of an opera glass whose objective has a focal length of 4 in.?

9. If the length of a microscope tube is increased after an object has been brought into focus, must the object be moved nearer to or farther from the lens in order that the image may be again in focus?

10. The magnifying power of a microscope is 1000, the tube length is 8 in., and the focal length of the eyepiece is $\frac{1}{2}$ in. What is the focal length of the objective?

11. What sort of lenses are necessary to correct shortsightedness? longsightedness?

[1] Laboratory experiments on the magnifying powers of lenses and on the construction of microscopes and telescopes should follow this chapter. See, for example, Experiments 47, 48, and 49 of the authors' manual.

CHAPTER XXI

COLOR PHENOMENA

Color and Wave Length

565. Comparison of the wave lengths of red and green light by soap films. Let a soap film be formed across the top of an ordinary drinking glass, care being taken that both the solution and the glass are as clean as possible. If now the glass is held so that the film is vertical, the water will run down to the lower edge of the film and cause it to become wedge-shaped. Let the film be placed before a sodium flame arranged precisely as in Fig. 430, p. 405. Alternating yellow and black bands will be seen, as in that experiment. In fact, the soap film plays exactly the same part as did the air film in the experiment of that section. Now let a beam of sunlight or the light from a projecting lantern pass through a piece of red glass at A, fall upon the soap film F, and be reflected from it to a white screen S (see Fig. 471). Let a convex lens L of from six to twelve inches focal length be placed in the path of the reflected beam in such a position as to produce an image of the film upon the screen S, i.e. in such a position that film and screen are at conjugate foci of the lens. A system of red and black bands (commonly called interference fringes) will be formed by the interference of the two beams of light coming from the front and back

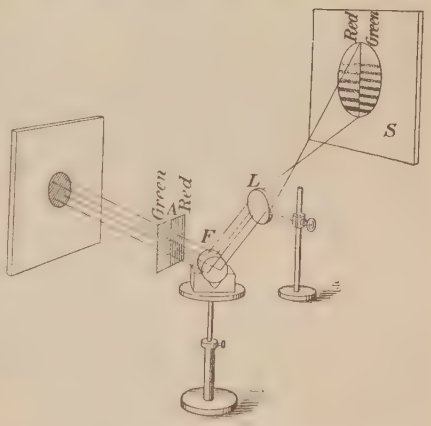


Fig. 471. Projection of soap-film fringes

surfaces of the film. Let now the red glass be held in one half of the beam and a piece of green glass in the other half, the two pieces being placed edge to edge, as shown at *A*. Two sets of fringes will be seen side by side on the screen. The fringes will be red and black on one side of the image and green and black on the other; but it will be noticed at once that the dark bands on the green side are closer together than the dark bands on the other side; in fact, seven green fringes will be seen to cover about the same distance as six red ones.[1]

Since it was shown in § 518 that the distance between two dark bands corresponds to an increase of one half wave length in the thickness of the film, we conclude, from the fact that the dark bands on the red side are farther apart than those on the green side, that red light must have a longer wave length than green light.

566. Wave lengths of lights of various colors. If we compare in this way red, yellow, green, blue, and violet lights, we find that the wave lengths are all different, — red being the longest, yellow next, green next, blue next, and violet the shortest. Actual measurements of wave length show that lights of the following wave lengths have the designated colors.

Red	.000068 cm.	Yellow	.000058 cm.
Green	.000052 cm.	Blue	.000046 cm.
	Violet	.000042 cm.	

These numbers represent unmistakable colors. Wave lengths between say .000048 cm. and .000050 cm. we call bluish green or greenish blue, and the wave lengths between .000059 cm. and .000061 cm. we call reddish yellow or orange.

Let the red and green glasses be removed from the path of the beam. The red and green fringes will be seen to be replaced by a series of bands brilliantly colored in different hues. These are due to the fact that the lights of different wave length form interference bands at

[1] If the conditions do not permit the projection of this experiment, simply look through the red and green glasses at the film when it is so placed as to reflect white light to the eye. The results will be the same.

different places on the screen. Notice that the upper edges of the bands (lower edges in the inverted image) are reddish, while the lower edges are bluish. We shall find the explanation of this fact in § 574.

567. Composite nature of white light. The last experiment showed that light of various colors can be obtained from white light, thus indicating that white light is itself a complex of colored lights. The following experiment will show what colors it contains.

Let a beam of sunlight pass through a narrow slit and fall on a prism, as in Fig. 472. It will be observed that the beam which enters the prism as white light is not only bent as it passes through the prism but is also broken up into a beam of many colors, so that when it falls on a white screen a colored band will be seen. Red, yellow, green, blue, and violet are clearly distinguishable, although each color merges, by insensible gradations, into the next. This band of color is called a *spectrum*.

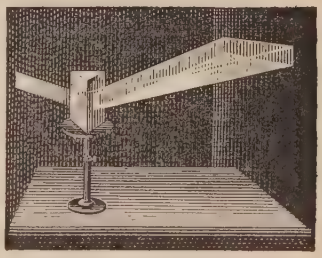


FIG. 472. White light decomposed by a prism

We conclude from this experiment that white light is a mixture of all the colors of the spectrum, from red to violet inclusive. The wave length of the extreme red is about .000076 cm., while the wave length of the extreme violet is about one half this amount, or .000038 cm.

568. Color of bodies in white light. Let a piece of red glass be held in the path of the colored beam of light in the experiment of the preceding section. All the spectrum except the red will disappear, thus showing that all the wave lengths except red have been absorbed by the glass. Let a green glass be inserted in the same way. The green portion of the spectrum will remain strong, while the other portions will be greatly enfeebled. Hence green glass must have a much less absorbing effect upon wave lengths which correspond to green than upon those which correspond to red and blue. Let the green and red glasses be held

one behind the other in the path of the beam. The spectrum will almost completely vanish, for the red glass has absorbed all except the red rays, and the green glass has absorbed these.

We conclude, therefore, that the color which a body has in ordinary daylight is determined by the wave lengths which the body has *not* the power of absorbing. Thus, if a body appears white in daylight, it is because it diffuses or reflects all wave lengths equally to the eye, and does not absorb one set of wave lengths more than another. For this reason the light which comes from it to the eye is of the same composition as daylight, or sunlight. If, however, a body appears red in daylight, it is because it absorbs the red rays of the white light which falls upon it less than it absorbs the others, so that the light which is diffusely reflected contains a larger proportion of red wave lengths than is contained in ordinary light. Similarly, a body appears yellow, green, or blue when it absorbs less of one of these colors than of the rest of the colors contained in white light, and therefore sends a preponderance of some particular wave length to the eye.

569. Color of bodies placed in colored lights. Let a body which appears white in sunlight be placed in the red end of the spectrum. It will appear to be red. In the blue end of the spectrum it will appear to be blue, etc. This confirms the conclusion of the last paragraph, that a white body has the power of diffusely reflecting all the colors of the spectrum equally.

Next let a skein of red yarn be held in the blue end of the spectrum. It will appear nearly black. In the red end of the spectrum it will appear strongly red. Similarly, a skein of blue yarn will appear nearly black in all the colors of the spectrum except blue, where it will have its proper color.

These effects are evidently due to the fact that the red yarn, for example, has the power of diffusely reflecting red wave lengths copiously, but of absorbing, to a large extent, the others. Hence, when held in the blue end of the spectrum, it sends but little color to the eye, since no red light is falling upon it.

570. Compound colors. It must not be inferred from the preceding paragraphs that every color except white has one definite wave length, for the same effect may be produced on the eye by a mixture of several different wave lengths as is produced by a single wave length. This statement may be proved by the use of an apparatus known as Newton's color disk (Fig. 473). The arrangement makes it possible to rotate differently colored sectors so rapidly before the eye that the effect is precisely the same as though the colors came to the eye simultaneously. If one half of the disk is red and the other half green, the rotating disk will appear yellow, the color being very similar to the yellow of the spectrum. If green and violet are mixed in the same way, the result will be light blue. Although the colors produced in this way are not distinguishable by the eye from spectral colors, it is obvious that their physical constitution is wholly different; for while a spectral color consists of waves of a single wave length, these colors produced by mixture are compounds of several wave lengths. For this reason the spectral colors are called pure colors and the others compound colors. In order to tell whether the color of an object is pure or compound it is only necessary to observe it through a prism. If it is a compound color, the colors will be separated, giving an image of the object for each color. If it is a pure color, the object will appear through the prism exactly as it does without the prism.

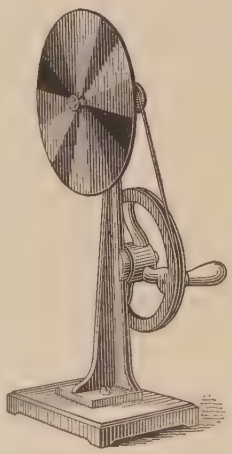


Fig. 473. Newton's color disk

By compounding colors in the way described above we can produce many colors which are not found in the spectrum. Thus mixtures of red and blue give purple or magenta; mixtures of black with red, orange, or yellow give rise to the various

shades of brown. Lavender may be formed by adding three parts of white to one of blue; lilac, by adding to fifteen parts of white, four parts of red and one of blue; olive, by adding one part of black to two parts of green and one of red.

571. Complementary colors. Since white light is a combination of all the colors from red to violet inclusive, it might be expected that if one or several of these colors were subtracted from white light, the residue would be colored light.

To test this experimentally let a beam of sunlight be passed through a slit s, a prism P, and a lens L, arranged as in Fig. 474. The lens should have a diameter of from four to six inches, and a focal length of from six to twelve inches. Let the lens be placed in the path of the refracted beam at a distance from the prism slightly greater than its focal length, and let the screen be moved back and forth until its position is the conjugate focus of the position occupied by the prism. At this point the band of colored light, which forms a spectrum at RV, the position conjugate to the slit s, will be recombined into a pure white image of the slit at S, the position which is conjugate to the prism face ab. Let a card be slipped into the path of the beam at R, so as to cut off the red portion of the light. The slit will appear a brilliant shade of greenish blue. This is the compound color left after red is taken from the white light. This shade of blue is therefore called the *complementary color* of the red which has been subtracted. Two *complementary colors* are therefore defined as any two colors which produce white when added to each other.

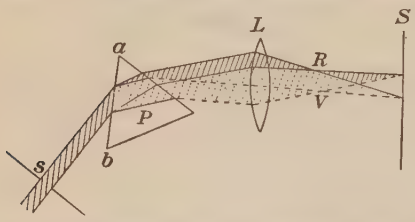


Fig. 474. Recombination of spectral colors into white light

Let the card be slipped in from the side of the blue rays at V. The slit will first take on a yellowish tint when the violet alone is cut out; and as the card is slipped farther in, the image will become a deep shade of red when violet, blue, and part of the green are cut out.

Next let a lead pencil be held vertically between R and V so as to cut off the middle part of the spectrum, i.e. the yellow and green rays. The remaining red, blue, and violet will unite to form a brilliant purple

image. In each case the color on the screen is the complement of that which is cut out.

572. Retinal fatigue. Let the gaze be fixed intently for not less than twenty or thirty seconds on a point at the center of a block of any brilliant color, — for example, red. Then look off at a dot on a white wall or a piece of white paper, and hold the gaze fixed there for a few seconds. The brilliantly colored block will appear on the white wall, but its color will be the *complement* of that first looked at.

The explanation of this phenomenon, due to so-called "retinal fatigue," is found in the fact that although the white surface is sending waves of all colors to the eye, the nerves which responded to the color first looked at have become fatigued, and hence fail to respond to this color when it comes from the white surface. Therefore the sensation produced is that due to white light minus this color, i.e. to the complement of the original color. A study of the spectral colors by this method shows that the following colors are complementary.

Red	Orange	Yellow	Violet	Green
Bluish green	Greenish blue	Blue	Greenish yellow	Purple

573. Color of pigments. When yellow light is added to the proper shade of blue, white light is produced, since these colors are complementary. But if a yellow pigment is added to a blue one, the color of the mixture will be green. This is because the yellow pigment removes the blue and violet by absorption, and the blue pigment removes the red and yellow, so that only green is left.

When pigments are mixed, therefore, each one *subtracts* certain colors from white light, and the color of the mixture is that color which escapes absorption by the different ingredients. Adding *pigments* and adding *colors*, as in § 570, are therefore wholly dissimilar processes and produce widely different results.

574. Colors of thin films. The study of complementary colors has furnished us with the key to the explanation of the fact, observed in § 566, that the upper edge of each colored band

produced by the water wedge is reddish, while the lower edge is bluish. The red on the upper edge is due to the fact that there the shorter blue waves are destroyed by interference and a complementary red color is left; while on the lower edge of each fringe, where the film is thicker, the longer red waves interfere, leaving a complementary blue. In fact, each wave length of the incident light produces a set of fringes, and it is the superposition of these different sets which gives the resultant colored fringes. Where the film is too thick the overlapping is so complete that the eye is unable to detect any trace of color in the reflected light.

In films which are of uniform thickness, instead of wedge-shaped, the color is also uniform, so long as the observer does not change the angle at which the film is viewed. With any change in this angle the thickness of film through which the light must pass in coming to the observer changes also, and hence the color changes. This explains the beautiful play of iridescent colors seen in soap bubbles, thin oil films, mother of pearl, etc.

QUESTIONS AND PROBLEMS

1. If a soap film is illuminated with red, green, and yellow strips of light, side by side, how will the distance between the yellow fringes compare with the distance between the red fringes? with the distance between the green fringes? (See table on p. 444.)

2. What will be the apparent color of a red body when it is in a room to which only green light is admitted?

3. Why do white bodies look blue when seen through a blue glass?

4. What color would a yellow object appear to have if looked at through a blue glass? (Assume that the yellow is a pure color.)

5. A gas flame is distinctly yellow as compared with sunlight. What wave lengths, then, must be comparatively weak in the spectrum of a gas flame?

6. If the green and the yellow are subtracted from white light, what will be the color of the residue?

7. Will a reddish spot on an oil film be thinner or thicker than an adjacent bluish portion?

Dispersion

575. Speed of light in glass dependent upon wave length. It was seen in the experiment of § 567 that when a ray of white light passes through a prism it is split up into a fan-shaped beam of colored light. This process of decomposing white light by refraction into its constituent elements is called *dispersion.*

An observation of the position of the colors in the spectrum shows that the color which is bent least is the red, while that which is bent most is the violet. In other words, the bending increases as the wave length decreases. But we saw in § 526 that the reason for the bending of a ray which strikes obliquely against the surface of a transparent medium is that the light undergoes a change in speed as it passes into the new medium, and that the greater the change in speed the greater the bending. Since, therefore, the blue light was bent more than the red, it follows that the speed of blue light in glass must be less than that of red light.

576. Chromatic aberration. It has heretofore been assumed that all the waves which fall on a lens from a given source are brought to one and the same focus. But since blue rays are bent more than red ones in passing through a prism, it is clear that in passing through a lens the blue light must be brought to a focus at some point v (Fig. 475) nearer to the lens than r, where the red light is focused, and that the foci for intermediate colors must fall in intermediate positions. It is for this reason that an image formed by a simple lens is always fringed with color.

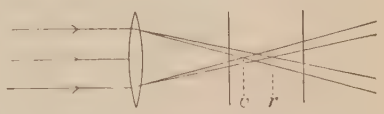


Fig. 475. Chromatic aberration in a lens

Let a card be held at the focus of a lens placed in a beam of sunlight (Fig. 475). If the card is slightly nearer the lens than the focus, the spot of light will be surrounded by a red fringe, for the red rays,

being least refracted, are on the outside. If the card is moved out beyond the focus, the red fringe will be found to be replaced by a blue one; for, after crossing at the focus, it will be the more refrangible rays which will then be found outside.

This dispersion of light produced by a lens is called *chromatic aberration.*

577. Achromatic lenses. The color effect caused by the chromatic aberration of a simple lens greatly impairs its usefulness. Fortunately, however, it has been found possible to eliminate this effect almost completely by combining into one lens a convex lens of crown glass and a concave lens of flint glass (Fig. 476). The first lens then produces both bending and dispersion, while the second almost completely overcomes the dispersion without entirely overcoming the bending. Such lenses are called *achromatic lenses.* They are used entirely in the construction of all good telescopes and microscopes.

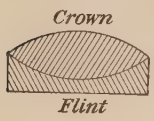


FIG. 476. An achromatic lens

578. The rainbow. A very beautiful spectrum with which every one is familiar is *the rainbow.* Its formation may be illustrated by the following experiment.

Let a spherical bulb F (Fig. 477) $1\frac{1}{2}$ or 2 in. in diameter be filled with water and held in the path of a beam of sunlight which enters the room through a hole in a piece of cardboard AB. A miniature rainbow will be formed on the screen around the opening, the violet edge of the bow being toward the center of the circle and the red outside. A beam of light which enters the flask at C is there both refracted and dispersed; at D

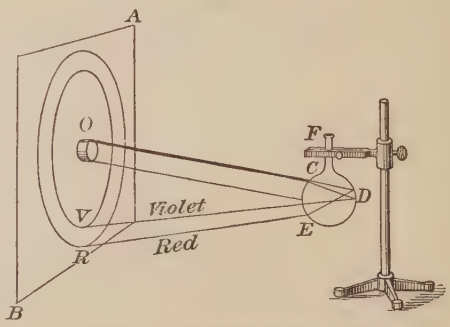


FIG. 477. Artificial rainbow

it is totally reflected; and at E it is again refracted and dispersed on passing out into the air. Since in both of the refractions the violet is bent more than the red, it is obvious that it must return nearer to the direction of the incident beam than the red rays. If the flask were a perfect sphere, the angle included between the incident ray OC and the emergent red ray ER would be 42°; and the angle between the incident ray and the emergent violet ray EV would be 40°.

The actual rainbow seen in the heavens is due to the refraction and reflection of light in the drops of water in the air, which act exactly as did the flask in the preceding experiment. If the observer is standing with his back to the sun the light which comes from the drops so as to make an angle of 42° (Fig. 478) with the line drawn from the observer to the sun must be red light; while the light which comes from drops which are at an angle of 40° from the eye must be violet light. It is clear that those drops whose direction from the eye makes any particular angle with the line drawn from the eye to the sun must lie on a circle whose center is on that line. Hence we see a circular arc of light which is violet on the inner edge and red on the outer edge.

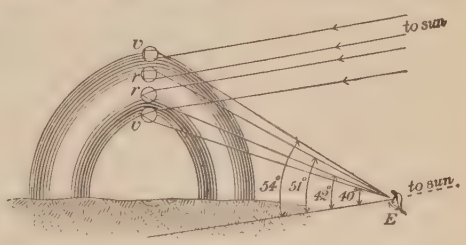


Fig. 478. Primary and secondary rainbows

579. The secondary bow. A second bow having the red on the inside and the violet on the outside is often seen outside of the one just described, and concentric with it. This bow arises from rays which have suffered two internal reflections and two refractions, in the manner shown in Fig. 478. Since in this bow the emergent ray crosses the incident ray, it will be seen that the color which suffers the largest refraction must make the largest angle with the incident ray. Hence the violet which comes to the eye must come from drops which are

farther from the center of the circle than those which send the red. The red rays come from an angle of 51°, and the violet rays from an angle of 54°.

QUESTIONS AND PROBLEMS

1. In what part of the sky will a rainbow appear if it is formed in the early morning?

2. Is a bow seen at 4 o'clock in the afternoon higher or lower than a bow seen at 5 o'clock on the same day?

3. Why is a rainbow never seen during the middle part of the day?

4. If you look at a broad sheet of white paper through a prism, it will appear red at one edge and blue at the other, but white in the middle. Explain why the middle appears uncolored.

Spectra

580. Continuous spectra. If a Bunsen burner is placed on the lecture table immediately behind a slit *s* (Fig. 479) a millimeter or two in width, and if the openings at the base of the burner where air is admitted are closed so that a white flame is produced, and if the slit is then viewed through a prism *P* held immediately in front of the eye *E* at a distance of not less than ten feet from the burner, the spectrum will be a continuous band of color passing from red at one end to violet at the other. If then the air is admitted at the base of the burner, and if a clean platinum wire is held in the flame directly in front of the slit, the white-hot platinum will also give a continuous spectrum. If an incandescent lamp

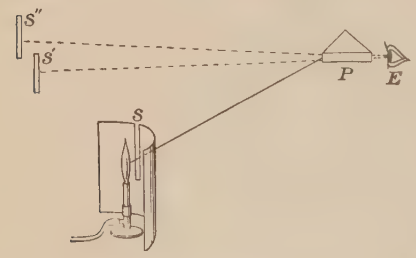


Fig. 479. Arrangement for viewing spectra

filament is observed in the same way, it will be found to give a continuous spectrum also.[1]

All incandescent solids and liquids are found to give spectra of this type which contain all the wave lengths from the extreme red to the extreme violet. The continuous spectrum of a luminous gas flame is due to the incandescence of solid particles of carbon suspended in the flame. The presence of these solid particles is proved by the fact that soot is deposited on bodies held in a white flame.

581. Bright-line spectra. Let a bit of asbestus or a platinum wire be dipped into a solution of common salt (sodium chloride) and held in the flame, care being taken that the wire itself is held so low that the spectrum due to it cannot be seen. The continuous spectrum of the preceding paragraph will be replaced by a clearly defined yellow image of the slit which occupies the position of the yellow portion of the spectrum. This image of the slit as viewed through the prism is no wider than the slit itself, thus showing that the light from the sodium flame is not a compound of a number of wave lengths, but is rather of just the wave length which corresponds to this particular shade of yellow. The light is now coming from the incandescent sodium vapor and not from an incandescent solid, as in the preceding experiments.

Let another platinum wire be dipped in a solution of lithium chloride and held in the flame. Two distinct images of the slit, s' and s'' (Fig. 479), will be seen, one in yellow and one in red. Let calcium chloride be introduced into the flame. One distinct image of the slit will be seen in the green and another in the red. (The yellow sodium image will probably be present also, because sodium is present as an impurity in nearly all salts.) Strontium chloride will give a blue and a red image, etc.

These narrow images of the slit in the different colors are called the characteristic *spectral lines* of the substances. The experiments show that incandescent vapors and gases give rise to *bright-line spectra,* and not continuous spectra like those produced by incandescent solids and liquids.

[1] By far the most satisfactory way of performing these experiments with spectra is to provide the class with cheap plate-glass prisms like those used in Experiment 50 of authors' manual, rather than to attempt to project line spectra.

582. Spectrum analysis. When studies of the sort just described are extended to other elements it is found that each element has its own characteristic spectrum. While some of these spectra, like that of sodium, are comparatively simple, others are much more complex; for example, mercury has two yellow lines close together, one brilliant green one, a faint blue one, and a violet one. Iron has more than four hundred bright lines scattered throughout the spectrum. Since the presence of a very small quantity of a substance in any mixture is sufficient to produce its characteristic lines in the spectrum, the method of detecting the presence of certain substances in mixtures of unknown composition by a careful study of the spectrum of the mixture has proved wonderfully efficient. Thus $\frac{1}{2000}$ mg. of barium, $\frac{1}{30,000}$ mg. of strontium, $\frac{1}{600,000}$ mg. of lithium, and $\frac{1}{14,000,000}$ mg. of sodium are sufficient to give rise to the characteristic lines of these substances when the substance is volatilized in a nonluminous Bunsen flame. Half a dozen new elements have been discovered through the presence of bright spectral lines which did not correspond to any previously known substances. This method of analyzing substances through a study of their spectra is called *spectrum analysis.* It was first used by the German chemist Bunsen in 1859.

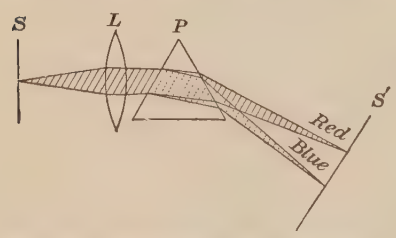


Fig. 480. Arrangement for obtaining a pure spectrum

583. Absorption spectra. Let a beam of sunlight pass first through a narrow slit S (Fig. 480) not more than $\frac{1}{2}$ mm. in width, and then through a prism P, and finally fall on a screen S', as shown in Fig. 480. Let the position of the prism be changed until a beam of white light is reflected from one of its faces to that portion of the screen which was previously occupied by the central portion of the spectrum. Then let a lens L be placed between the prism and the slit, and moved

back and forth until a perfectly sharp white image of the slit is formed on the screen. This adjustment is made in order to get the slit S and the screen S' in the positions of conjugate foci of the lens. Now let the prism be turned to its original position. The spectrum on the screen will then consist of a series of colored images of the slit arranged side by side. This is called a pure spectrum to distinguish it from the spectrum shown in Fig. 472, in which no lens was used to bring the rays of each particular color to a particular point, and in which there was therefore much overlapping of the different colors.

Now let a weak solution of permanganate of potash or chlorophyl[1] be held in the path of the light in front of the slit. The spectrum on the screen will be seen to be crossed by a series of dark bands. This must mean that the solution has the power of absorbing certain particular wave lengths, while allowing waves a little longer or a little shorter than these particular wave lengths to pass through.

A continuous spectrum from which certain wave lengths have been removed by absorption is called an *absorption spectrum.*

584. The solar spectrum an absorption spectrum. Let the solar spectrum, projected on a screen exactly as described in the last section, be carefully examined. If the slit and screen are exactly at conjugate foci of the lens, and if the slit is sufficiently narrow, the spectrum will be seen to be crossed vertically by certain dark lines.

These lines were first observed by the Englishman Wollaston in 1802, and were first studied carefully by the German Fraunhofer in 1814, who counted and mapped out as many as seven hundred of them. They are called after him, the Fraunhofer lines. Their existence in the solar spectrum shows that certain wave lengths are absent from sunlight, or, if not entirely absent, are at least much weaker than their neighbors. The solar spectrum is therefore an absorption spectrum. When the experiment is performed as described above it will usually not be possible to count more than five or six distinct lines.

[1] The chlorophyl solution may be made by stirring a handful of grass or clover vigorously in warm alcohol, and then pouring off the green liquid.

585. Explanation of the Fraunhofer lines. Let the solar spectrum be projected as in § 584. Let a few small bits of metallic sodium be laid upon a loose wad of asbestus which has been saturated with alcohol. Let the asbestus so prepared be held to the left of the slit, or between the slit and the lens, and there ignited. A black band will at once appear in the yellow portion of the spectrum, at the place where the color is exactly that of the sodium flame itself, or if the focus was sufficiently sharp so that a dark line could be seen in the yellow before the sodium was introduced, this line will grow very much blacker when the sodium is burned. Evidently then this dark line in the yellow part of the solar spectrum is due in some way to sodium vapor through which the sunlight has somewhere passed.

Fig. 481. Comparison of solar and iron spectra

The experiment at once suggests the explanation of the Fraunhofer lines. The white light which is emitted by the hot nucleus of the sun, and which contained all wave lengths, has had certain wave lengths weakened by absorption as it passed through the vapors and gases surrounding the sun and the earth. For it is found that *every gas or vapor will absorb exactly those wave lengths which it itself is capable of emitting when incandescent.* This is for precisely the same reason that a tuning fork will respond to, i.e. absorb, only vibrations which have the same period as those which it is itself able to emit. Since, then, the dark line in the yellow portion of the sun's spectrum is in exactly the same place as the bright yellow line produced by incandescent sodium vapor, or the dark line which is produced whenever white light shines through sodium vapor, we infer that sodium vapor must

be contained in the sun's atmosphere. By comparing in this way the positions of the lines in the spectra of different elements with the position of various dark lines in the sun's spectrum, many of the elements which exist on the earth have been proved to exist also in the sun. For example, the German physicist Kirchoff showed that the four hundred and sixty bright lines of iron which were known to him were all exactly matched by dark lines in the solar spectrum. Fig. 481 shows a copy of a photograph of a portion of the solar spectrum in the middle, and the corresponding bright-line spectrum of iron each side of it. It will be seen that the coincidence of bright and dark lines is perfect. Kirchoff concluded that iron, calcium, magnesium, nickel, barium, copper, sodium, and zinc certainly exist in the sun, and that gold, silver, mercury, aluminium, cadmium, tin, antimony, arsenic, strontium, lithium, and silicon do not exist there. The lines of silver, aluminium, arsenic, and silicon have, however, since been identified. The new element helium was first discovered in the sun; for certain lines had long been noted in the sun's spectrum which could not be identified with any elements on the earth, until Ramsey, in 1896, discovered a new element which he named helium because its lines exactly coincided with these hitherto unidentified lines in the sun's spectrum. In like manner, from a study of the spectra of the stars many of the elements which exist in their atmospheres have been determined.

586. Doppler's principle applied to light waves. We have seen (see Doppler's principle, § 458, p. 352) that the effect of the motion of a sounding body toward an observer is to shorten slightly the wave length of the note emitted, and the effect of motion away from an observer is to increase the wave length. Similarly, when a star is moving toward the earth each particular wave length emitted will be slightly less than the wave length of the corresponding light from a source on the earth's surface. Hence in this star's spectrum all the lines will be

displaced slightly toward the violet end of the spectrum. If a star is moving away from the earth, all its lines will be displaced toward the red end. From the direction and amount of displacement, therefore, we can calculate the velocity with which a star is moving toward or receding from the solar system. Observations of this sort have shown that some stars are moving through space toward the solar system with a velocity of 150 mi. per second, while others are moving away with almost equal velocities. The whole solar system appears to be sweeping through space with a velocity of about 12 mi. per second; but even at this rate it would be at least 1,000,000 years before the earth would come into the neighborhood of the nearest star, even if it were moving directly toward it.

QUESTIONS AND PROBLEMS

1. What evidence have we for believing that there is sodium in the sun?

2. What sort of a spectrum should moonlight give? (The moon has no atmosphere.)

3. If you were given a mixture of a number of salts, how would you proceed with a Bunsen burner, a prism, and a slit, to determine whether or not there was any calcium in the mixture?

4. Can you see any reason why the vibrating molecules of an incandescent gas might be expected to give out a few definite wave lengths, while the particles of an incandescent solid give out all possible wave lengths?

5. Can you see any reason why it is necessary to have the slit narrow and the slit and screen at conjugate foci of the lens in order to show the Fraunhofer lines in the experiment of § 584?

In many schools this course may properly be brought to an end with this chapter, but a final chapter, which deals largely with the recent fascinating and epoch-making discoveries in the field of "radiation," has been added. Even though the chapter is not used for class-room assignment, it is the hope of the authors that all of their readers will be sufficiently interested to devote a few moments at least to its perusal.

CHAPTER XXII

INVISIBLE RADIATIONS

Radiation from a Hot Body

587. Absorption of light waves produces heating. If an air thermometer, the bulb of which has been covered with lampblack so as to absorb light waves, be held anywhere within the spectrum of the sun, its temperature is found to rise. This indicates that light waves are transformed into heat when absorbed by ordinary matter. The effect may, however, be demonstrated more conveniently with some more delicate thermoscope than an air thermometer. The radiometer (Fig. 482) is much used for such purposes. It consists of a partially exhausted bulb within which is a little aluminium wheel carrying four vanes blackened on one face and polished on the other. When the instrument is held in sunlight or before a lamp, the vanes rotate in such a way that the blackened faces always move away from the source of radiation. This is because the blackened faces absorb ether waves better than do the polished faces, and thus become hotter. The heated air in contact with these faces then exerts a greater pressure against them than does the air in contact with the polished faces. The more intense the radiation the faster is the rotation.

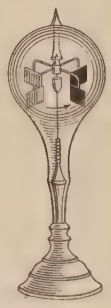


Fig. 482. The Crookes radiometer

588. Infra-red rays. If a radiometer or any other delicate thermoscope is moved from a position within the visible spectrum to a point just outside the red, it will continue to be

affected as strongly as at first. We conclude, therefore, that the sun sends out not only ether waves which are capable of affecting the optic nerve but also other waves which, though too long to affect the optic nerve, nevertheless are capable of producing strong heating effects. These rays, known as *infra-red* rays, are found to be emitted by all hot bodies, even though such bodies are not hot enough to produce light waves. They are found to possess all the properties of light waves, differing from them only in having a greater wave length. The fact that they may be reflected or refracted may be shown as follows.

Let two conjugate foci of a mirror or a lens be located by means of the light from a candle. Then let the candle be replaced by an iron ball heated almost to redness (Fig. 483). As soon as the radiometer is brought into the position which was occupied by the image of the candle, care being taken, of course, to screen it from the direct radiation from the ball, the increased rate of rotation will show that the invisible heat waves are focused by the mirror or lens precisely as were the light waves.

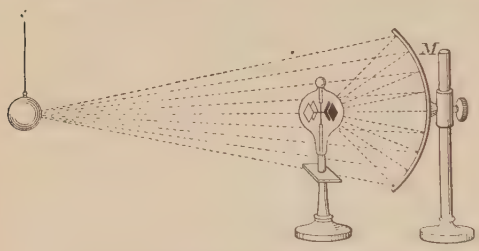


Fig. 483. Reflection of infra-red rays

Again, let a large flat bottle of water be inserted between the radiometer and a lamp or other source of heat. The rate of rotation will be much reduced, thus showing that while the water transmits practically all the light waves, it absorbs most of the infra-red waves. Let the bottle of water be replaced by a similar bottle filled with carbon bisulphide. The rate of rotation will be almost as great as at first. The carbon bisulphide therefore transmits the infra-red rays. Even if iodine is dissolved in the carbon bisulphide, so as to render it almost opaque to light waves, it will be found to be still transparent to infra-red waves.

589. Ultra-violet rays. If a photographic plate be exposed to the sun's spectrum, it is found upon being developed to be

blackened far beyond the limits of the violet. Hence the sun emits ether waves, which are shorter than those to which the eye can respond. These rays are called *ultra-violet* rays. When they are absorbed they produce heating effects, as do all ether waves, but so slight is their heating power that it is necessary to study them chiefly through the chemical effects which they produce upon photographic plates. In this respect they are far more active than are the longer yellow and red waves. In fact, red and infra-red rays are so deficient in ability to produce chemical effects that sensitive photographic plates are regularly exposed to the red light of the dark room without suffering any damage.

590. Limits of the sun's spectrum. Experiments like those of the preceding paragraphs have shown that the waves to which the eye responds are only a small fraction of all the wave lengths which are emitted by the sun or by any white-hot body. The ultra-violet spectrum has been studied down to a wave length of .00001 cm., which is only one fourth the wave length of the shortest violet waves. On the other hand, the infra-red spectrum has been investigated up to wave lengths of about .0061 cm., which is more than a hundred times the wave length of yellow light. In musical notation the visible spectrum comprises but about one octave, i.e. from .000038 to .000076 cm. The ultra-violet spectrum comprises at least two octaves, while the infra-red comprises seven. The sun's spectrum is therefore at least ten times as long, measured by wave lengths, as is the part which we see.

591. Radiation and temperature. All bodies, even such as are at ordinary temperatures, are continually radiating energy in the form of ether waves. This is proved by the fact that even if a body is placed in the best vacuum obtainable, it continually falls in temperature when surrounded by a colder body, such, for example, as liquid air. The ether waves emitted at ordinary temperatures are doubtless very long as compared with

light waves. As the temperature is raised, more and more of these long waves are emitted, but shorter and shorter waves are continually added. At about 525° C. the first visible waves, i.e. those of a dull red color, begin to appear. From this temperature on, owing to the addition of shorter and shorter waves, the color changes continuously, — first to orange, then to yellow, and finally, between 800° C. and 1200° C., to white. In other words, all bodies get "red-hot" at about 525° C. and "white-hot" at from 800° C. to 1200° C.

Some idea of how rapidly the total radiation of ether waves increases with increase of temperature may be obtained from the fact that a hot platinum wire gives out thirty-six times as much light at 1400° C. as it does at 1000° C., although at 1000° C. it is already white-hot. The radiations from a hot body are sometimes classified as heat rays, light rays, and chemical or actinic rays. The classification is, however, misleading, since all ether waves are heat waves, in the sense that when absorbed by matter they produce heating effects, i.e. molecular motions. Radiant heat is, then, the radiated energy of ether waves of any and all wave lengths.

592. Radiation and absorption. Although all substances begin to emit waves of a given wave length at approximately the same temperature, the total rate of emission of energy at a given temperature varies greatly with the nature of the radiating surface. In general, experiment shows that *surfaces which are good absorbers of ether radiations are also good radiators.* From this it follows that *surfaces which are good reflectors,* like the polished metals, *must be poor radiators.*

Thus, let two sheets of tin, 5 or 10 cm. square, one brightly polished and the other covered on one side with lampblack, be placed in vertical planes about 10 cm. apart, the lampblacked side of one facing the polished side of the other. Let a small ball be stuck with a bit of wax to the outer face of each. Then let a hot metal plate be held midway between the two. The wax on the tin with the blackened face will melt

and its ball will fall first, showing that the lampblack absorbs the heat rays faster than does the polished tin. Now let two blackened glass bulbs be connected, as in Fig. 484, through a U-tube containing colored water, and let a well-polished tin can, one side of which has been blackened, be filled with boiling water and placed between them. The motion of the water in the U-tube will show that the blackened side of the can is radiating heat much more rapidly than the polished side.

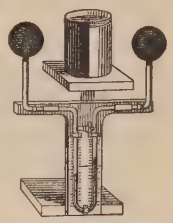


FIG. 484. Good absorbers are good radiators

593. Fluorescence. Let the sun's spectrum be thrown upon a screen, and let a piece of paper which has been coated with barium platino-cyanide be held just outside the violet end of the spectrum. It will be seen to glow with a yellowish green light, in spite of the fact that no yellow or green rays fall upon it.

This substance, therefore, possesses the property of absorbing waves of a certain wave length, and of sending out the absorbed energy in the form of light waves of a greater wave length. This property is possessed by many substances, such, for example, as sulphate of quinine, which glows with a pale blue light when exposed to the ultra-violet rays; or the uranium salts, which exhibit a brilliant green color under the same circumstances. Substances possessing this property are called *fluorescent* substances.

QUESTIONS AND PROBLEMS

1. When one is sitting in front of an open grate fire does he receive most heat by conduction, convection, or radiation?

2. The atmosphere is transparent to most of the sun's rays. Why are the upper regions of the atmosphere so much colder than the lower regions?

3. Sunlight in coming to the eye travels a much longer air path at sunrise and sunset than it does at noon. Since the sun appears red or yellow at these times, what rays are absorbed most by the atmosphere?

4. Glass transmits all the visible waves, but does not transmit the long infra-red rays. Hence explain the principle of the hotbed.

5. Which will be cooler on a hot day, a white hat or a black one?

6. Will tea cool more quickly in a polished or in a tarnished metal vessel?

7. Which emits the more red rays, a white-hot iron or the same iron when it is red-hot?

Electrical Radiations

594. Proof that the discharge of a Leyden jar is oscillatory. We found in § 484, p. 377, that the sound waves sent out by a sounding tuning fork will set into vibration an adjacent fork, provided the latter has the same natural period as the former. The following is the complete electrical analogy of this experiment.

Let the inner and outer coats of a Leyden jar A (see Fig. 485) be connected by a loop of wire *cdef*, the sliding cross piece *de* being arranged so that the length of the loop may be altered at will. Also let a strip of tin foil be brought over the edge of this jar from the inner coat to within about 1 mm. of the outer coat at C. Let the two coats of an exactly similar jar B be connected with the knobs n and n' by a second similar wire loop of fixed length. Let the two jars be placed side by side with their loops parallel, and let the jar B be successively charged and discharged by connecting its coats with a static machine or an induction coil. At each discharge of jar B through the knobs n and n' a spark will appear in the other jar at C, *provided the cross piece de is so placed that the areas of the two loops are the same.* When *de* is slid along so as to make one loop considerably larger or smaller than the other the spark at C disappears.

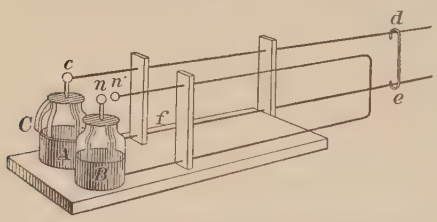


Fig. 485. Sympathetic electrical vibrations

The experiment, therefore, demonstrates that two electrical circuits, like two tuning forks, can be *tuned* so as to respond to each other sympathetically, and that just as the tuning forks will cease to respond as soon as the period of one is slightly altered, so this *electric resonance* disappears when the exact symmetry of the two circuits is destroyed. Since, obviously, this phenomenon of resonance can occur only between systems which have *natural periods* of vibration, the experiment proves that the discharge of a Leyden jar is a vibratory, i.e. an oscillatory,

phenomenon. As a matter of fact, when such a spark is viewed in a rapidly revolving mirror it is actually found to consist of from ten to thirty flashes following at equal intervals of about $\frac{1}{10,000,000}$ of a second. Fig. 486 is a photograph of such a spark.

FIG. 486. Oscillations of the electric spark

595. Electric waves. The above experiment demonstrates not only that the discharge of a Leyden jar is oscillatory, but also that these electrical oscillations set up in the surrounding medium disturbances, or waves of some sort, which travel to a neighboring circuit and act upon it precisely as the air waves acted on the second tuning fork in the sound experiment. Whether these are waves in the air, like sound waves, or disturbances in the ether, like light waves, can be determined by measuring their velocity of propagation. The first determination of this velocity was made by Heinrich Hertz in Germany in 1888. He found it to be precisely the same as that of light, i.e. 300,000 km. per second. *This result shows, therefore, that electrical oscillations set up waves in the ether.* These waves are now known as Hertz waves.

The wave length of the waves in the preceding experiment must evidently have been very long, viz. about $\frac{300,000,000}{10,000,000} = 30$ m., since the velocity of light is 300,000,000 m. per second, and since there are 10,000,000 oscillations per second; for we have seen in § 452, p. 349, that wave length is equal to velocity divided by the number of oscillations per second. By diminishing the size of the jar and the length of the circuit the length of the waves may be greatly reduced. By causing the electrical discharges to take place between two balls only a fraction of a millimeter in diameter, instead of between the coats of a condenser, electrical waves have been obtained as short as .3 cm, only fifty times as long as the longest measured heat waves.

596. The coherer. In the above experiment we detected the presence of the electrical waves by means of a small spark gap C in a circuit almost identical with that in which the oscillations were set up. This same means may be employed for the detection of waves many feet away from the source, but the instrument which is most commonly used for this purpose is the *coherer*. Its principle is illustrated in the following experiment.

Let a glass tube several centimeters long and 6 or 8 mm. in diameter be filled with fine brass or nickel filings, and let copper wires be thrust into these filings to within a distance of about a centimeter of each other. Let these wires be connected in series with a Daniell cell and a simple D'Arsonval galvanometer. The resistance of the loose contacts of the filings will be so great that very little current will flow through the circuit. Now let the static machine be started a few feet away. The galvanometer will show a strong deflection as soon as a spark passes between the knobs of the electrical machine. This is because the electric waves, as soon as they fall upon the filings, cause them to cohere or cling together, so that the electrical resistance of the tube of filings is reduced to a small fraction of what it was before. If the tube is tapped with a pencil, the old resistance will be restored, because the filings have been broken apart by the jar. The experiment may then be repeated.

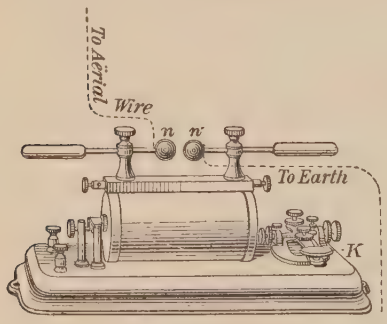


Fig. 487. Transmitter for wireless telegraphy

597. Wireless telegraphy. The last experiment illustrates completely the method of transmitting messages by wireless telegraphy. The transmitter, or oscillator, consists of an induction coil (Fig. 487) between the knobs of which, n and n', the oscillatory electric spark passes as soon as the circuit of the primary is closed by depressing the key K. This spark sends out waves into the ether, which are detected at the receiving station, in some cases hundreds of miles away,

by means of a coherer not differing at all in principle from that described in the last paragraph. This coherer C (Fig. 488) is in circuit with a relay R. When the electrical waves fall upon the coherer circuit the resistance of this circuit is greatly reduced, and hence the battery B pulls down the armature A and thus closes the circuit of the bell or sounder D through the contact point P. Hence the bell starts at once to ring. But in this operation of ringing the clapper M strikes against the coherer tube C and thus causes the filings to *decohere*. The spring S then draws back the armature A, and the instrument is in condition to receive another signal. Thus for every spark which is sent out by the coil at the sending station there is a click of the bell or sounder at the receiving station. It is found that the efficiency of both transmitter and receiver is very greatly increased by grounding one terminal of each and connecting to the other terminal a wire which runs up vertically into the air (dotted lines, Figs. 487 and 488). This wire is sometimes between one hundred and two hundred feet high. For sending signals across a schoolroom, wires ten or twelve feet high will be found an advantage, though they are not essential. Silver or nickel filings will be found to work best in the coherer. One Leclanché or dry cell is sufficient for B and B'.

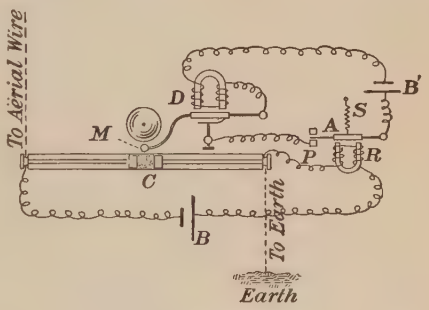


FIG. 488. Receiving apparatus for wireless telegraphy

598. The electro-magnetic theory of light. The study of electro-magnetic radiations, like those discussed in the preceding paragraphs, has shown not only that they have the speed of light, but that they are reflected, refracted, and polarized; in

fact, that they possess all the properties of light waves, the only apparent difference being in their greater wave length. Hence *modern physics regards light as an electro-magnetic phenomenon;* i.e. light waves are thought to be generated by the oscillations of the electrons, the minute electrically charged parts of the atoms. It was as long ago as 1864 that Clerk-Maxwell, of Cambridge, England, one of the world's most brilliant physicists and mathematicians, showed that it ought to be possible to create ether waves by means of electrical disturbances. But the experimental confirmation of his theory did not come until the time of Hertz' experiments (1888). Maxwell and Hertz together, therefore, share the honor of establishing the modern electromagnetic theory of light.

Cathode and Röntgen Rays

599. The electric spark in partial vacua. Let *a* and *b* (Fig. 489) be the terminals of an induction coil or static machine, *e* and *f* electrodes sealed into a glass tube 60 or 80 cm. long, *g* a rubber tube leading to an air pump by which the tube may be exhausted. Let the coil be started before the exhaustion is begun. A spark will pass between *a* and *b*, since *ab* is a very much shorter path than *ef*. Then let the tube be rapidly exhausted. When the pressure has been reduced to a few centimeters of mercury the discharge will be seen to choose the long path *ef* in preference to the short path *ab*, thus showing that *an electrical discharge takes place more readily through a partial vacuum than through air at ordinary pressures.*

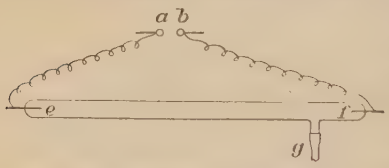


Fig. 489. Discharge in partial vacua

When the spark first begins to pass between *e* and *f* it will have the appearance of a long ribbon of crimson light. As the pumping is continued this ribbon will spread out until the crimson glow fills the whole tube. Ordinary so-called Geissler tubes are tubes precisely like the above, except that they are

WILLIAM CONRAD RÖNTGEN (1845-)

German physicist, born at Lennep, Rhine Province, Germany; received his doctor's degree at Zurich in 1868; became professor of physics at Giessen and afterwards at Würzburg, where in 1895 he made his great discovery and masterful study of Röntgen or X rays, — a discovery which must be regarded as the starting point, and, in a sense, the cause of the recent epoch-making investigations which have called into existence the "physics of the electron."

usually twisted into fantastic shapes, and are sometimes surrounded with jackets containing colored liquids, which produce pretty color effects.

600. Cathode rays. When a tube like the above is exhausted to a very high degree, say to a pressure of about .001 mm. of mercury, the character of the discharge changes completely. The glow almost entirely disappears from the residual gas in the tube, and certain invisible radiations called cathode rays begin to be emitted by the cathode (the terminal of the tube which is connected to the negative terminal of the coil or static machine). These rays manifest themselves first by the brilliant fluorescent effects which they produce in the glass walls of the tube, or in other substances within the tube upon which they fall; second, by powerful heating effects; and third, by the sharp shadows which they cast.

Fig. 490. Heating effect of cathode rays

Thus if the negative electrode is concave, as in Fig. 490, and a piece of platinum foil is placed at the center of the sphere of which the cathode is a section, the rays will come to a focus upon a small part of the foil and will heat it white-hot, thus showing that the rays, whatever they are, travel out in straight lines at right angles to the surface of the cathode. This may also be shown nicely by an ordinary bulb of the shape shown in Fig. 492. If the electrode A is made the cathode and B the anode, a sharp shadow of the piece of platinum in the middle of the tube will be cast on the wall opposite to A, thus showing that the cathode rays, unlike the ordinary electric spark, do not pass between the terminals of the tube, but pass out in a straight line from the cathode surface.

601. Nature of the cathode rays. The nature of the cathode rays was a subject of much dispute between the years 1875, when they first began to be carefully studied, and 1898. Some thought them to be streams of negatively charged particles shot off with great speed from the surface of the cathode, while

others thought they were waves in the ether, — some sort of invisible light. The following experiment furnishes very convincing evidence that the first view is correct.

NP (Fig. 491) is an exhausted tube within which has been placed a screen *sf* coated with some substance like zinc sulphide which fluoresces brilliantly when the cathode rays fall upon it; *mn* is a mica strip containing a slit *s*. This mica strip absorbs all the cathode rays which strike it; but those which pass through the slit *s* travel the full length of the tube, and although they are themselves invisible, their path is completely traced out by the fluorescence which they excite upon *sf* as they graze along it. If a magnet *M* is held in the position shown, the cathode rays will be seen to be deflected, and in exactly the direction to be expected if they consisted of negatively charged particles. For we learned in § 351, p. 261, that a moving charge constitutes an electric current, and in § 424, p. 326, that an electric current tends to move in an electric field in the direction given by the motor rule. On the other hand, a magnetic field is not known to exert any influence whatever on the direction of a beam of light or on any other form of ether waves.

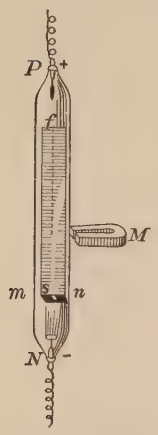


Fig. 491. Deflection of cathode rays by a magnet

When, in 1895, J. J. Thomson of Cambridge, England, proved that the cathode rays were also deflected by electric charges, as was to be expected if they consist of negatively charged particles, and when Perrin in Paris had proved that they actually impart negative charges to bodies on which they fall, all opposition to the projected particle theory was abandoned.

602. Size and velocity of cathode-ray particles. The most remarkable result of the experiments upon cathode rays is the conclusion that the rapidly moving particles of which they consist are not ordinary atoms or molecules, but are, instead, bodies whose mass is only about one two-thousandth the mass of the lightest atom known, namely the atom of hydrogen.

Moreover, the velocity with which these particles are projected through the tubes sometimes reaches the stupendous value of 60,000 mi. per second, i.e. one third the velocity of light. The calculation by which this conclusion is obtained is based upon a comparison of the amount of deflection given the rays by a magnet of known strength and the amount of deflection produced by an electric charge of known size.

603. New theories as to the constitution of matter. Furthermore, since experiments of the kind mentioned above always lead to the same value for the mass of the cathode-ray particle, no matter what may be the nature of the gas which is used in the bulb, and no matter what may be the nature of the metal which constitutes the cathode, physicists have been forced to the conclusion that these minute particles are constituents of each and every one of the different metallic elements, at least, and probably of all other elements also. It is largely because of these discoveries that the *electron theory* already referred to has been advanced by several of the greatest living physicists. According to J. J. Thomson's formulation of this theory these cathode particles are the primordial particles out of which the seventy odd atoms known to chemistry are built up, the chief differences between the different atoms of chemistry consisting in differences in the number of these particles which enter into them. The hydrogen atom, for example, is supposed to contain about 2000 of these so-called electrons, the oxygen atom 32,000, the mercury atom 400,000, and so on. But since the atoms themselves are probably electrically neutral, it is necessary to assume that they contain equal amounts of positive and negative electricity. Since, however, no evidence has yet appeared to show that positively charged electrons ever become detached from molecules, Thomson brings forward the hypothesis that perhaps the positive charges constitute the nucleus of the atom about the center of which the negative electrons are rapidly rotating.

According to this hypothesis, then, an atom is a sort of infinitesimal solar system whose members, the electrons, are no bigger with respect to the diameter of the atom than is the earth with respect to the diameter of the earth's orbit. Furthermore, according to this hypothesis, it is the vibrations of these electrons which give rise to light and heat waves; it is the streaming through conductors of electrons which have become detached from atoms which constitutes an electric current in a metal; it is an excess of electrons upon a body which constitutes a static negative charge, and a deficiency of electrons which constitutes a positive charge. (See § 330, p. 243.)

This theory undoubtedly contains many germs of truth. As yet, however, it is in the formative stage and ought to be regarded as a profoundly interesting speculation brought forward by men high in authority in the scientific world, rather than as an established doctrine. However, that such things as negatively charged corpuscles exist, and that they have a mass which is much smaller than that of an atom is now universally admitted.

604. X rays. It was in 1895 that the German physicist, Röntgen, first discovered that wherever the cathode rays impinge upon the walls of a tube, or upon any obstacles placed inside the tube, they give rise to another type of invisible radiation which is now known under the name of X rays, or Röntgen rays.

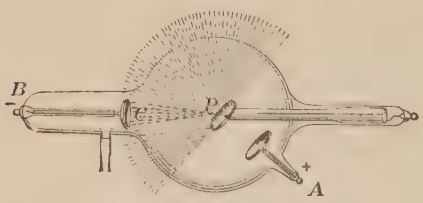


Fig. 492. An X-ray bulb

In the ordinary X-ray tube (Fig. 492) a thick piece of platinum P is placed in the center to serve as a target for the cathode rays, which, being emitted at right angles to the concave surface of the cathode C, come to a focus at a point on the surface of this plate. This is the point at which the X rays are generated and from which they radiate in all directions.

In order to convince oneself of the truth of this statement, it is only necessary to observe an X-ray tube in action. It will be seen to be divided into two hemispheres by the plane which contains the platinum plate (see Fig. 492). The hemisphere which is facing the source of the X rays will be aglow with a greenish fluorescent light, while the other hemisphere, being screened from the rays, is darker. The fluorescence is due to an effect which the X rays have upon the glass. In this respect they are like cathode rays.

605. Nature of X rays. While X rays are like cathode rays in producing fluorescence, and also in that neither of them can be reflected, refracted, or polarized, as is light, they nevertheless differ from cathode rays in several important respects. First, X rays penetrate many substances which are quite impervious to cathode rays; for example, they pass through the walls of the glass tube, while cathode rays ordinarily do not. Again, X rays are not deflected either by a magnet or by an electrostatic charge, nor do they carry electrical charges of any sort. Hence it is certain that they do not consist, like cathode rays, of streams of electrically charged particles. Their real nature is still unknown, but they are at present generally regarded as irregular pulses in the ether, set up by the sudden stopping of the cathode-ray particles when they strike an obstruction.

606. X rays render gases conducting. One of the notable properties which X rays possess in common with cathode rays is the property of causing any electrified body on which they fall to slowly lose its charge.

To demonstrate the existence of this property, let any X-ray bulb be set in operation within 5 or 10 ft. of a charged gold-leaf electroscope. The leaves at once begin to fall together.

The reason for this is probably that the X rays shake loose electrons from the atoms of the gas and thus fill it with positively and negatively charged particles, each negative particle being at the instant of separation an electron and each positive particle an atom from which an electron has been detached.

Any charged body in the gas, therefore, draws toward itself charges of sign opposite to its own, and thus becomes discharged.

607. X-ray pictures. The most striking property of X rays is their ability to pass through many substances which are wholly opaque to light, such, for example, as cardboard, wood, leather, flesh, etc. Thus, if the hand is held close to a photographic plate and then exposed to X rays, a shadow picture of the denser portions of the hand, i.e. the bones, is formed upon the plate. Fig. 493 shows a copy of such a picture.

Fig. 493. An X-ray picture of a living hand

Radio-activity

608. Discovery of radio-activity. In 1896 Henri Becquerel, in Paris, performed the following experiment. He wrapped up a photographic plate in a piece of perfectly opaque black paper, laid a coin on top of the paper, and suspended above the coin a small quantity of the mineral uranium. He then set the whole away in a dark room and let it stand for several days. When he developed the photographic plate he found upon it a shadow picture of the coin similar to an X-ray picture. He concluded, therefore, that *uranium possesses the property of spontaneously emitting rays of some sort which have the power of penetrating opaque objects and of affecting photographic plates, just as X rays do.* He also found that these rays, which he called *uranium rays*, are like X rays in that they discharge electrically charged bodies on which they fall. He found also that the rays are emitted by all uranium compounds.

609. Radium. It was but a few months after Becquerel's discovery that Madame Curie, in Paris, began an investigation of all the known elements, to find whether any of the rest of them

possessed the remarkable property which had been found to be possessed by uranium. She found that one, and but one, of the remaining known elements, namely, thorium, the chief constituent of Welsbach mantles, is capable, together with its compounds, of producing the same effect. After this discovery the rays from all this class of substances began to be called *Becquerel rays,* and all substances which emitted such rays were called *radio-active* substances.

But in connection with this investigation Madame Curie noticed that pitchblende, the crude ore from which uranium is extracted, and which consists largely of uranium oxide, would discharge her electroscope about four times as fast as pure uranium. She inferred, therefore, that the radio-activity of pitchblende could not be due solely to the uranium contained in it, and that pitchblende must therefore contain some hitherto unknown element which has the property of emitting Becquerel rays more powerfully than uranium or thorium. After a long and difficult search she succeeded in separating from several tons of pitchblende a few hundredths of a gram of a new element which was capable of discharging an electroscope more than a million times as readily as either uranium or thorium. She named this new element *radium.*

610. Nature of Becquerel rays. That these rays which are spontaneously emitted by radio-active substances are not X rays, in spite of their similarity in affecting a photographic plate, in causing fluorescence, and in discharging electrified bodies, is proved by the fact that they are found to be deflected by both magnetic and electric fields, and by the further fact that they impart electric charges to bodies upon which they fall. These properties constitute strong evidence that *radio-active substances project from themselves electrically charged particles.*

But an experiment performed in 1899 by Rutherford, of McGill University, Montreal, showed that Becquerel rays are

complex, consisting of three different types of radiation, which he named the *alpha*, the *beta*, and the *gamma* rays. The beta rays are found to be identical in all respects with cathode rays; i.e. *they are streams of electrons* projected with velocities varying from 60,000 to 180,000 miles per second. The alpha rays are distinguished from these by their very much smaller penetrating power, by their very much greater power of rendering gases conductors, by their very much smaller deflectability in magnetic and electric fields, and by the fact *that the direction of the deflection is opposite to that of the beta rays.* From this last fact, discovered by Rutherford in 1903, the conclusion is drawn that the alpha rays consist of *positively* charged particles; and from the amount of their deflectability their mass has been calculated to be about twice that of the hydrogen atom, i.e. about 4000 times the mass of the electron, and their velocity to be about 20,000 mi. per second. This difference in the masses of the alpha and beta particles explains why the latter are so much more penetrating than the former, and why the former are so much more efficient than the latter in knocking to pieces the molecules of a gas and rendering it conducting. A sheet of aluminium foil .005 cm. thick cuts off completely the alpha rays, but offers practically no obstruction to the passage of the beta and gamma rays.

The gamma rays are very much more penetrating even than the beta rays, and are not at all deflected by magnetic or electric fields. They are commonly supposed to be X rays produced by the impact of the beta particles on surrounding matter.

611. Crookes' spinthariscope. In 1903 Sir William Crookes devised a little instrument, called the spinthariscope, which furnishes very direct and striking evidence that particles are being continuously shot off from radium with enormous velocities. Radium itself in the dark glows with a light which resembles that of the glowworm, and when placed near certain substances, like zinc sulphide, it causes them to light up with a glow which is more or less brilliant, according to the amount of radium at hand. In the spinthariscope a tiny speck of radium *R* (Fig. 494)

is placed about a millimeter above a zinc sulphide screen S, and the latter is then viewed through a lens L, which gives from ten to twenty diameters magnification. The continuous soft glow of the screen, which is all one sees with the naked eye, is resolved by the microscope into a thousand tiny flashes of light. The appearance is as though the screen were being fiercely bombarded by an incessant rain of projectiles, each impact being marked by a flash of light, just as sparks fly from a flint when struck with steel. The experiment is a very beautiful one, and it furnishes very direct and convincing evidence that radium is continually projecting particles from itself at stupendous speeds. The flashes are probably due to the impacts of the *alpha*, not the *beta*, particles against the zinc sulphide screen, although this has not yet been definitely proved.

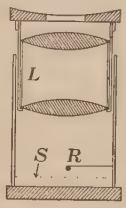


Fig. 494. Crookes' spinthariscope

612. The disintegration of radio-active substances. Whatever be the cause of this ceaseless emission of particles exhibited by radio-active substances, it is certainly not due to any ordinary chemical reactions; for Madame Curie showed, when she discovered the activity of thorium, that the activity of all the radio-active substances is simply proportional to the amount of the active element present, and has nothing whatever to do with the nature of the chemical compound in which the element is found. Thus, thorium may be changed from a nitrate to a chloride or a sulphide, or it may undergo any sort of chemical reaction, without any change whatever being noticeable in its activity. Furthermore, radio-activity has been found to be independent of all *physical* as well as chemical conditions. The lowest cold or greatest heat does not appear to affect it in the least. Radio-activity, therefore, seems to be as unalterable a property of the atoms of radio-active substances as is weight itself. For this reason Rutherford has advanced the theory that the atoms of radio-active substances are slowly disintegrating into simpler atoms, and has sought to explain in this way their extraordinary property

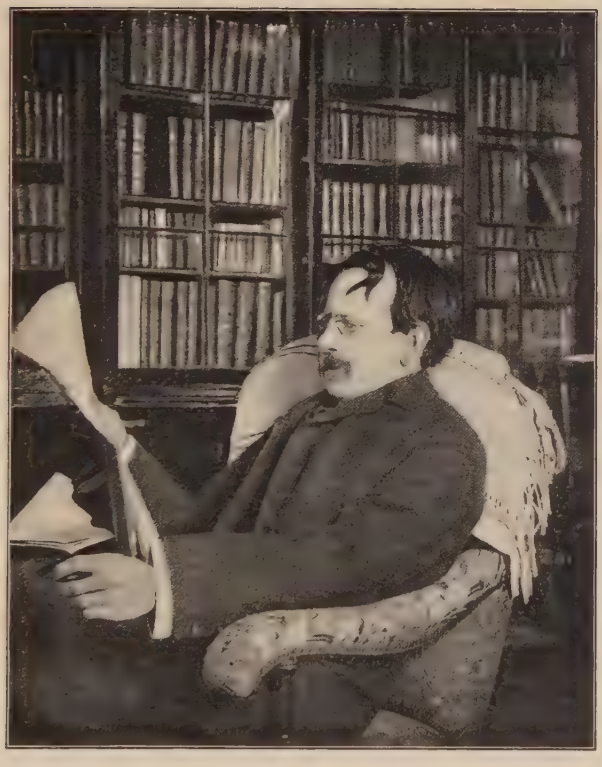
of projecting particles from themselves. This hypothesis has gained strong support from the discovery made by the Englishmen Ramsay and Soddy, in 1903, that the element helium seems to be continuously produced from radium. Just why these atoms of radio-active substances are disintegrating, and just how these new types of matter are being formed, must, of course, be largely a matter of speculation. But it is at least very suggestive to find that the three permanently radio-active substances thus far discovered, the only ones whose atomic weights have as yet been found,—namely, uranium, thorium, and radium,—possess the three heaviest atoms known. Now, according to modern notions of molecular motions, the molecules of all substances are in extremely rapid rotation. It appears, therefore, that these rapidly rotating systems of heavy atoms, such as characterize radio-active substances, not infrequently become unstable and project off a part of their mass. This projected mass is perhaps the alpha particle. What is left of the atom after the explosion is a new substance with chemical properties different from those of the original atom. This new atom is, in general, also unstable and breaks down into something else. This process is repeated over and over again till some stable form of atom is reached. Somewhere in the course of this atomic catastrophe some electrons leave the mass; these are beta rays.

613. The birth of radium. When a bit of radium is viewed in a spinthariscope we see that a multitude of little particles are being shot out every second. Nevertheless, in spite of this incessant projection of particles, the disintegration of radium is an extremely slow process. No one has as yet been able to detect with certainty any loss in the weight of a given specimen, or any diminution in its activity. Yet we may be certain that it is both losing weight and diminishing in activity, for otherwise the principle of conservation of energy, the corner stone of modern science, would be violated. The

reason that no change can be observed is that the number of atoms which become unstable and "explode" per second is extremely small as compared with the total number of atoms present; it is estimated in the case of radium to be not more than one in fifty-eight and a half billion. At this rate it would take as long as eighteen and a half years for a bit of radium to lose one hundredth part of its activity, and it would take it 1280 years to lose one half of its activity. On the other hand, these estimates, if correct, show that in a few thousand years practically all the radium now in existence will have ceased to be radio-active, i.e. will have ceased to be radium. Hence we are forced to conclude that radium is either being continually formed, or at least has been formed within the last few thousand years. Experiments made in 1905 render it extremely probable that radium is being continuously produced through the disintegration of uranium. Since the activity of uranium is less than one millionth of that of radium, the date of the birth of the uranium now on the earth may be pushed back as much as a few thousand million years. Although this carries us back to a period so remote that we can make no conjectures as to what physical or geological conditions then existed, the question which nevertheless forces itself upon us is, Where did the uranium come from? Did it, the heaviest of the elements, evolve through countless ages from simpler elements, as it now seems to be devolving upon the earth into simpler elements; and are some, possibly all, of the other elements only stages in this process, the heavier ones being perhaps formed from the lighter by the simple addition of more and more rings of rapidly rotating electrons? This is one of the great questions which we must leave for the physics of the future to decide; and it is not improbable that the answer will soon be forthcoming. All that we can say now is that *a certain few of the heavy elements are surely being slowly transmuted into other and lighter elements.*

614. The energy stored up in the atoms of the elements. In 1903 the two Frenchmen, Curie and Labord, made an epoch-making discovery. It was that radium is continually evolving heat at the rate of about one hundred calories per hour per gram. This result was to have been anticipated from the fact that the particles which are continually flying off from the disintegrating radium atoms subject the whole mass to an incessant internal bombardment which would be expected to raise its temperature. Curie and Labord's measurement of the exact amount of heat evolved per hour enables us to estimate how much heat energy is evolved in the disintegration of one gram of radium. It is about two thousand million calories, — fully three hundred thousand times as much as is evolved in the combustion of one gram of coal. Furthermore, it is extremely probable that similar enormous quantities of energy are locked up in the atoms of *all* substances, existing there in the form of the kinetic energy of rotation of the electrons. J. J. Thomson estimates that enough energy is stored in one gram of hydrogen to raise a million tons through a hundred yards. It is not improbable that it is the transformation of this subatomic energy into heat which is maintaining the temperature of the sun.

The most vitally interesting question which the physics of the future has to face is, Is it possible for man to gain control of this tremendous store of subatomic energy and to use it for his own ends? Such a result does not now seem likely or even possible; and yet the transformations which the study of physics has wrought in the world within a hundred years were once just as incredible as this. In view of what physics has done, is doing, and can yet do for the progress of the world, can any one be insensible either to its value or to its fascination?



JOSEPH JOHN THOMSON (1856-)

Most conspicuous figure in the development of the "physics of the electron"; born in Manchester, England; educated at Cambridge University; Cavendish professor of experimental physics in Cambridge since 1884; author of a number of books, the most important of which is the *Conduction of Electricity through Gases*, 1903; author or inspirer of much of the recent work, both experimental and theoretical, which has thrown light upon the connection between electricity and matter; worthy representative of twentieth-century physics.

INDEX

Aberration, chromatic, 451
Absolute temperature, 135
Absolute units, 6
Absorption, of gases, 125 ff.; spectra, 456; of light waves, 461; and radiation, 464
Acceleration, defined, 26; of gravity, 29
Activity, 161
Aëronauts, height of ascent of, 68, 70
Air, pressure in, 58; weight of, 58; compressibility of, 65
Air brake, 78
Alternator, 319
Amalgamation of zinc plate, 281
Ammeter, 268
Ampere, 267
Anode, 292
Arc light, 308; automatic feed for, 309
Archimedes, principle of, 51; portrait of, 52
Armature, ring type, 318, 321; drum type, 318, 322
Artesian wells, 50
Atmosphere, pressure of, 61; extent and character of, 68; height of homogeneous, 69; humidity of, 100
Atoms, energy in, 482

Back E.M.F. in motors, 328
Balance, 7
Balloon, 76
Barometer, mercury, 62; aneroid, 63; von Guericke's, 63; self-registering, 68
Batteries, primary, 280; storage, 296
Beats, 361
Becquerel, 477
Bell, electric, 302
Bellows, 79
Binocular vision, 440
Boiler, steam, 188
Boiling points, definition of, 204; effect of pressure on, 205; table of, 212
Boyle's law, stated, 66; explained, 85
Brooklyn Bridge, 145
Bunsen, 394
Buoyancy, of liquids, 52; of gases, 76

Caisson, 78
Calories, 175, 183; developed by electric currents, 306
Camera, pin-hole, 433; photographic, 433
Candle power, defined, 393; of incandescent lamps, 307; of arc lamps, 308
Capacity, electric, 251
Capillarity, 113 ff.
Cartesian diver, 75
Cathode, defined, 292
Cathode rays, 470
Cells, galvanic, 262; primary, 280; local action in, 281; theory of, 282; bichromate, 285; Daniell, 285; Leclanché, 287; dry, 288; combinations of, 288; storage, 296
Center of gravity, 22
Centrifugal force, 33
Charles, law of, 134
Chemical effects of currents, 292
Coefficient of expansion, of gases, 134, 137; of liquids, 140; of solids, 142
Coherer, 468
Cohesion, 106; properties depending on, 112
Coils, magnetic, properties of, 299; currents induced in rotating, 316
Cold storage, 214
Color, and wave length, 443; of bodies, 445; compound, 447; complementary, 448; of thin films, 449; of pigments, 449
Commutator, 320
Compass, 235

Component, 17
Condensers, 254
Conduction, of heat, 216; electric, 240
Conjugate points, 425, 429
Conservation of energy, 179
Convection, 219
Cooling, and evaporation, 100, 210; of a lake, 141; by expansion, 178; artificial, by solution, 207
Cooper-Hewitt lamp, 310
Crane, 160
Critical angle, 400
Crooks, 479
Crystallization, 124
Curie, 479
Currents, wind and ocean, 220; measurement of electric, 261 ff.; effects of electric, 292 ff.; induced electric, 312
Curvature, defined, 421; of mirror, 426

Daniell cell, 285
Declination, or dip, 235
Density, defined, 9; table of, 9; formula for, 10; methods of finding, 54 ff.; of air below sea level, 70; of saturated vapor, 94; of electric charge, 247
Dew-point, 99
Diffusion, of gases, 83 and 86; of liquids, 89; of solids, 105; of light, 396
Discord, 378
Dispersion, 451
Distillation, 210
Diving bell, 77; suit, 78
Doppler's principle, in sound, 352; in light, 459
Dynamo, principle of, 312; rule, 315; alternating-current, 318, 319; four-pole direct-current, 322; series-, shunt-, and compound-wound, 323
Dyne, 35

Eccentric, 188
Echo, 354
Edison, 386
Efficiency, defined, 170; of simple machines, 171; of water motors, 172; of steam engines, 191; of electric lights, 308 ff.; of transformers, 336
Eiffel Tower, 7
Elasticity, 108
Electric charge, unit of, 239; distribution of, 247
Electrical machines, 257
Electricity, static, 238 ff.; two-fluid theory of, 242; current of, 261 ff.
Electrolysis, theory of, 282; of water, 293
Electro-magnet, 301
Electromotive force, defined, 271; of induction, 315; back, in motors, 328; in secondary circuit, 331; at make and break, 332
Electron theory, 243, 473, 482
Electrophorus, 256
Electroplating, 294
Electroscope, 240, 245
Electrotyping, 295
Energy, defined, 162; potential and kinetic, 162; transformations of, 163; formula for, 165; of heat, 167; conservation of, 179; of sun, 181; expenditure of electric, 305; stored in atoms, 482
Engine, steam, 186; compound, 190; gas, 192
English equivalents of metric units, 5
Equilibrium, defined, 16; stable, 23; neutral, 24
Erg, 147
Ether, 408
Evaporation, effect of temperature on, 90; effect of air on, 96; cooling effect of, 100; freezing by, 102; effect of surface on, 103; of solids, 105; and boiling, 206; intense cold by, 210
Expansion, of gases, 83; of liquids, 89; unequal, of metals, 144; on solidifying, 200
Eye, 435

Falling bodies, 28, 29, 31
Faraday, 296, 312
Fields, magnetic, 231
Fluorescence, 465
Focal length, of convex mirror, 420; of concave mirror, 426; of convex lens, 428
Force, definition of, 12; method of measuring, 12; composition of; 14; resultant of, 14; component of, 17; centrifugal, 32; beneath liquid, 38; lines of, 230; fields of, 231

Foucault, 390, 412
Foucault currents, 333
Franklin, 249
Fraunhofer lines, 458
Freezing mixtures, 209
Freezing points, table of, 199; of solutions, 208
Friction, 167 ff.
Fundamentals, defined, 371; in pipes, 380

Galileo, 26, 29, 59, 388; portrait of, frontispiece
Galvanic cell, 262
Galvanometer, 266
Gas engine, 192
Gas meter, 80
Gay-Lussac, law of, 138
Geissler tubes, 470
Gilbert, 235, 238
Gram, of mass, 3; of force, 12
Gravitation, law of, 20
Gravity, variation of, 21; center of, 22
Guericke, Otto von, 63, 64, 73

Hardness, 112
Harmony, 366, 378
Heat, mechanical equivalent of, 174; unit of, 175; produced by friction, 176; produced by collision, 177; produced by compression, 177; specific, 182; of fusion, 196; transference of, 216
Heating, by hot air, 224; by hot water, 225
Heating effects of electric currents, 305
Helmholtz, 376
Henry, Joseph, 376
Hertz, 467
Hooke's law, 110
Horse power, 161
Humidity, 100
Huygens, 405
Hydraulic elevator, 48
Hydraulic press, 46
Hydrometer, 54, 55
Hydrostatic paradox, 41
Hygrometry, 97

Ice, manufactured, 213
Images, in plane mirrors, 416; multiple, 418; in convex mirrors, 419; size of, 422, 425, 430; in concave mirrors, 423, 427; virtual, 426, 430; real, 426; by convex lenses, 428; by concave lenses, 431
Incandescent lighting, 307
Incidence, angle of, 395
Inclined plane, 18, 156
Index of refraction, 412
Induction, magnetic, 229, 236; electrostatic, 241; charging by, 244; current, 312; coil, 330, 333
Inertia, 32
Intensity, of sound, 347; of light, 392
Interference, of sound, 361, 363; of light, 405
Ions, 282, 286, 292

Jackscrew, 157
Joule, 147, 176, 179, 307

Kelvin, 136
Kilogram, the standard, 4
Kinetic energy, 162, 165
Kirchoff, 459

Laminated cores, 334
Lamps, incandescent, 307; Nernst, 309
Lantern, projecting, 434
Leclanché cell, 287
Lenses, 428; formula for, 431; magnifying power of, 437; achromatic, 452
Lenz's law, 313
Level, of water, 44; carpenter's, 57
Lever, 151 ff.
Leyden jar, 255
Light, speed of, 388; intensity of, 392; reflection of, 395; refraction of, 398; nature of, 404; interference of, 405; dispersion of, 451; electromagnetic theory of, 469
Lightning, 249
Lines, of force, 230; isogonic, 235
Liquid-air machine, 212
Liquids, densities of, 9; pressure in, 38; transmission of pressure by, 45; incompressibility of, 65
Local action, 281
Locomotive, 189
Loudness of sound, 347

Machine, liquid-air, 212
Machines, general law of, 164; efficiencies of, 171; electrical, 257
Magdeburg hemispheres, 64
Magnet, natural, 227; poles of, 228; electro-, 301
Magnetism, 227 ff.; nature of, 232; terrestrial, 234
Magnifying power, of lens, 437; of telescope, 438; of microscope, 439
Manometric flames, 374
Mass, unit of, 3; measurement of, 9
Matter, three states of, 105
Maxwell, 86, 470
Mechanical advantage, 150
Mechanical equivalent of heat, 174
Melting points, table of, 199
Meter, standard, 3
Michelson, 389, 390, 412
Microscope, 439
Mirrors, 416; convex, 419; concave, 423; formula for, 431
Mixtures, method of, 184
Molecular forces, in solids, 107; in liquids, 113
Molecular motions, in gases, 82; in liquids, 88; in solids, 104
Molecular nature of magnetism, 232
Molecular velocities, 85, 87
Moments of force, 152
Momentum defined, 33
Mont Blanc, 68, 70
Morse, 303
Motion, uniformly accelerated, 25; laws of, 32; perpetual, 180
Motor, electric, principle of, 326; street-car, 327
Musical instruments, 384 ff.

Newton, law of gravitation, 20; portrait of, 20; laws of motion of, 32; and corpuscular theory, 405
Niagara, 166
Nodes, in pipes, 364; in strings, 370
Noise and music, 351
Nonconductors, of heat, 218; of electricity, 240

Oersted, 263
Ohm, 275, 276
Opera glass, 440
Optical instruments, 433
Organ pipes, 380, 384
Oscillatory discharge, 466
Overtones, 371, 381

Parachute, 77
Parallel connections, 289
Parallelogram law, 16
Pascal, 45, 61
Pendulum, force-moving 19; compensated, 143
Permeability, 230
Perrier, 62
Perrotin, 389
Phonograph, 386
Photometers, 393, 394
Pisa, tower of, 29
Pitch, cause of, 352; and wave length, 349
Polarization, of galvanic cells, 284; of light, 415
Potential, defined, 251; unit of, 253; measurement of, 254, 269, 272
Power, 161
Pressure, in liquids, 38; defined, 43; in air, 58; amount of atmospheric, 61; of saturated vapor, 95; effect of, on freezing, 202; in primary and secondary, 336
Prism, refraction through, 402
Pulley, 48, 148 ff.
Pump, air, 73; water, 74; force, 75

Quality of musical notes, 373

Radiation, thermal, 221; invisible, 461; and temperature, 463; and absorption, 464; electrical, 466
Radio-activity, 476
Radiometer, 461
Radium, discovery of, 476; birth of, 481
Rainbow, 452
Ramsey, 480
Rays, infra-red, 461; ultra-violet, 462; cathode, 471
Reflection, of sound, 354; of light, 395; total, 399, 401
Refraction, of light, 398; by atmosphere, 404; index of, 412; explanation of, 412
Regelation, 201
Relay, 303

Resistance, electric, defined, 273; specific, 273; unit of, 274; laws of, 274; internal, 276; measurement of, 277
Resonance, acoustical, 357; electrical, 466
Resultant, 14
Retinal fatigue, 449
Right-hand rule, 265, 300
Roemer, 388
Röntgen, 470, 474
Rowland, 179
Rumford, 393
Rutherford, 478

Saturation, of vapors, 90; of solutions, 122; magnetic, 234
Scales, musical, 366; diatonic, 367
Screw, 157
Series connections, 288
Shadows, 391
Shunts, 279
Singing flame, 378
Siphon, explanation of, 71; aspirating, 72
Soap films, 115, 443
Solar spectrum, 457, 463
Solution, of solids, 121; effect of evaporating, 123
Sonometers, 369
Sound, sources of, 343; speed of, 344; nature of, 346; musical, 351; reflection of, 354; foci, 356; interference of, 361
Sounder, 303
Sounding-boards, 360
Sources of light, 409
Spark, length and potential, 254
Spark, oscillatory nature of, 467; in vacuum, 470
Speaking tubes, 348
Specific gravity, 10
Specific heat, defined, 183; table of, 185
Spectra, continuous, 454; bright-line, 455
Spectrum analysis, 456
Speed, of sound, 343; of light, 388; of light in water, 410; of electric waves, 467
Spinthariscope, 479
Spring, intermittent, 72
Steam, engine, 187 ff.; turbine, 194
Stereoscope, 440
Strings, laws of, 369
Sun, energy derived from, 181
Sympathetic vibrations, of sound, 377; electrical, 466

Telegraph, 302, 304; wireless, 468
Telephone, 338 ff.
Telescope, astronomical, 438; terrestrial, 439
Temperature, measurement of, 129; absolute, 135; low, 136; critical, 211
Tenacity, 108
Thermometer, Galileo's, 129; mercury, 130; Fahrenheit, 131; gas, 132; maximum and minimum, 136; the dial, 144
Thomson, 243, 472, 482
Toepler-Holtz, 257
Torricelli, experiment of, 60
Transformer, 335, 337
Transmission, electrical, 337; of sound, 343 ff.; of light, 388
Transmitter, telephone, 340
Turbine, water, 173; steam, 194

Units, of length, 1, 2; of area, 2; of volume, 2; of mass, 2, 4; of time, 5; three fundamental, 5; C.G.S., 6; of force, 12, 35; of work, 147; of power, 161; of heat, 175; of magnetism, 229; of electricity, 239; of potential, 253; of current, 267, 296; of resistance, 274; of light, 393

Vacuum, sound in, 444; spark in, 470
Vaporization, heat of, 203
Velocity, defined, 25; of falling body, 27, 30; of sound, 343; of light, 388
Ventilation, 223
Vibration, numbers, 368; of strings, 369; forced, 360; sympathetic, 377
Visual angle, 436
Volt, 253
Volta, 253, 256
Voltmeter, 269

Water, density of, 3; city supply of, 49; maximum density of, 140; wheels, 172; expansion of, on freezing, 200

Watt, 162, 189
Waves, condensational, 349; water, 350; longitudinal and transverse, 351; light, transverse, 413; electric, 467
Wave length, defined, 348; formula for, 349; of yellow light, 408; of other lights, 444
Wave theory of light, 404
Weighing, method of substitution, 8
Weights, 7
Wheel, and axle, 156; gear, 158; worm, 158; water, 172
White light, nature of, 445
Windlass, 160
Wimshurst electrical machine, 259
Wireless telegraphy, 468
Work, defined, 146; units of, 147; principle of, 155

X rays, 474

Yard, 1
Yerkes telescope, 439

Zeiss binocular, 441